

MEMORANDUM

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Speculative attacks in the exchange market with a band policy: A
sequential game analysis

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Speculative attacks in the exchange market with a band policy: A sequential game analysis

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Abstract:

We model the exchange rate market for a country that initially follows a band policy, as a four-stage sequential game of complete information, where a stochastic shock is realized in the last stage. Given a fixed cost of leaving the band, we show that three types of equilibria may exist, corresponding to different expectations by the public about the government retaining the band, (1) in all future states, (2) only in some future states, and (3) in none of the states. In case (2) a speculative attack occurs but is countered when the cost of countering it is not too high, leading to adverse shifts in fundamentals. We study the government's incentives to abandon or retain the band, and to abandon the band preemptively before public expectations are revealed. The standard Krugman and Obstfeld models are obtained as special cases, respectively when we have regime collapse as a unique equilibrium, and when there are multiple equilibria and the game always progresses to stage 4.

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1. Introduction

In recent years the world has witnessed a number of "currency crises". For some of these "crises" the outcome has been that countries which initially pursued either a fixed exchange rate peg or a currency band, were forced to leave the peg or band. A particularly conspicuous case in recent history was the speculation wave that occurred in the fall of 1992, when first Finland, then Italy, Great Britain, Sweden and Norway left their currency bands.¹ In 1993, Mexico also experienced a similar currency crisis, and other Latin American countries followed. More recently (since mid 1997) we have gone through a world wide financial market crisis which started from the collapse of several Southeast Asean currency regimes and more recently those of Russia, Brazil, and the rest of Latin America. On other occasions there have been market pressures against the currency of one or several countries, without this resulting in a "collapse" or basic change in exchange rate policy (such as the case of Japan). In still other cases the initial policy has been abandoned even without a preceding major speculative attack.

Already a number of models exist in the literature to explain the phenomenon of currency crises. These are traditionally grouped into "first-" and "second-generation" models². The "first generation" was initiated by Krugman (1979) and Flood and Garber (1984). In the more recent "second generation" group the pioneer has been Obstfeld (1986, 1994, 1996), with contributions also from, among others, Bensaïd and Jeanne (1993), Andersen (1994), Ozkan and Sutherland (1994) and Davies and Vines (1995).

¹ In addition, in the following summer the bandwidths of the ERM countries, except the one between Germany and the Netherlands were widened from $\pm 2.5\%$ to $\pm 15\%$, following a wave of further speculative attacks. See Buitier, Corsetti and Pesenti (1997) for an excellent description of these events in 1992 and 1993.

² See Flood and Marion (1998) for a survey of much of this literature, for explanations on the origin of the "first generation" models, and for discussions of differences between "first" and "second-generation" models.

An important element in these models, as well as an important empirical phenomenon associated with many recent exchange rate regime breakdowns, is the appearance of **speculative attacks** whereby the country's currency for some period is subject to market expectations of currency depreciation.³

In the first-generation models, monetary policies are exogenous (i.e., not explicitly derived in terms of welfare optimization) and a devaluation involves no reputation cost. Leaving the initial currency regime here emerges as the unique equilibrium in face of rational and forward-looking speculators who maximize profits and exploit all potential arbitrage possibilities in international exchange markets. Such arbitrage possibilities arise as a result of expectations that the exchange rate will be devalued, due to inconsistencies between the current exchange rate policy and the trend in domestic credit.

The second-generation models by contrast embody the possibility of multiple equilibria whereby one equilibrium with a stable exchange rate exists alongside with equilibria where the peg or band is abandoned. Such multiplicity of equilibria is usually explained through the interaction between the public's expectations about the exchange rate, and the government's incentives to change it: When the public expects a low (high) probability of realignment, the government is thereby given incentives to choose to realign in "few" ("many") states, thus making both high and low probabilities of realignment compatible with rational expectations equilibrium. An often crucial factor, as in Obstfeld's (1996) exposition, is a dynamic consistency problem of the familiar Barro and Gordon (1983) type, making a "first-best" solution unreachable for the government, in the absence of binding

³In addition to the early work by Krugman (1979), speculative attacks have been explicitly dealt with in the literature by e.g. Buitier (1987), Dellas and Stockman (1993). Another early related paper, dealing with speculative attacks in the market for gold with anticipated government auctions, is Salant and Henderson (1978). See also the surveys by Agénor, Bhandari and Flood (1992) and by Blackburn and Sola (1993).

commitments. There a speculative attack is either successfully defended against, in which case the initial exchange rate regime is left intact (and possibly the government's reputation for toughness in the market enhanced); or the government immediately or ultimately succumbs to the attack, resulting in a change in the exchange rate regime. These models provide foundations for devaluation expectations to become self-fulfilling, in the sense that these expectations are validated by the monetary authorities ex-post even in the absence of macroeconomic imbalances.

In this literature, however, there exists in our opinion to date no fully satisfactory analysis of the strategic behavior of the market participants during a speculative attack event at different stages, as parts of different rational expectations equilibria. To be concrete, consider the widely cited models of Obstfeld (1994, 1996). Here the private sector expectations are generally assumed to be formed "ex ante", while the government's decision to retain or leave the band is made "ex post", after the realization of some stochastic variable.

The main objective of this paper is to attempt to remedy this weakness of the literature, by presenting a four-stage sequential game model of a strategic game between the government and private speculators. This represents a new and more elaborate approach to the issue of exchange rate crises, and we claim that it may account for a wide range of phenomena which have been observed in the 90's. Both the "first" and the "second" generation models are embodied in our approach as particular cases, but the analysis goes beyond the existing literature in several respects. In our model, the monetary authorities has the possibility of deciding whether or not to pursue an initial exchange rate band policy at each stage of the game.⁴ If the band policy is retained, the private sector and the government thereafter follow a complete-information, sequential-move game. By structuring it as a

⁴ In this paper we do not deal with the optimal width of the band or the optimal currency area. On this issue see for example Mundaca and Strand (1998), Sutherland (1995), McKinnon (1963), Mundell (1961) among others.

sequential game, the government now may make strategic decisions about the exchange rate at various stages of the game, and at each stage all future moves are predicted rationally. This implies a richer and more interesting structure than that of the static Obstfeld models, and a richer set of results.

By considering more explicitly the implications of distinguishing between the different stages at which decisions can be made, we find that leaving the band can be equilibrium strategies at several stages of the game: either at the very start of the game, prior to the onset of a damaging speculative attack; at a later stage but prior to the resolution of uncertainty about a shock to unemployment to be realized at the last stage of the game; or at the end of the game, after uncertainty is resolved. As will be clear later, our model encompasses Obstfeld's model as special case, in particular when the government decides to stave off a speculative attack and where multiple equilibria are possible. In addition it encompasses the "first-generation" models of Krugman and others, since a possible outcome is the collapse of the initial exchange rate regime as the unique equilibrium. Here, this happens when the government leaves the band, before uncertainty is resolved, and this is the unique equilibrium strategy.

Note that Flood and Marion (1998) have also, in a recent paper, tried to incorporate the "first" and "second" generation models into one single framework. Their approach is however quite different from ours. They use a modified (macroeconomic) "first generation" model where they show that macroeconomic policies which are inconsistent with the existing exchange rate regime can push the economy toward an inevitable currency crisis, but also demonstrate how a government currently following consistent macroeconomic policies can suddenly face a speculative attack triggered by a large shift in speculative opinion. However, the ability of a sudden shift in speculative opinion to trigger an attack is bounded by the position of fundamentals. Thus, an attack does not require a later

change in policies to make it profitable.⁵ Neither do they model explicitly the strategic behavior of the government and private sector at the different stages of the speculative attack process, something that is an important issue here.

2. Outline of the model and its relationship with “first-” and “second-generation” models.

Our model deals with a game consisting of four distinct stages. At the start of the game, the government pursues an exchange rate band policy and all agents in the economy know that a shock to employment will occur in the (“near”) future (and revealed in stage 4 of the game), and which may take two values: it may be “bad” by giving a negative shift to employment, or “not so bad”, and in case give a positive or smaller negative shift to employment. These two types of shocks define the two possible states of the economy in stage 4. Throughout we assume a fixed cost C (credibility cost) of leaving the band at any stage of the game. Uncertainty about what will be the state of the economy at the last stage of the game (stage 4) is then modelled as a stochastic shock to employment. The exchange rate band enters in the simplest possible way, in that staying within the band implies the restriction that the exchange rate be **at or below** the band’s upper edge.

In **stage 1** the government is assumed to have a first-mover advantage, and is able to immediately leave the band (“preemptive quitting”) before the public’s expectations are formed, in **stage 2**. If a speculative attack is initiated in stage 2, this attack is assumed to entail some costs to the government which cannot be avoided once the attack is under way, but which could have been avoided by preemptive quitting in stage 1. The government may therefore wish to use its immediate

⁵ Buitier, Corsetti and Pesenti (1997) suggest that when studying currency crisis, one should also consider a multi-country policy game, different degrees of inflationary bias, a political structure that attach a credibility to a devaluation and, more crucially, the presence of international externalities and policy spillovers that tie together the policy functions in the system. These are a very important issues that can be taken into account in future research.

quit option, provided that the equilibrium in the following stages 2-4 implies speculation with certainty or with a sufficiently high prior probability, as viewed from stage 1. This feature of our model is novel; in particular, it is not captured by the second-generation models.

Only if the government chooses to retain the band policy in stage 1 will the game progress to stage 2, where the private sector alone moves by forming its expectations. The public can here have three possible types of (rational) expectations about the further course of the game, given that the game progresses up to its last stage (stage 4). These are the following: (i) Expectations that the band policy is to be retained in both possible states in stage 4, i.e., independent of the realization of a shock to the economy in that stage (which is either a “bad” or a “not-so-bad” shock). These expectations correspond to what we here call N expectations. (ii) Expectations that the band will be abandoned in stage 4 but only when a “bad” shock occurs, which correspond to S expectations. (iii) Expectations that the government will leave the band in both states in stage 4, corresponding to Q expectations. Under either of the last two types of expectations, a speculative attack will always be initiated by the private sector in stage 2. As will be seen below, it is possible that the government leaves the band already in stage 3, thus ending the game. Such an event is also rationally predicted by speculators.

In **stage 3**, the government decides on whether or not to immediately succumb to the speculative attack and leave the band, whenever a speculative attack has been initiated in stage 2. We here assume that if a speculative attack is under way and the band not immediately abandoned, this attack can in principle be staved off until stage 4. We find that when the public expects the band to be abandoned with certainty in stage 4 (corresponding to Q expectations), the government’s optimal strategy is to always abandon the band already (at the latest) in stage 3, since the government thereby avoids the costs of countering the speculative attack in stage 3. This outcome corresponds to

Krugman's collapsing regime solution. A decision to defend against an attack in stage 3 can be an equilibrium strategy only when the outcome in stage 4 is to abandon the band in the bad but not in the good state, corresponding to an equilibrium with S expectations. Defending against the attack implies costs in the interim period, both direct out-of-the-pocket costs (e.g. loss of international reserves) for the government, and to the economy in the form of adverse shifts to employment (which consequently occur not due to the realization of any shock but are a result of the government's policies to protect the band against a speculative attack in stage 3). Note also that there is a fixed cost associated with a speculative attack being initiated in stage 2, and which cannot be avoided by quitting the band in stage 3. Finally, when the band is (rationally) expected to be retained in both states in stage 4 (corresponding to N expectations), no speculation will occur and the government is passive in stage 3, awaiting the stage 4 outcome.

In **stage 4**, the stochastic shock is revealed and uncertainty is resolved. If the government has not already left the band before stage 4, it then has the option to optimally choose to retain or quit the band, depending on the realization of the shock. If the band policy is retained, it is assumed to be credible, and no further interventions are required in this stage or later.

In our analysis we assume symmetric information and that the game is not repeated, such that reputation effects play no explicit role (except for its possible implicit representation by the fixed cost C of realignment). In contrast to Flood and Garber (1984) and Flood and Marion (1998), we do not explicitly model the actual speculation by private sector agents. A basic new insight from our approach is that at least two basic conditions must be fulfilled for a rational speculative attack to be initiated, and the government to not immediately succumb to the attack. First, speculators in the market must have (rational) expectations that the initial exchange rate policy will be changed (for certain or with some probability) at some future date. Secondly, the government must initially have

incentives to withstand the attack. As a consequence there must exist some future states where the band will, eventually, be successfully defended, and the gains from defending until that time are sufficiently great to make a defence rational. We assume that all speculators correlate on the same equilibrium, but the model embodies no explanation of a possible shift in everyone's expectations from the no-speculation to the speculation expectations, and no particular mechanism for formal coordination between speculators.

We show that quitting the band can be a rational strategy for the government at all three of the stages in which such a decision can be made (stages 1, 3 and 4), and for quite different reasons in each case. In stage 4 it occurs as a direct result of the adverse shock together with a preceding speculative attack (one of Obstfeld's equilibria); in stage 3 because the government finds it too costly (in terms of international reserves loss and increased unemployment) to stave off an ongoing speculative attack before the shock is realized (Krugman's solution); and in stage 1, because the government decides to preempt a speculative attack before it is initiated.

A decision by the government to abandon the band, either before a speculative attack can start (at stage 2), or before the shock to the economy is realized (at stage 4), as later demonstrated, follows from the government's anticipation of a currency crisis due to a speculative attack. The point is that such an attack in itself would lead to both costs of maintaining the currency band regime, and to shifts in fundamentals which in turn may imply higher unemployment, which together are of such magnitude that leaving the band is the government's rational strategy. Given that the government allows the game to go on to stage 2, and thus lets the private sector to form expectations about the future shock, those imbalances imply that the exchange rate band will be abandoned already in stage 3 before the shock is realized. This corresponds to Krugman's (1979) "first generation" model. Such a model is covered in our theoretical approach, given that the collapse of the initial exchange rate

regime before uncertainty is resolved is the unique equilibrium outcome. In fact we also show that with a Krugman type of currency crisis, the rational strategy of the government is, whenever possible, to abandon the band before a speculative attack is initiated, so as to avoid the potential costs of the attack.

In cases where it is rational for the government to withstand a possible speculative attack until the shock to employment to be realized in stage 4, our model is rather of the “second generation” type, where multiple equilibria are possible. A further insight, relative e.g. to Obstfeld’s models, is that in the case of our Q equilibrium (where the band will be left in both states in stage 4), the game will (as in the “first-generation” models described above) never actually progress to stage 4, but will instead be left earlier, at the latest in stage 3.

In the final section, we sum up the conclusions and possible directions for future research.

3. The model

2.1 Stages of the game

To illustrate the possible actions of the two agents (the government and the public) in each stage of the game, we refer in the following discussion to the game tree described in figure 1, which gives an overview of possible strategies available to the two players, the government and the public. The public can decide to speculate or not to speculate, given that the band policy is not abandoned already in stage 1. The government’s possible strategies imply retaining or leaving the band, in each of the stages 1, 3 and 4, conditional on the band not already having been left, and (in stages 3-4) conditional on public expectations. The numbers in the figure correspond to all possible nodes in the game tree, some of which will not be reached because they are shown not to be Nash equilibria.

The game ends whenever the band policy is given up by the government, and may end in stages 1 or 3. Otherwise the game progresses to stage 4. In stage 4, e.g. (Q,R) corresponds to the strategy of quitting the band given shock 1, and retaining the band given shock of type 2.

Stage 1. Such a stage is relevant when the government receives probabilistic information about the future shock before the public forms its expectations (in stage 2). The government is then assumed to select one of two actions, namely either (a) to declare that it leaves the band policy and let the currency float, or (b) to declare commitment to the initial band. We may call action (a) **preemptive quitting**. We will later see that the government may use this option when it foresees an imminent speculative attack. If this strategy is chosen, the game ends in this stage, and the government suffers the utility loss C of giving up the band. Given a preemptive quitting, the government subsequently sets the exchange rate to minimize its loss function depending on the stochastic shock that is realized in stage 4. As we show, for any of the shocks the government selects the first-best (natural) level of unemployment.

Such a stage 1 may be relevant when the government actually has an informational advantage, e.g. when the information to be revealed represents the possible future policies of other countries' central banks or governments, and such information is made available to the country's government or central bank before it is revealed to the general public. We will below argue that the inclusion of such a stage may be important for understanding a number of real-world situations. In particular, the Norwegian devaluation in 1986 could possibly be interpreted in such a light.⁶

⁶It is true that there was relatively heavy speculation against the NOK up until May 1986, when the conservative government yielded to a social democratic one. It could however be argued that the fact that immediately after the new government's capture of power, the public was left indecisive with respect to what equilibrium on which to correlate, in particular since the new government had made clear promises not to devalue. This indecision among the public was used by the new government, by immediately devaluing and thus, in our vocabulary, preempting the public.

Stage 2. If the government retains the band policy in stage 1, the game continues to stage 2. Here private-sector agents form expectations about what exchange rate will be realized in each of the two possible states in the final stage 4, given that the game would progress to that stage. Possible actions of the public in stage 2 can be classified as “nonspeculation” or “speculation”. Nonspeculation is chosen when the public expects the band policy to be retained, in both possible states in stage 4. This corresponds to what we define as **Nonquitting expectations**, hereafter **N**. Speculation may involve two types of expectations among the public. The first of these we call **Speculation expectations**, hereafter **S**. In this event the public expects the band to be abandoned when the more unfavorable shock occurs in stage 4, but not when the less unfavorable shock occurs. The second we call **Quitting expectations**, hereafter **Q**. Then the public would expect the government to quit the band in both states, given that the band policy is retained until stage 4. Note that the public in stage 2 also rationally predicts when the government will leave the band already in stage 3, thus ending the game in that stage.

Since the game described is one of complete information and fully rational players, whenever speculators have formed their expectations in stage 2 they can always calculate the future course of the game, by use of backward induction. In particular, speculators will always know with certainty what the government’s moves will be in stage 3, and in stage 4 given that the game progresses to that stage. In particular, this implies that when the expected value of the exchange rate that is realized in the final stage 4 (given that the game would progress to that stage) exceeds the upper edge of the initial band, a **speculative attack** is initiated against the home currency by private speculators. We assume that such an attack is initiated even in cases where the government abandons the band already in stage 3 (and this is rationally foreseen by speculators). In both these cases there will be potential gains to be made by speculating against the currency, in the (perhaps very short) intermittent period between stages 2 and 3.

Stage 3. Here only the government moves. Its possible actions are to either defend or to give up the band against a possible speculative attack. In the case where the private sector takes a no-speculation action, with N expectations, the equilibrium strategy of the government is to always retain the band policy, and no active defense of the band is necessary. In the case where the public takes a speculation action, with private sector expectations of types S or Q, the expected exchange rate in stage 4 exceeds the upper edge of the band. In this case, the government may or may not decide to defend the band against a speculative attack throughout stage 3. We will assume that such a defense is possible, but that the government then will incur certain costs, in terms of losses of international reserves and shifts in unemployment, to be described in more detail below.⁷ Alternatively, the government may abandon the band immediately in stage 3. No costs are then incurred, other than the cost C of quitting the band. We will below see that under Q expectations, when the speculators in stage 2 expect the band to be abandoned in stage 4 in both states, the government always immediately leaves the band in stage 3. Under S expectations, a choice exists for the government, to leave and stay in the band in this stage 3.

Note that the option of leaving the band in stage 3 is not part of the second-generation models. We will argue that this option is important, and that many real-world devaluations can be put in this category. In particular, Brazil's devaluation in January 1999 (which occurred shortly after the onset of a speculative attack) might be put in this category.

Stages 1-3 may be separated in real time, but could as well in principle describe just a sequence of events taking place within negligible time.

⁷Note that whenever the stage 4 expected exchange rate exceeds the upper band level, it would generally not be possible to defend the band throughout stage 3 with the use of interventions alone when investors are risk neutral. The domestic interest rate would then have to increase in order to fulfill the uncovered interest parity condition, in order to make investors indifferent between speculating and not. We will here not go deeply into such issues nor is this necessary for the model to be logically coherent, only assume that both interventions and interest rate increases are generally called for in face of speculation.

Stage 4. In this stage, a stochastic shock, either s_1 or s_2 , is revealed. This stage must occur at a strictly later time than stage 3.⁸The game then ends immediately. In each of the two possible states, the government either retains the band policy (and keeps it forever after), or leaves the band incurring cost C .The appendix provides a complete analytical solution to stage 4 outcomes, under each of the three types of expectations. It is important to notice that once stage 4 is reached, there will be no speculation, nor any need for further government interventions. Section 3 below presents the main results from this analysis.

Stage 4 is the third stage at which the band can be left; this is in fact the only actual leaving option covered by the Obstfeld model. We argue that this stage clearly also is important; e.g., the Mexican devaluation in 1994 appears to come under this category.

2.2 *The optimization problem*

Denote the exchange rate (the number of domestic currency unit per unit of foreign currency) of the economy in question by x , the fixed strong (lower) edge of the initial exchange rate band by x^L , and the weak (upper) edge by x^U , all in logs. We here have a symmetric currency band with respect to its central parity. In the following we will assume that the band policy is retained as long as the government keeps $x \leq x^U$.⁹ The issue of allowing the exchange rate to move below x^L then does not enter into the formal analysis.¹⁰ If the band is retained until stage 3, the government then has the

⁸In particular, when the government uses a high interest rate as part of its set of instruments to hold off the speculative attack, the time period up until the shock occurs cannot be infinitely short.

⁹We do not consider the case of realignment, i.e. changing the band's edges or central parities. More specifically, once the exchange rate moves outside the initial band ($x > x^U$), it is assumed that the currency band policy is abandoned.

¹⁰Thus, we are only considering cases where there is pressure for depreciation in the domestic currency. In practice, this implies that the exchange rate levels relevant for our analysis lie in the weaker part of the band, i.e. above the central parity. The central parity itself, however, plays no role in the following discussion.

following loss function: ¹¹

$$(1) \quad V(u, Ex) = a_1 (u_3 - u_n)^2 + a_2 E[(u_4 - u_n)^2] + b_1 [Ex - x^U]^2 + b_0,$$

where u_3 and u_4 represent the rate of unemployment in stages 3 and 4 respectively. (1) implies that the government's cost associated with countering a speculative attack in stage 3 consists of the following variable items: the costs associated with possible excess unemployment created in stage 3, which is a simple quadratic cost function related to the realized unemployment rate in excess of the natural rate u_n in that stage (the term starting with a_1); similar expected costs of unemployment in stage 4 (the term starting with a_2); and the costs of countering the speculative attack in stage 3, which are assumed to be a quadratic function of the future (stage 4) expected exchange rate as a deviation from the weak edge of the band, x^U (the term starting with b_1). Notice then that only after the public has formed expectations (at stage 2) and has decided to speculate (i.e. if $Ex > x^U$) having expectations either S or Q, the constants b_1 and b_0 will take strictly positive values; otherwise these coefficients take the value of zero. $b_0 > 0$ represents an assumption that immediately upon the onset of a speculative attack (once $Ex > x^U$), the government is committed to use a fixed amount of international reserves to defend the currency band, represented by b_0 . Thus, if the government decides to leave the band immediately in stage 3, b_0 will be the only cost in this stage. If the attack on the other hand is countered, there will be costs in addition to b_0 , of defending the band throughout stage 3. $Ex - x^U$ is taken to represent the magnitude of the speculative attack in stage 3. More interventions, and other possible policy measures (such as interest rate increases), are generally necessary to optimally defend the band during stage 3. The costs to the government of these

¹¹This is a "minimalist" formulation of such a loss function; in particular, exchange rate movements per se are not valued negatively, as they are in some formulations (e.g. Obstfeld (1996)). We here have the flexibility of allowing the exchange rate to move inside the band. The movements of the exchange rate outside the band, and the position of the exchange rate with respect to the upper bound of the band, are also important.

measures will depend on the size of the speculation, and are comprised of two components. First there is a cost of increased unemployment in stage 3, represented by the term starting with a_1 . Secondly, there is a direct economic cost (e.g. the cost of lost reserves) represented by the term starting with b_1 . (1) then embodies an assumption that defending the band against larger attacks becomes disproportionately more costly, as it will e.g. require greater use of foreign exchange reserves beyond b_0 by the central bank. It is most natural to view the terms containing b_1 and b_0 (representing the direct costs of the speculative attack) to be incurred in stage 3.

Let $u(s_i)$ denote the actual level of unemployment in state i in stage 4. $u(s_i)$ is represented by the following relationship:

$$(2a) \quad u(s_i) \equiv u(x(s_i)) = -h[x(s_i) - x^U] + u_n + u_0 + s_i; \quad i=1,2.$$

u_0 is defined just below and h is a positive constant and indicates how the unemployment rate will be affected by the exchange rate $x(s_i)$ in stage 4. Note that only in stage 4 is $s_i \neq 0$, and unemployment affected by the realization of the shock. This means that in stage 3, unemployment will be:

$$(2b) \quad u_3 = u_n + u_0,$$

In (2a), unemployment is related to the distance of the exchange rate (x) from the weakest edge of its band. Notice first that if the exchange rate moves inside its band ($x < x^U$) the first term is positive, and it is negative when x is let to fall outside its band ($x > x^U$). The idea is that a more depreciated domestic currency (higher x) will stimulate economic activity by increasing employment ex post (after the shock is realized in stage 4). We assume that in stage 1, and at the start of stage 3 (and the

band was not left in stage 1), the unemployment rate equals u_n , such that the exchange rate is set at $x = x^U$.¹²

We assume that when $Ex > x^U$ (Ex being the private sector's rationally expected exchange rate in stage 4) and the government has decided to defend the band until stage 4, in the interim period between stages 3 and 4 unemployment is also affected directly by the (linear) shift u_0 in the unemployment rate, as follows:

$$(3) \quad u_0 = g[Ex - x^U],$$

where g is a positive constant. u_0 is here also affected by the size of the speculation. What we have in mind is that a shift in unemployment may be caused by the government's optimal response to the speculative attack (the size of $Ex - x^U$). To be concrete, we think that every time there is a speculative attack, the government defends the exchange rate regime against such an attack by increasing interest rates (in proportion to $Ex - x^U$) in the interim period (between stages 3 and 4), before the uncertainty is finally resolved. Such increases in interest rates may among other things put firms out of business and thereby shift employment downward. It is important to notice that u_0 takes the value of zero when the government leaves the band, either in stage 1 (before public expectations are formed), or at the very start of stage 3.

Thus, $Ex - x^U > 0$ will create costs to the government in three separate ways: in terms of loss of international reserves (through parameters b_0 and b_1), and in terms of increased unemployment, in the period between stage 3 and stage 4 (through parameter g).

¹² This implies that before there is anticipation of a future shock to employment, the government could choose its first-best level of employment by going to the weakest edge of the initial band. In these cases a first best is thus compatible with retaining the band.

In stage 3, unemployment equals u_n before a speculative attack starts. The attack moves this rate to $u_3 = u_0 + u_n$ in that stage, before the shock is realized and before the actual exchange rate for stage 4 set. This implies that $u_3 - u_n = u_0$, implying that the first term on the right-hand side of (1) can be replaced by $a_1 g^2 [Ex - x^U]^2$. Thus (1) can be simplified to

$$(1a) \quad V(u, Ex) = a_2 [\alpha(u(s_1) - u_n)^2 + (1-\alpha)(u(s_2) - u_n)^2] + b_2 [Ex - x^U]^2 + b,$$

where $b_2 = b_1 + a_1 g^2$, α and $1-\alpha$ are the probabilities of shocks 1 and 2, and where we have substituted for possible realizations of u_i in the term starting with a_2 . Remark that the term starting with b_2 incorporates two separate effects, namely the costs of actual interventions, and the costs associated with excess unemployment, both in stage 3. For given a_1 and b_1 , interventions will represent the overwhelming share of these costs when g (representing the degree to which fundamentals shift) is small, while the opposite may be the case when g is large.

The government may decide to leave the band already in stage 3 once a speculative attack has started. It then suffers a loss equal to $C + b_0$, where C is the direct cost of leaving the band. C is exogenous in the model, not directly justified by the analysis below, and represents loss of credibility for the government.¹³ We remark that C is assumed to be independent of the time at which the band is left, and of other circumstances.¹⁴

¹³ See e.g. De Kock and Grilli (1993) for an analysis where the cost of leaving the band is derived explicitly, in a repeated game formulation.

¹⁴ More generally C could be a variable dependent on several factors, e.g. on whether the economy has already sustained a speculative attack or not, and the magnitude of it. A speculative attack could in principle inflict damage on reputation even when the band policy is successfully defended, and it could in addition reduce the gains associated with a future fixed exchange rate policy. This might in case imply that C is smaller given S expectations in stage 4, than under N expectations. An analysis of this type is presented by Drazen and Masson (1995), who argue that realignments can be less costly in terms of reputation loss, when the state of the economy is particularly bad.

3. Stage 4 equilibria

3.1 Introduction

Given that the band is retained up to stage 4, all possible outcomes in this stage must be accounted for. The appendix contains the full mathematical solution to the model in stage 4. In this section we will present the main results of that analysis, in the form of propositions 1-5. The formal proofs of these propositions are described in the appendix.

Out of the 8 possible strategy combinations of the government in stage 4 (see figure 1), we will find that only three constitute Nash equilibrium strategies in the sequential game. These equilibria, which we in the following call N, S and Q equilibria respectively, correspond to the three types of expectations the public may hold, namely N, S and Q expectations. Under N expectations, the band is defended in both states at stage 4. Under S expectations, the band is defended in state 2, but not in state 1. Under Q expectations, the band is left in both states. The purpose of this section is to derive the conditions under which each of the equilibria exists, and the properties of each of them given existence. We will also study conditions under which two or all three of the equilibria may exist simultaneously (i.e. multiple equilibria).

3.2 Existence and properties of N equilibria

N equilibria exist when the public expects the band to be retained in both states in stage 4, and the government at the same time has incentives to maintain the band in both states. In the appendix we readily show the following result:

Proposition 1: An N equilibrium exists in stage 4 if and only if :

$$(4) \quad a_2[s_1]^2 \leq C.$$

(4) constitutes the condition under which the government wishes to retain the band in the worst possible state in stage 4, given that there was no speculative attack in stage 3. With no speculative attack there are not costs due to speculation nor shifts in unemployment in stage 3, and the shock simply changes the rate of unemployment to a level equal to $u_n + s_1$ in stage 4 when state 1 occurs. When state 2 occurs, unemployment is u_n whenever $s_2 \leq 0$, and $u_n + s_2$ whenever $s_2 > 0$. Thus whenever s_2 is negative, the government will here set the exchange rate inside its band in that state, i.e., $x(s_2) < x^U$. Whenever s_1 is positive, however (as is always the case in state 1), the government will set the exchange rate at the weakest edge of the band, and some inefficient unemployment will result. This is disliked by the government and could be avoided by leaving the band and letting the exchange rate float freely, incurring a cost C . In our model a free float is assumed to always lead to an optimal unemployment rate of u_n . The band is retained in state 1 whenever the cost of sustaining $u_n + s_1$ is smaller than the cost of leaving the band, leading to (4). When (4) holds and the band is retained in state 1, the band will always be retained in state 2 since $s_2 < s_1$.¹⁵

For the government not to have incentives to abandon the band, (4) needs to hold, which means that the dislike of having some inefficient unemployment has to be always smaller than the cost C of leaving the band. In terms of figure 1, this solution implies that the action pair (R,R) at stage 4 is the unique government's optimal response to the privat sector's nonspeculation either when shock of type one or two occurs. This result corresponds to node 12 of the game tree.

¹⁵ Note that in this stage the shock (either s_1 or s_2) becomes known with certainty. One can then say that its probability of occurrence equals one.

3.3 Existence and properties of Q equilibria

A Q equilibrium is characterized by public expectations that the band will be left in both states in stage 4, provided that the game has progressed until this stage. We will now study the possible stage 4 outcomes in this case. To find the conditions under which this type of equilibria may apply, we may simply concentrate on the government's incentive to leave the band in the more favorable state 2.

From the appendix we derive the following result:

Proposition 2: A Q equilibrium exists if and only if the following condition holds:

$$(5) \quad a_2 \left[\frac{\alpha g s_1 + (h - \alpha g) s_2}{h - g} \right]^2 > C.$$

In this case, the expression inside the main bracket in (5) is the excess unemployment level in state 2, given that the government in this state would retain (i.e., set the exchange rate at the upper edge of) the band. Condition (5) then states that the costs of retaining the band in state 2, on the left-hand side, exceed the costs of leaving the band, on the right-hand side. Obviously, given that the government leaves the band in state 2, the band will be left also in the less favorable state 1.

3.4 Existence and properties of S equilibria

In an S equilibrium, the band will be left in one state but not in the other in stage 4, again given that the game has progressed until this stage. Now speculation is triggered in stage 2 whenever the exchange rate in expectation, that will result in stage 4, exceeds the upper edge of the exchange rate band, that is $Ex > x^U$. The condition for this to happen is $\alpha s_1 + (1 - \alpha) s_2 > 0$, which implies that the

expected value of the forthcoming shock is adverse, and which here is assumed to hold.

Unemployment will partly be affected by u_0 (as indicated by equation (3)), as part of the costs to the government of staving the speculative attack off (when it increases interest rates), but also by the size of the shock and the relative position of the exchange rate set by the government with respect to the upper edge of its band. This type of equilibrium then exists when the band is defended in the worst state (state 2) in stage 4, and not in the other state.

Thus, in state 1 the band is left and the exchange rate is set above x^U (outside its band) to offset the effect of the size of the speculative attack on unemployment, represented by u_0 . Given this, the rate of unemployment, u_1 , can be set at u_n , minimizing (1).¹⁶ Thus, if state 1 occurs, the optimization of employment is the only concern for the government.

For state 2, we have two possible types of solutions under an S equilibrium, namely (a) x_2 is set strictly inside the band, and (b) x_2 is set at the upper edge of the band. We deal with these two cases in turn. Again, all formal derivations are found in the appendix.

(a) $x(s_2) < x^U$. $x(s_2)$ is then kept strictly inside the band. In this case we have the following result:

Proposition 3: An S equilibrium exists with $x(s_2)$ strictly inside the band, $x(s_1)$ outside the band and the government setting $u_i = u_n$ ($i=1,2$), i.e. at the first-best level, if and only if the following conditions are fulfilled:

$$(6) \quad -(h-\alpha g)s_2 > \alpha g s_1$$

¹⁶ More precisely, for an equal change in the actual ex-post change in the exchange rate (which is determined by the government) to the ex-ante change in the exchange rate (expected by the speculators), the effect on unemployment of the former has to be larger than the effect of the former on the latter.

$$(7) \quad a_2 \left(\frac{[h-(1-\alpha)g]s_1 + (1-\alpha)g s_2}{h-g} \right)^2 > C.$$

Note that since $s_1 > 0$, in this case $s_2 < 0$ implying that the type 2 shock must be "favorable". (6) expresses the condition that s_2 be sufficiently favorable (and unemployment is reduced directly), that the government chooses to stay inside the band when a type 2 shock occurs. (7) expresses a condition that s_1 is sufficiently unfavorable (unemployment is increased directly), that the government will want to quit the band in stage 4 given a shock of type 1, after having sustained and countered a speculation attack in stage 3 compatible with this equilibrium. The expression inside the large bracket in (7) is the unemployment rate (in excess of u_n) that would result, if the government in stage 4 instead decides that the exchange rate stays inside (or at the edge of) the band when a type 1 shock occurs. By (1), the entire term on the left-hand side of (7) is then the stage 4 cost associated with such unemployment, as viewed by the government. For the decision of leaving the band when shock 1 occurs, that cost must be greater than the cost of leaving the band, C . If not, the government would retain the band policy in stage 4, given that a type 1 shock occurs, and no S equilibrium could exist in this case.

(b) $x(s_2) = x^U$. Now the government chooses a solution for the exchange rate in stage 4, at the edge of the exchange rate band given a (more favorable) type 2 shock. We then find the following:

Proposition 4: An S equilibrium exists with $x(s_2)$ at the edge of the band, $u_2 > u_n$ and $u_1 = u_n$, if and

only if (5) holds with opposite inequality, and the following two additional conditions hold:

$$(8) \quad a_2 \left(\frac{h}{h - \alpha g} s_1 \right)^2 > C.$$

$$(9) \quad a_2 \left(\frac{\alpha g}{h - \alpha g} s_1 + s_2 \right)^2 < C.$$

In this case (6) obviously cannot hold, since if it did, we would be back to Proposition 3 above. (8) is here the equivalent to (7). The term inside the large bracket is again the rate of unemployment (in excess of “natural unemployment”) that now would result from staying at the edge of the band in state 1. If the costs of sustaining the exchange rate regime are sufficiently large, leaving the band will become an optimal strategy in this case. (9) is a new condition, which did not appear under Proposition 3. We must now also namely worry about the incentives to leave the band in state 2, when the shock is less unfavorable than the state 1 shock. The term inside the large bracket is again the excess unemployment rate, and the left-hand side of (9) the total cost associated with this unemployment as viewed by the government. For the band not to be abandoned such that $x(s_2)$ equals x^U , the latter cost must be less than the cost of leaving the band, C .

Note that in figure 1, an S equilibrium corresponds to the government action pair (Q, R) in stage 4, following speculation by the private sector in stage 2 and government nonquitting in stage 3. Given that stage 4 is reached, the game consequently ends up in node 15 in figure 1.

Note also differences between our unique equilibria, and those of Obstfeld. While Obstfeld’s model embodies the possibilities of unique N and Q equilibria with our notation, it does not appear to

embody the possibility of a unique S equilibrium; this always in case coexists with (at least) the Q equilibrium.

3.5 Coexistence of equilibria

The conditions derived above, for existence of each of the three types of equilibria (N, S and Q) gives a basis for investigating the possibility that two or all three of these equilibria may exist simultaneously, for any given parameter set, and for deriving the parameter sets possibly permitting such coexistence. We may derive the following general result:

Proposition 5: In the model described in sections 3.1-3.4, there exist parameter values permitting coexistence of a) an N equilibrium with an S equilibrium; b) an N equilibrium with a Q equilibrium; c) an S equilibrium of type (b) with a Q equilibrium; and d) an N equilibrium, an S equilibrium of type (b), and a Q equilibrium simultaneously.

This result is easily verified by inspecting the conditions under which each of the three types of equilibria exist. The key to understanding the possibility of coexistence of equilibria is the function (3), whereby the unemployment rate is shifted by the public's expectations of a future devaluation. The parameter g in equation (3) expresses the strength of this effect. When $g=0$, equilibrium is always unique. When g increases, the unemployment rate is moved more by the public's expectation of a devaluation, and a devaluation, in either one or both states, becomes self-fulfilling in more cases when no-devaluation equilibria also exist. Note also that, for a given g , whenever shocks are sufficiently small, only N equilibria exist, and whenever they are sufficiently great, only Q equilibria exist. In a middle range, and in particular when s_1 is "much more adverse" than s_2 , S equilibria exist.

In terms of figure 1, a Q equilibrium corresponds to the action pair (Q, Q) in stage 4 by the government, corresponding to node 16 in the game tree. As we will see in section 4 below, however, the rational strategy for the government is then to quit the band (at the latest) in stage 3. This node will consequently not be reached, as part of a Nash equilibrium of the entire game.

4. Stage 3 equilibria

The solutions for stage 4 above were derived under the assumption that the government had not already left the band at an earlier stage. We will now study optimal strategies for the government in stage 3 of the game, taking as given the equilibrium strategies in stage 4 as described in section 3 above. In stage 3, the government has the option to either leave or retain the band policy. When the band is left in stage 3, the game ends in this stage. When the band is not left in stage 3, the game goes on to stage 4, with outcomes as described in section 3 above. The following result is then immediate.

Proposition 6: Assume that the government has not abandoned the band in stage 1, and the public holds N expectations in stage 2. Then the government will not abandon the band policy in stage 3.

Proof: By construction, staying within the band in stage 4 is an optimal response to N expectations. Thus, the government cannot gain by leaving the band in stage 3. No speculative attack occurs, since $E x \leq x^U$, and there are thus no costs related to speculation. The government then needs to only consider the stage 4 payoffs. Q.E.D.

Proposition 7: Assume that the government has not abandoned the band in stage 1, and the public holds (rational) S expectations, $x(s_2) < x^U$ (we have case (a) in subsection 3.4 above), and $u_i = u_n$

($i=1,2$). Then the government will then counter a speculative attack in stage 3, if and only if (6)-(7) hold, together with the following condition:

$$(10a) \quad b_2 \left(\frac{\alpha s_1 + (1-\alpha)s_2}{h-g} \right)^2 \leq (1-\alpha)C.$$

To realize this result, note that the left-hand side of (10a) expresses the total costs (in terms of lost foreign reserves and increased unemployment) of withstanding a speculative attack in stage 3. The right-hand side of (10a) expresses the ex ante expected opportunity cost of leaving the band in stage 3 before the shock is realized (stage 4). One could also say that $(1-\alpha)C$ is the expected cost saving for the government by not leaving the band in stage 4, which will be the case if s_2 occurs.

Proposition 8: Assume that the government has not abandoned the band in stage 1, the public holds rational S expectations, and $x(s_2) = x^U$ (we have case (b) in subsection 3.4 above). Then the government will counter a speculative attack in stage 3 if and only if (8)-(9) hold, together with the following condition:

$$(10b) \quad b_2 \left(\frac{\alpha}{h-\alpha g} s_1 \right)^2 \leq (1-\alpha) \left[C - a \left(\frac{\alpha g}{h-\alpha g} s_1 + s_2 \right)^2 \right].$$

The left-hand and right-hand sides of (10b) here have similar interpretations as those of (10a), except that in deriving the expected opportunity cost of leaving the band at stage 2, one must now subtract the cost associated with inefficient unemployment at the weakest edge of the band in state 2 in stage 4. This cost is given by the last term inside of the square bracket on the right-hand side.

(10a)-(10b) express the conditions under which the government will counter a speculative attack,

and not quit the band in stage 3, given that the public rationally holds S expectations, something that also requires that (6)-(7), respectively (8)-(9), hold in cases (a) and (b). This implies that an S equilibrium in such a case will materialize, and the game will end up in node 15 in the game tree of figure 1.

Proposition 9: Assume that the government has not abandoned the band in stage 1, and that the public either holds rational Q expectations; or rational S expectations and conditions (10a)-(10b) do not hold in each of the respective cases (a) and (b). Then the government will abandon the band in stage 3.

Proof: Consider first the option of countering a speculative attack under Q expectations. Once an attack has materialized, the cost of countering the attack is given by the left-hand side of (10a) in stage 3 (which is positive). Define this cost by $C(1)$. In addition the government will incur a cost of C in stage 4 (since the band will anyway be left with certainty). Consider next the option of not countering a possible speculative attack. The cost of adopting this option at the beginning of stage 3 must equal C plus b_0 , which is the very initial costs of intervening in the exchange rate market once the market has decided to speculate, and no additional cost in stage 4. If $C(1) + C > C + b_0$, the attack will not ever be countered. When (10a)-(10b) do not hold under S expectations the result is immediate, from Proposition 7. Q.E.D.

This proposition implies the rather obvious result that a Q equilibrium can never actually materialize in stage 4, since the possibly ensuing speculative attack will never be countered. Moreover, an S equilibrium in stage 4 cannot materialize when the conditions (10a)-(10b) do not hold in cases (a) and (b). In both cases the government will instead immediately leave the band in stage 3, and we end up in node 7. None of the end nodes (13) - (16) of the game tree of figure 1 is then reached.

5. Stage 1 equilibria

Stage 1 of the game is relevant when the government is able to decide whether to leave or retain the band in stage 1, before private-agent expectations are formed in stage 2. We will here consider two different approaches to this issue. The first, and perhaps the better founded theoretically, is what we call a Bayesian approach, where the government in the event of multiple equilibria at stage 2, assigns subjective probabilities to each of these at stage 1. The second is what one may perhaps call a forward induction approach, where the government by its choice of action in stage 1 may try to influence the formation of expectations at stage 2, in multiple equilibrium cases. We study each of these in turn.

5.1 Bayesian equilibria at stage 1

Here the government assigns subjective probabilities β_N , β_S and β_Q (summing to one), to the events that the public sector holds respectively N, S and Q expectations in stage 2. We interpret these probabilities such that in a case where one equilibrium is excluded, the respective prior probability is zero. Consequently also, in unique equilibrium cases, the prior probability that the respective (unique) expectations are held in stage 2 is unity. With multiple equilibria, the prior probabilities attached to the each of the possible equilibria may take any value between zero and one.¹⁷ Define $A(N) = \alpha a(s_1)^2$,

¹⁷Our discussion here departs from an assumption that in the case of multiple stage 2 equilibria, the government cannot know for certain what expectations will actually result in stage 2, and skips the perhaps crucial issue of exactly how expectations among the public are formed at that stage. This is a problematic subject and one on which traditional game theory has little to offer. A possible story is the “informational cascades” explanation, inherent in the analyses of Banerjee (1992) and Bikhchandani, Hirshleifer and Welch (1992), where individuals (here, speculators) imitate each others’ behavior. The story could go something like the following: Assume that an N equilibrium exists together with one or both of the two other potential equilibria (S and Q), and that in the initial situation (in stage 1) speculators (rationally) hold N expectations. Assume then that some group of speculators, for some unknown reason, take positions against the home currency, and that this is observed by other speculators. This would in turn serve as a signal to other speculators that the initial equilibrium no longer is relevant, and that the relevant equilibrium instead is the speculation equilibrium. The government might have no way of foreseeing for certain whether such a shift would occur, and would simply have to assign subjective probabilities to the event.

$s_2 \leq 0$; $A(N) = \alpha a(s_1)^2 + (1-\alpha)a(s_2)^2$, $s_2 > 0$, where $A(S)$ is the expected cost of retaining the band in the event that the public rationally holds N expectations. Define also the right-hand and the left-hand sides of (10a) and (10b), by $A(i)$ and $B(i)$ respectively, $i=a,b$. Then we have the following result.

Proposition 10: Assume that either an S equilibrium or a Q equilibrium exists alone or both coexist, possibly together with an N equilibrium, and the government holds prior probabilities β_N, β_S and β_Q associated with the tree types of equilibria. Then the government will quit preemptively in stage 1 if and only if

$$(11a) \quad \beta_N[C-A(N)] + \beta_S[A(i)-B(i)-b_0] - \beta_Q b_0 < 0$$

when (10a) or (10b) holds,

$$(11b) \quad \beta_N[C-A(N)] - (\beta_S + \beta_Q)b_0 < 0$$

when (10a)-(10b) do not hold.

The proof of this proposition is straightforward. Here (11a) is relevant in cases where the government will choose not to leave the band in stage 3, and (11b) when the government chooses to leave the band in stage 3, under S expectations in stage 2. In both cases the expression to the left of the inequality sign is the total net expected gain of retaining the band relative to quitting it in stage 1, in terms of the subjective probabilities given to the different public expectations in stage 2. E.g., the expression in the first square bracket in (11a) is the gain from retaining the band in stage 1, given that the subsequent equilibrium is of type N , while the last square bracket in (11a) is the equivalent

expression in the event of an S equilibrium.

Among the implications of Proposition 10 are the following:

- When the government has a "very high" subjective probability that the band will be left in stage 3, it will always be advantageous to realign preemptively, in order to avoid the fixed cost associated with a speculative attack. This will be the case always when either a Q equilibrium or an S equilibrium when (10a)-(10b) do not hold, but no N equilibrium, exist alone or together, or when an N equilibrium also exists but is assigned a sufficiently low prior probability.

- When an S equilibrium exists alone (i.e., $\beta_S = 1$), and a speculative attack will be countered in stage 3 (i.e., (10a) or (10b) holds), and $b_0 > 0$, there exist cases where the condition

$$(12) \quad A(i)+B(i) < C < A(i)+B(i)+b_0$$

holds. Here the last inequality is relevant for the stage 1 decision, and implies that the government will strictly prefer to quit the band preemptively in stage 1, to prevent the speculative attack from occurring and making the government incur the fixed cost b_0 associated with its occurrence. The first inequality by contrast indicates the stage 3 decision, given that the game is let to progress to this stage, and expresses that the band then will not be left. The option of preemptive quitting then implies that the band will be left in more cases than otherwise.

- Coexistence of an S or a Q equilibrium, or both, together with an N equilibrium, and where the government attaches positive prior probabilities to each, i.e. β_N , β_S and/or β_Q are positive. Since the first square bracket in each of (11a)-(11b) is always positive, preemptive quitting in stage 1 will not

be chosen when β_N is sufficiently close to one. The game will then go on to stages 2-4, and it is possible that an S equilibrium may occur, and condition (12) is fulfilled, or that a Q equilibrium occurs. This would imply total overall costs that are greater than those resulting from preemptive quitting. In such cases the government could perhaps be said to have been "overly optimistic" in stage 1, by attaching a low prior probabilities to the "unfavorable" S and Q equilibria. When on the other hand β_S is close to one and (12) holds, preemptive quitting will always be chosen. This will make it impossible to realize a (favorable) N equilibrium, which would have been a possible outcome of stages 2-4 of the game. Since the game then ends with stage 1, there will of course be no way of observing which equilibrium might have resulted in the subsequent stages.

5.2 A "forward induction" approach

Instead of the Bayesian approach taken above, a possible alternative approach is to view the government as having some influence on the formation of expectations among speculators in stage 2, by its action in stage 1, to retain the band instead of using the quitting option.¹⁸ Such a line of argument may be relevant only in cases with multiple equilibria, and where an N equilibrium is one of these. To make the point, assume that there are two equilibria, and the other is a Q equilibrium. Then clearly, the government would gain by leaving the band in stage 1, if it knew that a Q equilibrium would result, and it would gain by staying in if it knew that an N equilibrium would result. When the private sector also knows this, and in addition knows that the government has a first-mover advantage, a failure to quit the band in stage 1 may be taken to indicate a high "belief" on behalf of the government, that the N equilibrium will result. This may serve as a signal to speculators, to correlate or focus on this equilibrium.

¹⁸Admittedly, the argument in this section is speculative and tentative. A major formal objection is that the government's "signal" to choose non-realignment in stage 1, gives no credible signal related to its type or future actions. There are thus in the model no compelling reasons for the private sector not to ignore this signal.

Given that the government in such a way is assumed able to steer the public's expectations formation, the outcomes of the game may be different from that described in section 5.1 above. Potentially, when the government is completely successful at such a steering of expectations, the outcome may be the one described in section 5.1, where the probability β_N associated with an N equilibrium, equals one in some multiple equilibrium cases, in particular when an N equilibrium coexists with either a Q equilibrium or an S equilibrium leading to abandoning the band in stage 3. The implication of this is that the government will choose to retain the band policy in more cases in stage 1.

6. Conclusions and final comments

Our model describes a four-stage, sequential-game, model of the foreign exchange market for a country that, at the outset, follows a band policy for its exchange rate, and faces an uncertain future shock to employment. The government can react to this future shock by leaving the band immediately in stage 1 or in stage 3 respectively (before or after public expectations are revealed), or it can stick to the band policy until the state is revealed in stage 4. In the latter stage, the government can either retain or abandon the band in either or both states. When the public rationally foresees that the band will be abandoned (in one or both of the states), a speculative attack is triggered prior to the revelation of the uncertain variable, as part of a rational expectations equilibrium. We assume that it is possible for the government to counter the attack, and thus keep the exchange rate within the band in the period until the shock is observed, but that this is costly both directly and since employment shifts adversely. We show that whenever the game moves beyond stage 1, there are three possible types of outcomes of this game. First, it may progress to stage 4 and the band policy then retained in both states, in which case there is no speculation. Secondly, a speculative attack may be triggered, and countered by the government, in stage 3, and the government leaves the band in the

worse but not in the better of the two states in stage 4. Thirdly, the government may leave the band immediately in stage 3 (before the shock is realized), thus preventing prolonged speculation. As a general rule, the first type of equilibrium occurs when the shock in either state is small; the second type when the shock in the bad state is “relatively serious” and significantly worse than the less adverse shock, and the costs of countering a speculative attack are not too large; and the third type when both shocks are adverse and relatively large, or when the bad shock is large and it is at the same time expensive to counter speculation. We also find that there is considerable overlap of the parameter spaces under which each of the equilibria may exist. In particular, parametric cases can be found where all three outcomes are simultaneously possible. In general, coexistence of equilibria is more likely, the more unemployment is shifted by expectations of future exchange rate changes.

The government may however also choose to leave the band policy already in stage 1. This may be advantageous since it will avoid costs associated with an anticipated speculative attack in stage 3, had the game progressed to that stage.

An important conclusion from our analysis is that for a prolonged speculative attack to materialize, two basic conditions must be fulfilled simultaneously. First, the public must rationally expect the government to stick to the initial (band) policy until the state is known, and subsequently leave the policy but only in the bad state. Secondly, the government must have incentives to withstand the attack, once it has been initiated. We show that the latter condition never holds when the government’s costs of countering a speculative attack are sufficiently high. The government then instead implements a free float, immediately after the attack has started, in order to minimize the costs associated with the attack.

We find that the solution to the game may differ according to whether the government or the public

moves first upon learning the probability distribution of the future stochastic shock. The game starts in stage 1 when the government moves first, and in stage 2 when the public moves first. When moving first, there are cases where the government will leave the band immediately, while the band policy would be retained when the public instead moves first. This occurs for one thing when the public holds S expectations in stage 2 and (12) is fulfilled. It also occurs in some cases with multiple equilibria, where an N equilibrium is one of these, but where the government's prior probability (in stage 1) that an N equilibrium will occur in stage 2 is small. In such cases the government may wish to immediately leave the band in order to avoid the fixed cost of a speculative attack, should it be triggered in stage 2. But this precludes the possibility of a favorable N equilibrium, which might have been the result if the public had moved first, and which would have made the government better off *ex post*. In such cases the government will suffer an *ex post* loss by moving first, although it gains in *ex ante* expected terms.

To sum up, the government can leave the band in three different stages in our model, and for very different reasons. Working backwards, the government will as part of a rational expectations equilibrium with a prolonged speculative attack (an S equilibrium), leave the band in the worst state in stage 4, but not in the less adverse state. In stage 3 the government will always leave the band when the cost of countering a speculative attack is sufficiently great; and also always when the government foresees that it will always leave the band in stage 4 (in a Q equilibrium). Finally, in stage 1 the government will leave the band in a number of cases in order to avoid the costs associated with the initiation of a speculative attack, as discussed in the paragraph above. Note that the overall outcome when a bad state actually occurs in stage 4 will always be less favorable when the attack is countered than when the band is abandoned in stage 3. The reason is that in addition to the fixed costs of quitting the band, the government in the former case also suffers the costs of withstanding the attack, which are not suffered in the latter (when the band is abandoned). The

reason why the band is defended, is that in such circumstances the prior gains from successfully having defended the band, for the event that the good state would occur in stage 4, more than outweighs this loss.

Our model is based on a number of simplifying assumptions. There is no assumed dynamic inconsistency problem, as the government is at any time assumed free to choose its most preferred level of employment unencumbered by private-sector expectations. As a consequence, multiplicity of equilibria in our model does not require an assumption of such inconsistency, and is thus in contradiction to the view of some recent authors, notably Obstfeld (1994, 1996). In our model multiplicity follows only because a speculative attack which is countered, itself causes certain real variables (“fundamentals”) to move, which in turn make the government more likely than else to quit the band.¹⁹ This property of the model makes equilibria with (actual or threatened) speculative attacks self-fulfilling, often alongside equilibria with no speculative attacks.

Formally, the band only enters with its upper edge and by possibilities to move below this upper edge, and still stay within the band. The formal difference from a model of a single currency peg is thus minimal, and appears only in the N equilibrium when $u_2 < 0$, and in the S equilibrium when (10) holds, and in both cases only in state 2. While this may be a reasonable starting point for an analysis of “currency crises” (in particular since basically all such “crises” for countries with band policies have involved pressures to the respective bands' upper edges), a lot is left unstudied by such an analysis. E.g., our assumption that moving close to the upper edge of the band (while still inside of

¹⁹In fact the possibility of multiple equilibria in our model does not even require uncertainty about the stage 4 state. To see this, assume that u_1 occurs with probability one. Then (5) is still the condition that an R equilibrium exists, while the condition that a Q equilibrium exists is now given by $a\{[h/(h-g)]s_1\}^2 > C$. These two conditions are simultaneously fulfilled over a strictly positive range for u_1 whenever $g > 0$. Here an equilibrium with actual speculation of course does not exist, i.e., a node in the corresponding game tree at which speculation occurs can never be reached. In this case the role played by the construction of a solution with speculation in stage 4 becomes particularly clear: it serves as a deterrent against the onset of a speculative attack that would have evolved, had the government not immediately quit the band.

the band) will not affect market expectations about future exchange rate movements (or realignment possibilities) is unrealistic (Mundaca (1999)).

Our model is as noted a "minimalist" one in terms of simplicity, provided that we wish to accommodate the joint possibilities of currency speculation and multiple equilibria. Linearity of the relationships (2) and (4), and a two-point distribution of the uncertain stage 4 variable, preclude more than one possible equilibrium with actual speculation, but in other respects do not fundamentally alter the main qualitative analysis above. The model is likely to change more fundamentally if we instead drop the assumptions of complete information and a single repetition of the game. Indeed, a repeated game formulation is likely to be necessary in order to formally justify the existence of the cost C of leaving the band. A perhaps crucial assumption behind our model is that of complete information, and common knowledge among speculators which equilibrium is played in the case of multiple equilibria. In a recent related paper, Morris and Shin (1998a) consider a refinement of the basic (static) Obstfeld model where they assume that speculators are unsure about other speculators' beliefs about fundamentals. They demonstrate that a small such uncertainty may lead to a unique equilibrium, i.e., one with either speculation or no speculation. It remains to be seen in subsequent work whether a similar result can arise here. Other kinds of informational asymmetries should be considered in future extensions. Relevant extensions may be to cases where the private sector is unsure about the government's cost of leaving the band, or about the effect of a speculative attack on the unemployment rate. With asymmetric information one may also visualize equilibria with actual speculation even when there is no uncertainty about the future state, only about the government's type; and equilibria where the government credibly signals its type by choosing a solution strictly inside of the band even when this is not efficient in the complete information case. Such extensions may be fruitful topics for future work.

The issue of exactly when a speculative attack is initiated in the market is also important for understanding the nature and effects of such attacks. In our model the attack is initiated upon the arrival of some new information about the future state of the fundamentals of the economy. In practice information may be changing more gradually. Morris and Shin (1998b) study such a case, and derives some results about the timing of the onset of a speculative attack as a function of the deterioration of fundamentals (which follow Brownian motion processes over time), given a certain degree of dispersion of information among potential speculators. The effects of such a gradual deterioration of fundamentals could be studied in the context of a model more similar to ours.

A number of other potentially important issues have been overlooked in our framework. Buiter, Corsetti and Pesenti (1997) argue that “theories based on country-specific imbalances (including “self-fulfilling” shifts in expectations) have a hard time in rationalizing the simultaneous and unanticipated eruption of financial tensions in virtually all countries participating in the ERM”.

They argue that multi-country issues are very important, perhaps with different degrees of aversion to unemployment and inflation across countries, different degrees of established credibility against devaluation for the different currencies, and stress the presence of international externalities and policy spillovers across countries.

We will argue that, while limited in some respects, the richness of our model still gives wide room for interpreting recent currency crisis phenomena. In particular, we feel that there are many plausible cases where the government may have a first-mover advantage, and where actual policies correspond to the government using such an advantage.²⁰ The government may then choose to leave the initial exchange rate policy even without any prior observation of shifts in fundamentals nor any prior

²⁰Note however that the game with the government moving first is formally identical to a game where the government and the public move simultaneously in stages 1-2, provided that fixed costs are always avoided when the band is immediately abandoned.

manifestation that the private sector expectations are coordinated on a speculative regime. A possible example in recent monetary history from Norway could be the major band realignment in the spring of 1986, as already explained above.

Another possible application of the model is to the Norwegian band regime collapse in the fall of 1992, and perhaps also to other collapses that occurred during the same period. To put the Norwegian case in the context of the model, speculators could here be taken to move first, by staging speculative attacks against both the Norwegian and the Swedish currencies in the summer and early fall of 1992. These attacks were initially withstood by both governments. This raised interest rates and currency reserve outflows and generally worsened the economic situation in both countries, in the fall of 1992. Thus “fundamentals” moved, more or less directly as a result of the speculative attack being countered by the governments. For the Norwegian government, countering a speculative attack could make sense if the government later (in some but not all states) would turn out to be able to defend the band. For Norway, a possible future Swedish collapse was reasonably viewed as an exogenous event, taking place with probability between zero and one, and correspond to the bad shock in our model. Thus when Sweden left the band in the fall of 1992, Norway soon followed. The presumption is then that if Sweden had not left the band policy, Norway would neither. This implies that, using our game tree in figure 1, in this particular game the public held S expectations, and that the game ended up in node 15, with a bad (type 1) shock and the government leaving the band.²¹

²¹ Note that, in hindsight, there was nothing to indicate that Norway was in need of a realignment, when the speculative attack started in the summer of 1992. We thus do not agree with Bordo and Schwartz's (1996) claim that all such collapses, historically, are initiated by adverse movements in fundamentals. True enough, however as noted, fundamentals probably moved adversely as a result of the speculative attack, thus making a collapse more likely later in the fall of 1992.

Appendix: A characterization of stage 4 equilibrium solutions

We consider the possible outcomes (a)-(f) in turn, at stage 4 of the game.

(A.1) The public has decided not to initiate a speculative attack: They have N expectations and s_1 is realized. In this case the public does not expect the government to leave the band in neither of the two states realized in stage 4. This implies that $Ex \leq x^U$. Thus, no speculation occurs, no costs associated with a possible speculative attack are incurred ($b_0 = b_1 = 0$) and $u_0 = 0$. We then have $u(x(s_1)) = -h(x(s_1) - x^U) + u_n + s_1$, from (2a). Here an unconstrained maximization of utility V by the government would imply that the exchange rate x_1 is set to keep the unemployment rate at the first-best level u_n , implying $x(s_1) = x^U + s_1/h > x_n$. Since $s_1 > 0$, however, setting the exchange rate at this level would imply leaving the band, which would yield a utility loss for the government of C . Retaining the band policy, by contrast, implies setting $x \leq x^U$. Given this constraint, the government prefers the solution at the upper edge of the band, i.e., $x(s_1) = x^U$, yielding a loss of $a(s_1)^2$. From (1a), note that this solution is a Pareto inferior to the other solution but for an equilibrium with N expectations to exist, the government must optimally choose the latter solution, $x(s_1) = x^U$ with inefficient unemployment $u_1 = u_n + s_1$. This implies the following constraint on the parameters:

$$(4) \quad a_2(s_1)^2 < C.$$

(A.2) The public has decided not to initiate a speculative attack: They have N expectations, and s_2 is realized. Now $u(x(s_2)) = -h(x(s_2) - x^U) + u_n + s_2$. Assume first $s_2 \leq 0$. Then $x(s_2) = x^U + s_2/h < x^U$, which is inside the band and will thus be chosen by the government.²² Here $u(x(s_2)) = u_n$, and the government is at its first-best solution with costs equal to zero. Assume next $0 < s_2 < s_1$. Then the solution is as under point (a) above, i.e. $x(s_2) = x^U$ is chosen. The loss for the government is now $a(s_2)^2$, which is smaller than in state 1. Consequently, the required incentive constraint on the government not to realign must hold, provided that (4) holds.

(B) The public has decided to initiate a speculative attack: They have S expectations. Under S expectations, for the government to behave rationally, it will leave the band when s_1 occurs, but not when s_2 occurs. We should here distinguish between two subcases, namely (B.1) $x_2 < x^U$, and (B.2) $x_2 = x^U$.

(B.1) $x(s_2) < x^U$. Note in neither of the two states, the upper level x^U of the band constrains the government in setting the exchange rate *ex post*. It is then optimal for the government, in both states, to choose the first-best level of u , i.e., $u_i = u_n$. This optimal decision together with (2a) give us the following two equations:

$$(13) \quad u_n = -h(x(s_i) - x^U) + u_n + u_0 + s_i, \quad i = 1, 2.$$

Using (3) and (13) and the fact that $Ex = \alpha x(s_1) + (1-\alpha)x(s_2) > x^U$, we obtain the following two-

²²We are here assuming that $x(s_2)$ is not so low as to be below the lower limit of the band.

equation system in the two variables $x(s_1)$ and $x(s_2)$:

$$(14) \quad h(x(s_i) - x^U) = g[\alpha x(s_1) + (1-\alpha)x(s_2) - x^U] + s_i, \quad i = 1, 2,$$

with the following solutions when $u_i = u_n$:

$$(15) \quad x(s_1) = x^U + \frac{h-(1-\alpha)g}{h(h-g)}s_1 + \frac{(1-\alpha)g}{h(h-g)}s_2$$

$$(16) \quad x(s_2) = x^U + \frac{\alpha g}{h(h-g)}s_1 + \frac{h-\alpha g}{h(h-g)}s_2.$$

A basic stability condition is $h > g$, i.e., ex post movements in unemployment due to changes in the exchange rate determined by the government are larger than the ex ante movements in unemployment caused by for equal changes in the exchange rate but expected by the speculators.

For $x(s_2) < x^U$ to hold, from (16) we find that the following conditions should be satisfied:

$$(6) \quad -(h-\alpha g)s_2 > \alpha g s_1.$$

While $x(s_1) > x^U$ in (15) implies the following condition:

$$(17) \quad [(h - (1 - \alpha)g]s_1 > -(1-\alpha)g s_2.$$

When $s_1 > 0$, given that (6) holds, s_2 must be negative. We here conclude that for shocks that reduce unemployment, the government must set the exchange rate away from the weak edge of the band, that is $x(s_2) < x^U$. The better the shock is, the further inside the band the exchange rate should be set. Note also that since the system solves uniquely for $x(s_1)$ and $x(s_2)$, the equilibrium given S expectations is unique.

For an S equilibrium to materialize, the government must choose to leave the band given s_1 in stage 4. For this to hold, we need (10) and the following condition. Note first that if the government were not to realign when a shock of type s_1 occurs, it would choose x at the upper edge of the band, with a corresponding unemployment rate $u(x(s_1)) = u_0 + s_1 + u_n$. Considering this, the condition under which the band will be left in state 1 is $V(u(x(s_1))) > C$. From (1) and (13)-(15), this can be expressed as:

$$(7) \quad a_2 \left(\frac{[(h-(1-\alpha)g)s_1 + (1-\alpha)g s_2]}{h-g} \right)^2 > C.$$

(B.2) $x(s_2) = x^U$. From (14), we obtain:

$$(18) \quad h(x(s_1) - x^U) = \alpha g(x(s_1) - x^U) + s_1,$$

from which we find the solution for $x(s_1)$:

$$(19) \quad x(s_1) = x^U + s_1/(h-\alpha g).$$

Now the government faces two potentially binding incentive constraints, namely $V(u_0+s_1) > C$ (implying a realignment in state 1), and $V(u_0+s_2) < C$ (implying no realignment in state 2), from which as $u_i(x(s_i)) - u_n = u_0 + s_i$ should hold when the band is retained in both states. These conditions can be expressed respectively as:

$$(7) \quad a \left(\frac{h}{h-\alpha g} \right)^2 (s_1)^2 > C.$$

$$(8) \quad \left(\frac{\alpha g}{h-\alpha g} s_1 + s_2 \right)^2 < C.$$

(C)The public has Q expectations. Under Q expectations, the public expects the government to abandon the currency band policy in both states in stage 4. Given that it leaves the band, the government chooses its first-best level of u , namely u_n , in both states. The general expressions for $x(s_1)$ and $x(s_2)$ are then still given by (15)-(16), i.e., the same general expressions as under S expectations. The only difference from that case is that while the solution for $x(s_2)$ then fell inside of the band, now this solution level of $x(s_2)$ falls outside the band.

In deriving the conditions for Q expectations to be confirmed at equilibrium, note that we only need

to consider the government's incentive constraint for state 2.²³ This constraint is expressed as $V(u_0+s_2) > C$. Using (16), this can be written as

$$(9) \quad a_2 \left(\frac{[\alpha g s_1 + (h - \alpha g) s_2]}{h - g} \right)^2 > C.$$

Discussion of possible coexistence of equilibria

i) Consider first the possible coexistence of N and S equilibria. The only condition for an N equilibrium is (4), which is a condition for s_1 only (i.e., it is independent of s_2). Whenever $g > 0$, we easily see that (4) may hold, and at the same time both (7) and (8) be violated, for relevant values of s_2 (remember our basic assumption that $\alpha s_1 + (1 - \alpha) s_2 > 0$).

ii) Next, consider N and Q equilibria. In the first case only (4) must hold, and in the second case only (9). Again we easily see that parameters can be found for coexistence whenever $g > 0$.

iii) Consider coexistence of S and Q equilibria. In case (A), no parameters for such coexistence can be found. The reason for this is that in these two cases, the optimal exchange rates $x(s_1)$ and $x(s_2)$ are derived from the same pair of equations (15)-(16) in the appendix), which yield unique solutions for the x_i . This implies that different parameter values are required to generate different solutions, and that both S and Q solutions cannot be derived under the same parameters. In case (B), the relevant conditions to compare are (8) and (9), which together imply that the band will be retained with a type 2 shock given an S equilibrium but not given a Q equilibrium. We easily see that such parameters can be found; this may e.g. be exemplified by setting $s_2 = 0$.

iv) Finally, consider the possibility of coexistence of all three types of equilibria, for cases with pairwise coexistence. Again we easily find that this is possible. Using the previous example of $s_2 = 0$, (4), (8) and (9) will hold simultaneously e.g. when g is close to h , α is "not too small" and s_1 has a value that obeys (4).

²³The reason is of course that whenever a realignment now is chosen in state 2, it is also automatically chosen in state 1, since the gains from realigning are always greater in state 1, while the cost is the same in both states.

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