Option values in sequential markets

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Abstract

We consider competitive behaviour in sequential markets when current success or failure may affect the profitability of future market opportunities. The analysis is conducted in a set up which may be interpreted as two private-value, sealed-bid, second-price sequential auctions. We demonstrate that whether agents price higher or lower than in the corresponding static context depends on the relative magnitudes of the 'winner's option value' and the 'loser's option value' of participating in the later market. Pricing strategies may be non-monotonic in the number of participants, and behaviour may be most aggressive for intermediate degrees of competition.

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1. Introduction

In some markets, opportunities are best thought of as arriving in sequence. Examples are found in the construction and manufacturing sectors, where individual jobs or orders are often large relative to a firm's total output. Other examples include oil and gas drilling rights and government contracts for the supply of public services. In such markets, agents not only have to consider the attractiveness of the current market opportunity - they will have to take into account also how present success or failure affects the attractiveness of future opportunities. For example, a firm which is contemplating bidding for a particular contract may find that, due to scale economies or learning or reputation effects, winning the contract will improve its competitive position in the future. On the other hand, if firms have limited output capacities, then undertaking one project may mean having to let go of another. Consequently, allowing someone else to win the contract will reduce competition in the immediate future.

Our purpose with this paper is to systematically explore competitive behaviour in such sequential markets. In particular, we would like to be able to distinguish between circumstances in which the existence of future market opportunities leads to more, respectively less, aggressive behaviour than when such opportunities are not taken into account. We suggest that this may be done by considering the relative importance of what we term, respectively, the 'winner's option value' and the 'loser's option value' of participating in later markets. Loosely speaking, the winner's option value equals the gain a market participant expects to obtain from later opportunities conditional on him competing successfully for the current opportunity. Similarly, the loser's option value is the expected gain from future participation, given that the agent was currently unsuccessful. Relative to behaviour in the corresponding static context, an agent competes more aggressively if and only if the winner's option value exceeds the loser's option value.

To illustrate this idea, consider the following set up. There are two market opportunities, the current and the future. Each time the market is open a group of participants compete for a single object by simultaneously making price offers. To highlight the effects of inter-temporal linkages we assume that participants are symmetric ex ante and that their valuations of the objects are independently distributed. (By 'valuation' we mean the willingness to pay for an object were the purchasing opportunity considered in isolation. In the reverse case, in which producers compete to sell, the corresponding entity would be costs of production.) Valuations of the first item are drawn before the opening of the first market, and, since we allow for the possibility that some time may elapse between market opportunities, valuations of the second item are drawn after the closing of the first market. The distribution of a participant's second-object valuation may depend on whether or not he obtained an item at the first opportunity.

Our interest focuses on behaviour at the initial market opportunity. Consider first the case in which the winner's option value is less than the loser's option value; assume, for example, that participants, due to diseconomies of scale or capacity constraints, want at most one unit. Then the winner's option value is nil, and, as long as there is any probability of obtaining a gain at the next opportunity, the loser's option value is positive. It is easily demonstrated that equilibrium strategies call for the players to make price offers below their valuations (allowing a high-valuation competitor to win the early round reduces competition for the item which will be available later). Consider next the case in which the winner's option value exceeds the loser's option value; for example, if the two objects are identical, but there are synergies, the winner's option value is positive. However, the loser's option value is negligible when there are sufficiently many participants, since, apart from the winner (who may have an even higher valuation than in the first round), an identical set of players meet again in the second round. One can show that equilibrium strategies involve bidding sufficiently above valuation (participants compete aggressively for the first item in the hope of obtaining another item later on for which they have a higher, expected valuation).

Based on the general notions of the winner's and the loser's option values, we explore competitive behaviour in sequential markets by considering how the option values depend on technologies and market structure. In particular, and as illustrated by the above examples, we investigate alternative assumptions about the number of participants and the inter-temporal correlation of valuations across market opportunities. In doing so we synthesize results in the received literature, as well as derive new insights.

Our framework may be interpreted as two private-value, sealed-bid, second-price sequential auctions. Although intentionally simple, the framework is general enough to encompass a number of earlier models of sequential auctions. With the exception of Donald B Hausch (1988), who considers a model in which valuations are independent of earlier success or failure (so that the winner's and the loser's option values are the same), the literature on sequential auctions has tended to make assumptions that eliminate the winner's option value. For example, Robert J Weber (1983) describes the classic model in which each participant buys at most one unit (in our framework, this corresponds to the case in which first-market losers retain their valuations while the first-market winner values the second item at nil). In such a model participants shade their price offers in early rounds due to the option value of participating in later rounds (in the last round participants always submit price offers equal to their valuation). On the other hand, ceteris paribus the selling price declines as the number of remaining participants goes down. Since items are assumed to be identical in the eyes of the participants, at equilibrium the two countervailing effects cancel out and so, in expected terms, all gains to waiting are arbitraged away and the sale price is the same across markets.

Dan Bernhardt and David Scoones (1994) and Richard Engelbrecht-Wiggans (1994) replace the assumption that items are identical with the assumption that items are stochastically equivalent. Otherwise, the models are similar to Weber's; in particular, participant valuations are identically and independently distributed and each participant wants one item only.[1] Under this alternative assumption expected prices are not necessarily constant, and may be declining. For example, since second-item valuations are independent of first-item valuations, the expected value of participating in the second auction is equal for all first-auction losers. Consequently, in the first auction high-valuation participants shade less, and low-valuations participants shade more, compared to in Weber's model (in that model, due to the fact that all losers retain their first-item valuation, the option value of participating in the second round is higher for high-valuation participants than for low-valuation participants). Bernhardt and Scoones also consider cases when the distributions of valuations differ between the two auctions and point out that the degree of shading in the first auction depends on the dispersion of the distribution of second-auction valuations. In particular, if second-item valuations were identical for all players the expected gain of participating in the second auction would be zero, and consequently each participant would submit a first-auction price offer equal to his valuation. Accordingly, the seller's expected revenue is greater if the item with the highest dispersion in valuations is offered first. As we demonstrate below, this conclusion depends crucially on the assumption that the first winner's valuation of the second item drops to zero with certainty.

Introducing the possibility that participants may want more than one unit has ambiguous effects on equilibrium bidding behaviour. The Weber, Bernhardt-Scoones and Engelbrecht-Wiggans type incentive to shade bids in the first round decreases and may vanish altogether when also the first winner has an option value of participating in the later round. If the option value is higher when a participant wins the first item than when he loses, the participant will price above his valuation in the first round. If this is true for sufficiently many participants (and, in particular, for high-valuation participants), prices may decline even when (in probabilistic terms) second-item valuations are higher than first-item valuations. On the other hand, the fact that also the first winner participates in the second round enhances competition and consequently dampens (and may even reverse) the tendency to a price decline.

Jane Black and David de Meza (1992) consider a model in which participants may want more than one unit. There are two auctions and in the second auction

[1] In Engelbrecht-Wiggans' model, there is independence of valuations across objects but not across individuals.

all first-auction losers retain their valuations, while the first-auction winner's valuation of the second item is drawn from a distribution that may be conditioned on his first-item valuation. The distribution of first-item valuations stochastically dominates that of second-item valuations and, more importantly, second-item valuations never exceed first-item valuations. Therefore, the winner's option value is always zero in their model also. Consequently, participants price below their valuations in the first auction while the presence of the winner increases competition in the second auction. As a result, the selling price increases in expected terms.

In the next section we present our model and characterize equilibrium bidding behaviour. In section 3 we state the result that, when second-item valuations do not depend on who won the first item, participants price at their valuation in the first market also. In the particular case in which either first-item and second-item valuations are identical, or they are independently and identically distributed, (expected) selling prices are identical across the two markets.

In section 4 and 5 we consider two special cases of degenerate second-item valuations. In section 4, participant valuations of a first item are independent of whether this item is obtained at the first or the second opportunity (this corresponds to Weber's assumption that items are identical). How much participants value a second unit may, however, differ from their first-unit valuations. In this case participants price above their valuations when there are sufficiently many present. In section 5 we consider instead the case of no (positive or negative) synergies, in which the first-item and second-item valuations are identical. However, we allow for the possibility that first-item valuations may depend on whether the item is obtained early or later on.

Section 6 is devoted to the case when all second-item valuations are stochastic. We investigate how strategies in the first market depends on the first-order and second-order moments of the respective distribution functions. An interesting result is that pricing strategies may be non-monotonic in the number of participants. In particular, when the distribution of a participant's second-item valuation is more dispersed when he won the first item than when he did not, pricing is often most aggressive for intermediate numbers of participants (in which case participants bid above their valuations), while when competition becomes very fierce, participants bid less aggressively. Consequently, the expected selling price in the first market price may exceed that in the second market for intermediate levels of competition, while price dispersion is reduced as competition increases even further.

2. The model

For each participant i, $i = 1, 2, \ldots, n$, we define X_i to be the 'valuation' for the item offered for sale in the first market; that is, X_i would be participant i's maximum willingness to pay in the corresponding static or single-market context. Participant i's valuation for the second item may depend on whether or not he was successful in the first market, i.e. whether participant i is after his second or first item. We denote by X_i^W participant i's valuation of the second item conditional on i being the winner in the first market and, correspondingly, X_i^L is i's valuation of the second item conditional on i being among the losers in the first market.

The X_i's are independently and identically distributed according to the distribution function $F(x) = \Pr\{X_i \leq x\}$, with density $f(x)$ on the support $[0; \overline{X}]$. Similarly, the X_i^s's are independently distributed according to distribution functions $F_x^s(x^s) = \Pr\{X_i^s \leq x^s \mid X_i = x\}$, $s = L; W$. We make no specific a priori assumptions about $F_x^L(\cdot)$ and $F_x^W(\cdot)$; indeed, our analysis will be concerned mainly with exploring alternative assumptions about the distribution of second-market valuations.

Each of the two markets has the same move structure: First valuations are drawn, then buyers simultaneously set prices and, lastly, the item is allocated to the participant with the highest submitted price offer who pays a price equal to the second-highest price offer in the market. The assumption that participants do not necessarily know their own valuations of the second object until after the first market is closed is the same as in e.g. Black and de Meza (1992), Berhardt and Scoones (1994) and Engelbrecht-Wiggans (1994). Ian L Gale and Hausch (1992) make the alternative assumption that all uncertainty is resolved before the first market opens. Since we are concerned here with markets in which some time may elapse between the occurrence of each market opportunity, the Gale-Hausch assumption seems less appropriate.

In the second market, pricing at valuation is a (weakly) dominant strategy (Vickrey, 1961).[2] Let $Y_i = \max_{j \neq i}\{X_j\}$ be the highest competing bid to i's in the first market and, correspondingly, Y_i^L and Y_i^W are the highest competing bids to i's in the second round when i is, respectively, loser and winner in the first market. $G(y) = [F(y)]^{n-1}$ is the distribution function of Y_i while $E_{x;y}$ is the expectations operator conditioned on the event $\{X = x; Y = y\}$. Then, dropping subscripts, we may write a participant's payoff when he has valuation x and sets price b in the first market and all other participants submit price offers according to their

[2] Note that, since agents have dominant strategies in the second auction, the possible release of information concerning the outcome of the first auction prior to the start of the second auction is not an issue.

(symmetric) equilibrium strategies $b_j = \beta(x_j);\ j \neq i$, as[3]

$$\pi(b;x) = \int_0^{G^{-1}(b)} \left\{ x - \beta(y) + E_{x;y}\max\left\{X^W - Y^W; 0\right\}\right\} dG(y) + \int_{G^{-1}(b)}^{\overline{X}} E_{x;y}\max\left\{X^L - Y^L; 0\right\} dG(y) \qquad (2.1)$$

The first element on the right hand side of (2.1) represents the expected payoff in the event that i wins the first item, and equals the expected gain obtained in the first market plus the expected gain obtained in the second market, conditional on i winning in the first. The second element is the expected payoff in the event that i looses in the first market, and equals the expected gain obtained in the second market conditional on i not obtaining an item in the first.

At the symmetric equilibrium, the first-order condition for the maximum of (2.1) with respect to b may be written

$$\beta(x) - x = OV_x^W - OV_x^L, \qquad (2.2)$$

where we have used that, due to symmetry, $b = \beta(x)$ and consequently $G^{-1}(b) = x$, and $OV_x^s \equiv E_{x;x}\max\{X^s - Y^s; 0\}$, $s = L; W$. The left-hand side of (2.2) measures the discrepancy between an agent's 'valuation' and his willingness to pay. An agent prices above his valuation of the first item if, loosely speaking, the expected gain in the second auction is greater in the case when he wins the first auction than when he does not. In the former case he will be competing in the second auction to obtain his second unit; in the latter he is after his first unit in the second auction also. Conversely, an agent prices below his valuation if the gain from obtaining a second unit is less than the gain from obtaining the first unit in the second auction. We will call the first of these expected gains 'the winner's option value' and the second 'the loser's option value'.

It should be noted that both the winner's and the loser's option values are evaluated at the event that the highest-valuation competitor has the same valuation, and is furthermore following the same strategy. In this event, a participant in effect has it in his power to determine the outcome in the first market, and

[3] There are no participation costs. As demonstrated in von der Fehr (1994), such costs may have profound effects on equilibrium strategies in sequential auctions. In the present set up, in which valuations are not neccessarily perfectly correlated across auctions, such effects are likely to be smaller. In future work we will extend the present model to explore the importance of participation costs.

wins the first item if he marginally raises his price offer. The likelihood that this event occurs is of no significance. Consequently, the incentive to let price offers deviate from valuations may survive even in environments in which a participant has no market power. Below we shall see that this is indeed the case.

Let $H_s(z)$ be the distribution function of the second-order statistic for valuations in market s, $s = 1; 2$; that is, $H_s(z)$ is the probability that the second-highest valuation in market s is no higher than z. The expected equilibrium selling prices in each of the two markets are then given by

$$Ep_1 = \int_0^{\bar{x}} \beta(x) dH_1(x); \tag{2.3}$$

$$Ep_2 = \int_0^{\bar{x}} x dH_2(x): \tag{2.4}$$

Clearly, a difference in selling prices across markets may be due either to differences in behaviour or to differences in the distributions of valuations. In particular, we may decompose the difference between expected selling prices in the following manner:

$$Ep_1 - Ep_2 = \int_0^{\bar{x}} [\beta(x) - x] dH_1(x) + \int_0^{\bar{x}} [H_2(x) - H_1(x)] dx, \tag{2.5}$$

where we have applied integration by parts to arrive at the expression for the second element on the right-hand side of (2.5). In expected terms, the first-market selling price may exceed the second-market price either if price offers for the first item are consistently above valuations of that item or if first-market valuations exceed second-market valuations. As we shall see below, either of these circumstances may occur alone; in particular, even when, in expected terms, the second-highest valuations are the same, participants may price above their valuations in the first market; and, conversely, participants may price at their valuations in the first market even if the distribution of valuations differ between the two markets.

3. When the future does not matter

We start our analysis by considering circumstances in which participants employ the same strategies as they would in a static context, so that the future does not affect current behaviour. This occurs when the winner's and the loser's option values are the same, that is, if the distribution of second-auction valuations does not depend on which participant won the first item, as in Hausch (1988). In particular, we have:

Proposition 1. If $\forall x', x'' : F^L_{x'}(x'') = F^W_{x'}(x'')$; then $\beta(x) = x$, $\forall x$.

The assumption that the F^L and F^W distributions are identical includes the case in which $X^L = X^W = X$, as well as the case of X, X^L and X^W being independently and identically distributed. In both cases, the expected selling prices are the same in the two auctions, i.e. $Ep_1 = Ep_2$. Note the difference between this result and those obtained by Weber (1983), Bernhardt and Scoones (1994) and Engelbrecht-Wiggans (1994). Weber showed that, in expected terms, selling prices are the same when objects are identical. Bernhardt and Scoones and Engelbrecht-Wiggans proved that this is no longer true when valuations are not perfectly correlated across objects; in particular, selling prices differ even if objects are 'stochastically equivalent', in the sense that first-auction and second-auction valuations are independently and identically distributed. The reason is that buyers want at most one unit. Consequently, there is no winner's option value and participants discount their price offers below valuations. The degree of discounting depends on the correlation between first-auction and second-auction valuations and it is only in the case that correlation is perfect that all gains to waiting are arbitraged away.

Bernhardt and Scoones also show that in their model selling prices typically depend on the order of auctions. This is not so in the case considered here; in particular, bidding behaviour is independent of the correlation between X and X^s, $s = L; W$, as long as X^L and X^W are identically distributed. On the other hand, expected selling prices generally differ between markets. In particular, under the assumption in Proposition 1 (2.5) reduces to

$$Ep_1 - Ep_2 = \int_0^{\bar{X}} [H_2(x) - H_1(x)]\, dx; \qquad (3.1)$$

implying that the difference in selling prices depends on the second-order statistics of the two distributions of valuations only. The second-order statistics differ either if valuations are systematically higher for one item than for the other or if valuations are differently dispersed. We briefly consider each of these possibilities in turn for the specific case that participant valuations are distributed independently between markets, i.e. so that $F^L_x(x') = F^W_x(x') = F^{LW}(x')$, $\forall x; x'$.

If valuations of the first item are systematically higher than valuations of the second, then obviously the selling price in the first market is in expected terms higher than in the second market. In particular, if first-item valuations (first-order) stochastically dominate second-item valuations, i.e. $F(x) < F^{LW}(x)$, all x, we have $H_1(x) = F(x)^n + n[1 - F(x)]F(x)^{n-1} < F^{LW}(x)^n + n[1 - F^{LW}(x)]F^{LW}(x)^{n-1} = H_2(x)$, and consequently $Ep_1 - Ep_2 = \int [H_2(x) - H_1(x)]\,dx > 0$.

As the number of participants increases, there is an increasing probability that the selling price will be determined by a participant with a high valuation. For sufficiently many participants, therefore, in expected terms the first-market selling price exceeds the second-market selling price whenever the distribution of first-item valuations is more dispersed than the distribution of second-item valuations, and vice versa.

4. Identical items

We consider next the case in which the items offered in the two markets are identical, so that a participant's valuation of his first unit is independent of in which market the item is obtained. That is, conditional on i missing the first opportunity $X^L = X$, or $F_x^L(x^L) = 0$ when $x^L < x$ and $F_x^L(x^L) = 1$ when $x^L \geq x$. This corresponds to the Weber (1983) assumption, insofar as losers retain their valuations. Unlike Weber however, we allow for the possibility that the first-market winner may value a second unit.

Let $G_y^L(y^L) = \Pr\{Y^L \leq y^L \mid X < Y = y\}$ and $G_y^W(y^W) = \Pr\{Y^W \leq y^W \mid X > Y = y\}$ denote the distribution functions of Y^L and Y^W, respectively, conditional on the realization of Y. Under the above assumption $G_y^W(y^W) = 0$ when $y^W < y$ and $G_y^W(y^W) = 1$ otherwise; that is, since first-market losers retain their valuations, the winner in the first market meets the same highest-valuation competitor in the second market as he did in the first. Furthermore,

$$G_y^L(y^L) = \begin{cases} F_y^W(y^L)\left[\frac{F(y^L)}{F(y)}\right]^{n-2} & \text{if } y^L < y \\ F_y^W(y^L) & \text{otherwise} \end{cases} \tag{4.1}$$

The highest competing second-market valuation only exceeds the highest competing first-market valuation in the event that the competitor who won the first item draws a higher valuation of his second unit than he had for his first. The event in which, in the second market, all competitors have valuations below the valuation of first-market winner, occurs only if both the first-market losing competitors (who carry over their valuations) and the winning competitor (who is after his second unit) have sufficiently low valuations.

We obtain the following expressions for the winner's and the loser's option values in this case:

$$OV_x^W = \int_x^{\bar{X}} [u - x]\, dF_x^W(u) = \int_x^{\bar{X}} [u - x]\, dG_x^L(u) \tag{4.2}$$

$$OV_x^L = \int_0^x [x - u]\, dG_x^L(u) \leq \int_0^x [x - u]\, dF_x^W(u) \tag{4.3}$$

When $n = 2$, the inequality in (4.3) holds with equality and (2.2) reduces to

$$\beta(x) - x = E(X^W \mid X = x) - x, \text{ or} \tag{4.4}$$
$$\beta(x) = E(X^W \mid X = x) \tag{4.5}$$

More generally, for $n \geq 2$ we have

$$\beta(x) - x = E(Y^L \mid Y = x) - x, \text{ or} \tag{4.6}$$
$$\beta(x) = E(Y^L \mid Y = x) \geq E(X^W \mid X = x) \tag{4.7}$$

In the first market, the optimal price offer equals the expected value of the highest competing second-market price offer, conditional on a competitor winning the first item and having a valuation equal to the participant's own. Such a price offer is optimal because it equals the price the participant expects to pay in the second market if he is after his first item. In expected terms, the highest competing second-market price offer, conditional on a competitor with the same valuation winning the first item, is never smaller than the participant's own valuation of a second unit.[4]

From (4.2) we see that the winner's option value does not depend on the number of participants; it depends only on the likelihood that the first-market winner draws a second-item valuation greater than his own first-item valuation. The loser's option value, on the other hand, does depend on the number of participants. In particular, as can be seen from (4.1) and (4.3), the loser's option value is decreasing in the number of participants and disappears as the number of participants goes to infinity. Consequently, price offers will be increasing in the number of participants, n. In particular, if there are synergies, i.e. there is a positive probability that the valuation of a second unit exceeds the valuation of the first, then for large enough n, a participant with valuation x prices above his valuation:

[4] See Black and de Meza (1992), section 2, for corresponding results for the two special cases (i) $X^W = kX$, where $0 < k < 1$, and (ii) $X^W \succeq X$ and $\partial F_x^W(\cdot)/\partial x \leq 0$. Black and de Meza also demonstrate that the equilibrium is unique.

Proposition 2. Assume $F_x^W(x) < 1$. Then $\exists\, n(x) : n > n(x)$ implies $\beta(x) > x$:

Proof. For given x (assuming that $F_x^W(x) < 1$), the expression in (4.2) is strictly positive, independently of n. On the other hand, the left hand side of the inequality in (4.3) can be made arbitrarily small by letting n become large. Consequently, for n sufficiently large $OV_x^W - OV_x^L > 0$, and the result follows. ■

Two observations may be made on the basis of the above result. First, as the number of participants increases, behaviour may become so aggressive that prices are consistently above valuations. The reason is that as a loser you can expect nothing from the second auction when competition is sufficiently fierce; as a winner on the other hand, you may experience a favourable second-item draw and consequently obtain a positive gain. Second, the impact of future market opportunities on current behaviour is not strategic; that is, it does not necessarily go away as participants lose market power. In fact, in this case it is precisely when competition is at its fiercest that price offers are furthest above valuations.

One might conjecture that, since price offers are increasing in the number of participants, *ceteris paribus* the selling price in the first market may tend to exceed that in the second market. It turns out that matters are not that simple. What we can show is that the valuation distribution effect taken in isolation tends to a downward-sloping price path. In particular, we have:

$$H_2(z) = H_1(z) + n[n-1]\,F(z)^{n-2} \int_x^{\bar{X}} [F(u) - F(z)]\, F_u^W(z)\,dF(u) \tag{4.8}$$

Loosely speaking, a low second-highest valuation in the second market occurs both in the event of a low second-highest valuation in the first market and in the event that the winner of the first item has a low valuation of his second item. Note that if there are sufficiently strong synergies, in particular if $X^W \geq X$, the two second-order statistics are identically distributed. Then (2.5) reduce to

$$Ep_1 - Ep_2 = \int_0^{\bar{X}} [\beta(x) - x]\,dH_1(x); \tag{4.9}$$

and hence the difference between selling prices depends on behaviour only. In this case, since the winner draws a higher valuation of the second item, the loser's option value is zero while the winner's option value is positive. Consequently, first-market bids exceed valuations and selling prices tend to fall.

More generally, we may express the difference between selling prices as follows:

$$
\begin{aligned}
Ep_1 - Ep_2 = n[n-1] \Bigg\{ & \int_0^\infty \int_x^\infty \left[1 - F_x^W(u)\right] F(x)^{n-2}[1 - F(x)]\, du\, dF(x) \\
& - \int_0^\infty \int_u^\infty F_x^W(u) F(u)^{n-2}[1 - 2F(x) + F(u)]\, dF(x)\, du \Bigg\}
\end{aligned} \tag{4.10}
$$

In the example just considered, we had $F_x^W(u) = 0$ for $u < x$, and hence the second element on the right-hand side of (4.10) is zero and the whole expression becomes positive. At the other extreme, the Weber (1983) assumption corresponds to setting $F_x^W(u) \equiv 1$ and selling prices become identical in expected terms, as they should. More generally, selling prices may be both increasing and decreasing between the two markets. The following result gives a sufficient condition for a decreasing price sequence:

Proposition 3. $\partial F_x^W(u)/\partial x \geq 0 \Rightarrow Ep_1 \geq Ep_2$:

Proof. The first-element of the right-hand side of (4.10) is clearly non-negative. Consequently, the overall expression is non-negative if the second element is non-negative also. Using integration by parts, we find:

$$
\begin{aligned}
& - \int_0^\infty \int_u^\infty F_x^W(u) F(u)^{n-2}[1 - 2F(x) + F(u)]\, dF(x)\, du \\
= & - \int_0^\infty \left[F_\infty^W(u) - F_u^W(u) \right] F(u)^{n-1} du \\
& + \int_0^\infty \int_u^\infty \frac{\partial F_x^W(u)}{\partial x} F(u)^{n-2}[1 - F(x) + F(u)]\, F(x)\, dx\, du \\
= & \int_0^\infty \int_u^\infty \frac{\partial F_x^W(u)}{\partial x} F(u)^{n-2}[1 - F(x)][F(x) - F(u)]\, dx\, du
\end{aligned} \tag{4.11}
$$

A sufficient condition for the last expression to be non-negative is $\partial F_x^W(u)/\partial x \geq 0$. ■

To demonstrate that prices may in fact also increase, consider the following example: Assume that first-item and second-item valuations are positively related, in particular, so that $\partial F_x^W(u)/\partial x \leq 0$, but that the valuation of a second item never exceeds that of the first, i.e. $X^W \leq X$, or $F_x^W(u) = 1$ when $u > x$. These assumptions are made in Black and de Meza (1992). Then the first-element on the right-hand side of (4.10) is zero while the second is negative from (4.11). In this case only the loser's option value is positive. Consequently participants discount their prices below valuations so that the behaviour effect tends to an increasing price sequence. The valuation distribution effect works in the opposite direction, but since valuations are positively correlated a high realization of the second-order statistic in the first market makes it likely that the second-order statistic is high in the second market as well.

The above discussion may be related to the literature on the so-called 'price decline anomaly'. The term is coined to characterize the seemingly robust empirical finding that when identical items are offered for sale in sequence selling prices tend to decline with later items.[5] The theoretical literature on the issue has offered a number of possible explanations for this phenomenon, including attitudes towards risk (MacAfee and Vincent, 1993), participation costs (von der Fehr, 1994), superadditive valuations (Franco, 1997) and the existence of a buyer's option, allowing the successful participant to purchase as many of the remaining units as he wants at the winning price (Black and de Meza, 1992). The present analysis focuses on scale economies and market concentration. In particular, for identical objects a downward sloping price path is more likely when the number of participants is large, participants have high valuations of a second unit and there is a weak, or negative, correlation between the valuation of the first unit and the valuation of a second.

5. No (positive or negative) synergies

In the previous section we considered a case in which items were identical but participants might value a second item differently from the first; that is, $X^L = X$ while X^W is distributed according to some distribution function F^W. We turn now to the diametrically opposite case in which a participant values each item obtained equally whereas the valuation of the first item nevertheless may depend on in which market it is obtained; in particular, $X^W = X$ (i.e. $F_x^W(x^W) = 0$ when $x^W <$

[5] The empirical evidence is not unequivocal (Vanderporten, 1992) although a number of studies have found that prices decline in sequential auctions, including Ashenfelter (1989) and McAfee and Vincent (1993) (wine auctions), Ashenfelter and Genesove (1993) (real estate auctions), Lusht (1994) (commerical properties), Beggs and Graddy (1997) (arts auctions), and Engelbrecht-Wiggans and Kahn (1997) (cattle auctions).

x and $F_x^W(x^W) = 1$ when $x^W \geq x$) while X^L is distributed according to some distribution function F^L. This assumption may be reasonable in procurement auctions for example, when some time elapses between each auction. Having finished the first project, a firm may be able to undertake a second project with the same equipment and the same staff at the same level of cost. For first-auction losers, the opportunity set may however change between the first and the second auction, due e.g. to commitments to other projects, or to changes in staff or equipment. For these latter firms the cost of undertaking the second project may then differ from those they would have incurred on the first.

Under such a constant-returns-to-scale assumption, the distribution of the highest competing second-market bid, conditional on the participant not winning the first item, is given by

$$G_y^L(y^L) = \begin{cases} 0 & \text{if } y^L < y \\ F^L_y(y^L)^{n-2} & \text{otherwise} \end{cases} \tag{5.1}$$

where

$$F^L_y(y^L) \equiv \int_0^y F_u^L(y^L)\frac{dF(u)}{F(y)}. \tag{5.2}$$

Since the competitor who won the first item enters the second market with the same valuation as in the first, the highest competing second-market price offer cannot be below the highest competing first-item valuation, i.e. $Y^L \geq Y$. The highest competing second-market price offer will exceed the highest competing first-item valuation in the event that the second-item valuation of some competitor, who was not successful in the first market, exceeds the highest competing first-item valuation.

In the event that a participant does win the first item, the distribution of the highest competing second-auction valuation is given by

$$G_y^W(y^W) = F_y^L(y^W)F^L_y(y^W)^{n-2}. \tag{5.3}$$

The winner's and the loser's option values become, respectively,

$$OV_x^W = \int_0^x [x - u]\, dG_x^W(u) = \int_0^x F_x^L(u)F^L_x(u)^{n-2}du \tag{5.4}$$

$$OV_x^L = \int_x^{\bar{X}}\int_x^u [u - \xi]\, dG_x^L(\xi)dF_x^L(u) = \int_x^{\bar{X}} \left[1 - F_x^L(\xi)\right] F^L_x(\xi)^{n-2}d\xi \tag{5.5}$$

The last equality in (5.4) follows from applying integration by parts and substitution from (5.3). Similarly, in (5.5) we have applied integration by parts, interchanged the order of integration and substituted from (5.1) to arrive at the last expression.

The winner's option value is higher the more likely it is that losers draw low second-item valuations. Consequently, the winner's option value is high when valuations of the second item are systematically lower than valuations of the first item. The loser's option value, on the other hand, is large when there is a high probability that the participant himself draws a high second-item valuation while all other (i.e. lower valuation) losers draw low valuations. If there is a strong correlation between first-item and second-item valuations such an outcome is likely and hence the loser's option value is large in this case. Unlike in the previous section, both the winner's and the loser's option values disappear as the number of competitors becomes large.

Substituting the above expressions into (2.2), we find

$$\begin{aligned}\beta(x) - x &\leq F^L_x(x)^{n-2} \left\{ \int_0^x F^L_x(u)du - \int_x^{\overline{X}} \left[1 - F^L_x(u)\right] du \right\} \\ &= F^L_x(x)^{n-2} \left[x - E_x X^L\right]. \end{aligned} \tag{5.6}$$

When there are two participants only, (5.6) holds with equality and consequently participants price above their valuations if and only if their first-item valuations exceed their expected, second-item valuations as losers. If first-item and second-item valuations are uncorrelated or negatively correlated, high-valuation participants price above their valuations, and vice versa. Participants generally price lower when there are many participants than when there are only two. As the number of participants becomes very large, however, prices tend to valuations. We summarize these results in the following proposition:

Proposition 4. Assume $X^W = X$. Then

$$\begin{aligned} \text{i)}\ \beta(x) - x &= [x - E_x X^L] \text{ if } n = 2 \\ \text{ii)}\ \beta(x) - x &< [x - E_x X^L] \text{ if } n > 2 \\ \text{iii)}\ \beta(x) - x &\to 0 \text{ as } n \to \infty. \end{aligned}$$

It is difficult to characterize precisely the relationship between first-market and second-market selling prices, except in special cases. The second-order statistic for valuations of the second item becomes

$$H_2(z) = \int_0^{\bar{x}} F^L_u(z)^{n-1} dF(u)^n + [n-1] \int_0^{x} F^L_u(z)^{n-2} \left[1 - F^L_u(z)\right] dF(u)^n \quad (5.7)$$

In the particular case that X and X^L are independently and identically distributed, (5.7) reduces to

$$H_2(z) = F(z)^{n-1} + [n-1]F(z)^{2[n-1]}[1-F(z)] < H_1(z). \quad (5.8)$$

Since the winner retains his valuation, a low second-order statistic is less likely in the second market than in the first whenever the other valuations are identically distributed in both markets. Consequently, the valuation distribution effect tends to an increasing price path. Furthermore, from (5.4) and (5.5) we see that in this case the winner's option value is increasing, while the loser's option value is decreasing, in the participant's own valuation. Therefore, participants with sufficiently high valuations will price above their valuations while lower-valuation participants price below. Hence the behaviour effect tends to a downward-sloping price path also, at least when the number of participants is sufficiently large.

6. The impact of competition

The preceding analysis has suggested that there is no straightforward relationship between competition and behaviour in sequential markets. In section 4, we demonstrated that the forward-looking nature of pricing behaviour is not a strategic phenomenon that necessarily disappears as competition toughens; when items are identical there is always an incentive to price above the static valuation as long as there are synergies. The analysis presented in section 5 hinted that, although when items differ sufficiently option values may become very small as the number of competitors gets large, the relationship between the degree of competition and pricing behaviour is not necessarily monotone. We now proceed with a more detailed analysis of the impact of competition on pricing behaviour.

In the two previous sections we considered the opposite extremes in which in the one case the losers, and in the other the winners, retain their valuations from the first market opportunity to the next. We now turn to intermediate cases in which, contrary to in the above two sections, neither of the distributions of X^L and X^W contain mass points; in particular, we assume that F^L and F^W have positive densities everywhere on their support. Then the distribution functions for the highest competing bid that a winner and a loser (both with valuations x) will face in the second market are, respectively,

$$G^W_x(y) = F^L_x(y)F^{\,L}{}_{x}(y)^{n-2}; \tag{6.1}$$
$$G^L_x(y) = F^W_x(y)F^{\,L}{}_{x}(y)^{n-2}. \tag{6.2}$$

From these expressions, and by the application of integration by parts and interchange of the order of integration, the winner's and the loser's option values may be written:

$$\begin{aligned}
OV^W_x &= \int_0^{\bar{x}}\int_0^u [u - \mu]\, dG^W_x(\mu)dF^W_x(u) \\
&= \int_0^{\bar{x}} F^{\,L}{}_{x}(u)^{n-2}F^L_x(u)\left[1 - F^W_x(u)\right] du \qquad (6.3) \\
OV^L_x &= \int^{\bar{x}}\int_0 [u - \mu]\, dG^L_x(\mu)dF^L_x(u) \\
&= \int_0^{\bar{x}} F^{\,L}{}_{x}(u)^{n-2}F^W_x(u)\left[1 - F^L_x(u)\right] du \qquad (6.4)
\end{aligned}$$

We see, as we would expect from the discussion in the two previous sections, that the winner's option value is large when there are positive synergies (i.e. $1 - F^W_x(u)$ is large for small values of u), and when losers have systematically lower valuations of the second item than of the first (i.e. $F^L_x(u)$ is large for small values of u). Note that, due to the symmetry of the two option values, these are exactly the circumstances in which the loser's option value tends to be low and, consequently, pricing behaviour is aggressive.

Substituting (6.3) and (6.4) into the first-order condition (2.2) we obtain

$$\beta(x) - x = \int_0^{\bar{x}} F^{\,L}{}_{x}(u)^{n-2}\left[F^L_x(u) - F^W_x(u)\right] du. \tag{6.5}$$

It turns out that the characterization of pricing behaviour is particularly simple when there are two players only:

Proposition 5. For $n = 2$, $\beta(x) - x = E_x X^W - E_x X^L$.

Proof. Integration by parts on $E_x X^s = \int u dF_x^s(u)$; $s = L, W$, yields

$$E_x X^W - E_x X^L = \int_0^{\bar{x}} \left[F_x^L(u) - F_x^W(u) \right] du; \tag{6.6}$$

and the result follows from (6.5). ■

Consequently, in the case in which there are two market participants, whether a participant sets the price in the first round above, equal to or below his valuation depends on the first-order moments of the distributions of second-item valuations only. Note however, that also in this case the evolution of selling prices from the first market to the next does depend on the higher-order moments of the two distributions (for identical distributions we are in the case covered by Proposition 1). Assume for example, that $F_x^L(x') = F(x')$ (so that valuations of a first item are independently and identically distributed across markets), that the $F_x^W(\cdot)$ distribution is independent of x also and is more dispersed than the $F^L(\cdot)$ distribution, but that the means are the same, i.e. $EX^W = EX^L$. The winner's option value reflects the gain in the case of a favourable draw of his second-item valuation while the loser's option value reflects the gain in the case that the winner experiences an unfavourable draw of the second-item valuation. With two players, expected gains are the same independently of whether a participant wins or looses in the first market, and consequently players price at valuations. However, since the selling price equals the losing bid and, due to the greater dispersion of the F^W distribution, the expected value of the second-order statistic is lower in the second market than in the first the expected second-market selling price is lower as well.

If the number of participants exceeds two behaviour depends on higher order moments of the F^L and F^W distributions also. It is difficult to obtain clear results in the most general cases. We therefore concentrate on cases in which the following definition can be applied:

Definition 1. For given $X = x$, the distribution of X^s is more dispersed than the distribution of X^t, denoted $F_x^s(\cdot) \succ F_x^t(\cdot)$, if there exists $\xi(x)$ such that for $x' \lessgtr \xi(x)$, $F_x^s(x') \gtrless F_x^t(x')$.

When $E_x X^s = E_x X^t$ the above condition is the 'one sign change condition', as defined in Shaked (1980). $F^s(\cdot) \succ F^t(\cdot)$ is also satisfied for a mean-preserving spread in the sense of Rotschild and Stiglitz (1970).

In the analysis, below we will make repeatedly use of the following auxiliary result:

Lemma 1. For the real numbers $a \quad b \quad c$, let $\varphi(\xi) : [a; c] \to [-1; 1]$ be an

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the F^W distribution. Proposition 7, which we state without proof, covers the case in which the least dispersed distribution has the lowest mean:

Proposition 7. : If $F_x^W(\cdot) \succ (\prec) F_x^L(\cdot)$ and $E_x X^W > (<) E_x X^L$,

$$\text{i) } \beta(x) - x > (<)\ 0 \text{ for all } n;$$
$$\text{ii) } \beta(x) \to x \text{ as } n \to \infty.$$

Clearly, if the winner's valuation both has a higher mean and a greater dispersion than the loser's valuation the winner's option value exceeds the loser's option value and prices exceed valuations. The opposite occurs if it is the loser's valuation that has both the higher mean and the greater dispersion.

A more difficult case is when the winner's valuation has a higher mean but a lower dispersion that the loser's valuation. Although we cannot in general say whether a player will then price above or below his valuation, we can say something about how behaviour is affected by the degree of competition in the market. Define $\beta_n(x)$ to be the equilibrium bid of a participant with valuation x when there are n players in the market. We can prove that, as a function of the number of players, equilibrium option values are either monotone or single peaked:

Proposition 8. : Assume $F_x^W(\cdot) \succ (\prec)\, F_x^L(\cdot)$. Then either

$$\text{i) } 0 \leq \beta_{n+1}(x) - x < \beta_n(x) - x, \text{ all } n, \text{ or}$$
$$\text{ii) } \beta_{n+1}(x) - x \gtrless (\lessgtr)\, \beta_n(x) - x;\ n \lessgtr \mathbf{b}, \text{ for some } \mathbf{b} > 2, \text{ or}$$
$$\text{iii) } 0 \geq \beta_{n+1}(x) - x > \beta_n(x) - x, \text{ all } n.$$

Proof. We prove the proposition for $F^W \succ F^L$ (again the opposite case may be proved by reversing signs in the argument below). From (6.5), and applying Lemma 1, we find for $n \geq 3$,

$$\begin{aligned}
\beta_{n+1}(x) - \beta_n(x) &= \int_0^x F^L{}_x(u)^{n-2}\left[F_x^L(u) - 1\right]\left[F_x^L(u) - F_x^W(u)\right] du \\
&< F^L{}_x(\mathbf{b}) \int_0^x F^L{}_x(u)^{n-3}\left[F_x^L(u) - 1\right]\left[F_x^L(u) - F_x^W(u)\right] du \\
&= F^L{}_x(\mathbf{b}) \left[\beta_n(x) - \beta_{n-1}(x)\right];
\end{aligned}$$

where $\mathbf{b} = \arg\left\{u \mid F_x^L(u) - F_x^W(u) = 0\right\}$. It follows that if $\beta_n(x) - \beta_{n-1}(x) < 0$ then $\beta_{n+1}(x) - \beta_n(x) < 0$ also. Consequently, the sequence $\beta_{n+1}(x) - \beta_n(x)$ can change sign at least once, and only from positive to negative values. Since $\beta_n(x) - x \to 0$ as $n \to \infty$, we must have $\beta_n(x) - x \geq (\leq) 0$ if the sequence is always decreasing (increasing). ■

To illustrate the various possibilities, consider the following family of distributions: Fix two distribution functions F^L and F^W and let X_Δ^W be a random variable distributed according to $F_{x\Delta}^W(x') = F_x^W(x' - \Delta)$. Increasing Δ is equivalent to increasing the mean of the distribution of the winner's valuation without affecting higher order moments. Let F^L and F^W be chosen such that $E_x X^L = E_x X^W$ and $F_{x\Delta}^W \succ F^L$ for all relevant values of Δ. Then we know from Proposition 5 that $\beta_2(x) - x$

atively easy to characterize behaviour in the first market. Note however, that as participants acquire items in early rounds, they are no longer symmetric with their less fortunate co