Fees, Reputation, and Rating Quality*

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Abstract

I explore how the compensation of a rating agency affects its incentive to rate accurately. A sequence of short-lived firms can hire a long-lived agency to rate their projects. The agency can privately acquire costly information about the projects. The agency might be committed to acquiring information or be fully strategic. In equilibrium, the agency demands a fee only if a rated project is sold. I consider regulations that require the agency to collect part or all of its fee whenever hired. Whenever the agency is impatient, these regulations result in less informative ratings. I discuss policy implications.

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1. **Introduction**

Despite the increasing number of ratings issued each year and record revenues as a result, neither Moody’s nor S&P hired sufficient staff or devoted sufficient resources to ensure that the initial rating process ... produced accurate credit ratings. (US Senate (2011): 304).

This excerpt is taken from a United States Senate report. The report was not alone in criticizing the largest rating agencies for assigning inaccurate ratings in the years leading up to the last financial crisis. Such criticism has spurred a wave of proposals for reforming the credit rating process. The way rating agencies are paid for their services has received the most attention.

Currently, all the largest rating agencies receive fees from the issuers of the securities they rate. These fees are paid only if the issuer obtains the desired rating and decides to publish it. This peculiar feature of the credit rating market has attracted considerable attention. As early as 2008, Andrew Cuomo, who then served as Attorney General of New York, proposed that issuers of Residential Mortgage-Backed Securities be required to pay an agency regardless of whether the issuer received the desired rating.

Should rating agencies be required to charge their fees whenever they are hired, regardless of the rating assigned? In order to answer this question, I develop a dynamic model of the rating process. In every period a new firm has a project of unknown quality. Before taking the project to potential investors, a firm can hire an infinitely-lived rating agency to collect information about the project and rate it. The agency can (privately) incur the cost necessary to learn the quality of the project and rate it accordingly. The alternative is to shirk. If the agency shirks, it must assign a high rating. The agency has an unobservable type, as in the

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1 See Benmelech and Dlugosz (2009), White (2010) and Haan and Amtenbrink (2011).

2 E.g. the Dodd Frank Act requires the Securities and Exchange Commission (SEC) to carry out an assessment of “appropriate methods for paying fees to the nationally recognized statistical rating organizations” (US Senate (2010): 514).


ations-three-principal.
class of models pioneered by Kreps and Wilson (1982) and Milgrom and Roberts (1982). The agency can be committed to learning the quality of every project, or be fully strategic.

I first consider a market in which the agency decides the amount and the structure of the rating fee in each period. The agency can either charge a fee only if the rating assigned is high (i.e. a contingent fee); demand the entire fee whenever a firm hires the agency (i.e. an up-front fee); or opt for combination of these two structures. I show that contingent fees are an equilibrium outcome. Then, I compare the market described above with regulated markets in which the agency can still choose any fee amount, but the fee structure is imposed, once and for all, by a regulator.

When the rating agency weighs heavily current revenues at the expenses of future revenues, i.e. when the agency is impatient, an unexpected result holds. Regulations that require part or all of the fee to be collected at the time when the agency is hired reduce the probability that the agency will collect an informative signal, and as a result reduce the informative content of the ratings as well as total surplus. When instead the rating agency is sufficiently patient, a regulation requiring up-front fees maximizes total surplus. Interestingly, in order to maximize total surplus, a regulator never needs regulations that require a combination of contingent and up-front fees.

In order to explain these results, I focus on the regulation requiring up-front fees. This regulation has a static and a dynamic effect. The static effect works as expected. As long as fees are contingent, learning the quality of a project, and rating it accordingly, entail an opportunity cost: the agency receives no compensation whenever it assigns a low rating. Up-front fees eliminate this opportunity cost, and weaken the static incentive to assign high ratings without collecting information about the projects.

A regulation requiring up-front fees also has a dynamic effect. Up-front fees reduce the value of reputation and weaken the strategic agency’s incentive to collect costly informative signals. To see why this is the case, it is useful to consider the expected revenue of the agency in a given period, given its fee, its reputation, and the anticipated choice between effort and shirking. All else being equal, this expected revenue is the same regardless of whether the
fee is up-front or contingent. If the fee is up-front, then a strategic agency’s revenue is equal
to the agency’s expected revenue. If the fee is instead contingent, the expected revenue of a
shirking strategic agency is higher than the expected revenue of the agency. These observations
imply that a strategic agency can earn more under contingent fees than under up-front fees
and, most importantly, this difference in revenues widens as the reputation for commitment
becomes stronger and fees become larger. A regulation that imposes up-front fees results in
the revenue of a strategic agency being less responsive to variations in reputation. This, in
turn, weakens the agency’s incentive to assign correct ratings and maintain its reputation for
commitment.

For small discount factors, the dynamic effect is relatively strong and the static effect
relatively weak, and as a result a strategic agency is more likely to exert effort under contingent
fees than under up-front fees. If the discount factor is small, a strategic agency has weak
incentives to collect informative signals regardless of the fee structure. When the agency is
expected to shirk, its rating has little value for a firm, the fee is low, and the static effect
is weak. At the same time, the dynamic effect is strong. As a strategic agency is expected
to shirk and assign a high rating, the agency’s reputation becomes considerably stronger
whenever it assigns a low rating. As discussed above, the prospective of a strong reputation
is more valuable to a strategic agency under contingent fees than it is under up-front fees.

The main result of this article shows that regulations which prohibit contingent fees can
curb an agency’s incentive to acquire information. This result hinges on the assumption that
the agency incurs a cost to obtain an informative signal. If the agency can collect the signal
at no cost, up-front fees eliminate all incentives to shirk and ensure that the agency assigns
correct ratings.

The results described so far might seem discouraging: I consider regulations that at first
sight might appear always desirable, and I show that they are counterproductive exactly in
the cases in which we should expect the rating process to be least efficient to begin with (i.e.
when the agency has a small discount factor and when the cost to collect the signal is non-
negligible). A more constructive interpretation of the results is however possible. I show that
as long as a regulator mandates public supervision of the resources that the agency devotes
to the rating process, up-front fees always improve the rating process. The result provides a
novel rationale for devoting resources to the supervision of rating agencies. This is particularly
relevant given recent efforts by regulators to facilitate the external evaluation of procedures
followed by rating agencies.\footnote{Sections 932 and 939E of the Dodd Frank Act (US Senate (2010)), contain, among other things, provisions that require (a) the SEC to monitor rating agencies and sanction those that do not have sufficient resources to perform their task; (b) rating agencies to document their internal controls over procedures; and (c) the creation of a professional organization that would set standards for the profession of rating analysts.}

This article is the first to compare rating agency compensation schemes in a dynamic frame-
work. The literature has considered the merits of contingent and up-front fees in static frame-
works. Bolton, Freixas and Shapiro (2012) and Kovbasyuk (2013) consider the case of agen-
cies that acquire information at no cost. If agencies incur exogenous losses whenever they
lie, up-front fees are socially desirable (Bolton, Freixas and Shapiro (2012)). If instead agen-
cies internalize the effect of their ratings on the buyer’s utility, contingent fees are preferable
(Kovbasyuk (2013)). In my model, the cost of dishonesty is endogenous and operates through
the reputation mechanism.

The case of costly and unobservable information acquisition has been considered before
by Kashyap and Kovrijnykh (2013). In a static framework, they look at a broad range of
compensation schemes. Bolton, Freixas and Shapiro (2012) and Ozerturk (2013) consider
costly and publicly observable information acquisition in a static setting. Up-front fees are
socially desirable if the agency acquires information before setting its fee (Ozerturk (2013)).
In contrast, up-front fees can reduce total surplus if information acquisition takes place after
the fees are set (Bolton, Freixas and Shapiro (2012)).

The existing results on the role of reputation in the market for ratings follow a general
theme: reputation motives do not necessarily ensure honest ratings. Mathis, McAndrews and
Rochet (2009) show that when an agency is paid contingent fees, reputation incentives ensure
honest ratings only if the agency’s revenues from rating complex products is not too large.
Even if fees are up-front and information is costless, an agency might inflate its ratings if each firm requires ratings for multiple projects (Frenkel (2015)).

The model developed in this article is based on the model of Mathis, McAndrews and Rochet (2009). My analysis differs from their work as I introduce a moral hazard component to the rating process and compare the rating process under different compensation schemes.

My analysis also provides a rationale for the widespread adoption of rating fees contingent on the rating assigned. In my model, contingent fees are not the unique equilibrium outcome in an unregulated market, but the agency chooses different fees only in the cases in which they ensure the same payoff and the same ratings as contingent fees do. In my model, contingent fees are an equilibrium outcome because investors do not observe the fees paid by firms for their ratings. Faure-Grimaud, Peyrache and Quesada (2009) provide an alternative rationale for the adoption of up-front fees. Their result rests on the assumption that firms have private information about their projects.

My work is also related to the literature on regulation of markets for financial advice (Inderst and Ottaviani (2012a), Inderst and Ottaviani (2012b)). This literature considers the compensation of experts advising single investors, while I focus on a setting in which advice, in the form of ratings, is publicly observed.

The article is structured as follows. Section 2 describes the model. Section 3 presents the equilibria in unregulated and regulated markets. Section 4 contains the policy analysis. Section 5 extends the analysis to the case of supervised effort. Section 6 concludes. All the proofs are in the appendices.

2. THE MODEL

My model builds on Mathis, McAndrews and Rochet (2009). The game has infinite periods and three types of risk-neutral agents: a monopolistic rating agency that is present in every period, and a sequence of one firm and \( n \geq 2 \) investors, which are active for one period only. In
each period $t$, the active firm (‘firm $t$’) owns a project of unobservable quality $q_t$. Quality can be high ($H$) or low ($L$). Everyone knows that qualities are independent, identically distributed and

$$Pr\{q_t=H\} = \lambda \in (0,1) \quad \forall t.$$  

Every project yields 1 if its quality is high, 0 if its quality is low, and requires an investment equal to its expected gross return $\lambda$.

At the beginning of each period $t$, the agency privately communicates its fee $\{\phi_t, \gamma_t\} \in \mathbb{R}_+ \times [0,1]$ to firm $t$. The agency sets the non-negative fee amount $\phi_t$ and the fee structure, captured by $\gamma_t$. If firm $t$ hires the agency, the firm pays the up-front share $\gamma_t \phi_t$ to the agency regardless of the rating received. The contingent share $(1-\gamma_t) \phi_t$ is paid only if the agency assigns a high rating, as described below. In line with these definitions, the agency may demand an up-front fee ($\gamma_t=1$), a contingent fee ($\gamma_t=0$), or a combination of the two ($\gamma_t \in (0,1)$).

Firm $t$ observes the fee and decides whether to hire the agency or sell the project without a rating. If hired, the agency decides privately whether to scrutinize the firm’s project (i.e. ‘exert effort’), or to shirk. The agency observes a signal $s \in \{h,l\}$. Signal realization $l$ corresponds to observing hard information, while signal realization $h$ corresponds to observing no information. If the agency exerts effort, the signal is perfectly informative: $s=h$ if $q_t=H$ and $s=l$ if $q_t=L$. If the agency shirks, the signal is completely uninformative: $s=h$ regardless of the project’s quality.

The agency either discloses the hard information (i.e. assigns rating $r_t=l$) or does not disclose any information (i.e. assigns rating $r_t=h$). If (and only if) the project receives rating $r_t=h$, the agency receives the contingent share of the fee, $(1-\gamma_t) \phi_t$. The nature of the signal requires the agency to provide evidence only when it assigns a low rating. As noted in Frenkel

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6By ruling out firms’ private information about the quality of the project, I can abstract from the issue of signaling through the decision to obtain a rating.
(2015), as the agency is hired by the active firm, it seems reasonable that the burden of the proof is heavier when the agency assigns a low rating.

The agency has a type, determined once and for all at the beginning of the game. The agency is committed with probability $\mu_0 \in (0,1)$, and strategic otherwise. The agency knows its type, whereas the other agents only know the prior distribution. A committed agency exerts effort, at no cost, whenever hired, and reports honestly: if $s = l$ then $r = l$.\textsuperscript{7} A strategic agency chooses whether to exert effort, at a cost $\kappa$, in order to maximize the discounted sum of its instantaneous payoffs.\textsuperscript{8} The probability that a strategic agency exerts effort in period $t$ is denoted $e_t \in [0,1]$. A strategic agency also reports honestly. This is equivalent to assuming that a strategic agency discloses hard information whenever it observes such information. This assumption does not entail a loss of generality. Even if the agency had the option to keep hard information undisclosed, it would never need to do so. Instead, the agency could shirk, and observe $s = h$.

**Assumption 1.** $0 < \kappa < (1 - \lambda) \lambda$.

If no information is acquired, ex ante total surplus in each period is equal to zero. If information is acquired, ex ante total surplus in each period is $\lambda(1 - \lambda) - \kappa$. Assumption 1 ensures that cost $\kappa$ is lower than the expected increase in total surplus generated as a result of the acquisition of information about a project’s quality.

Investors observe rating $r_t \in \{h, l, \emptyset\}$, where $r_t = \emptyset$ if the agency is not hired, and simultaneously post non-negative bids. If at least one bid is strictly larger than zero, the project is sold to one of the investors who made the largest bid, and the investor who obtains the project finances it.\textsuperscript{9} Figure 1 shows the time line in each period $t$.

The outcome of the project is denoted $y \in \{0, 1, \emptyset\}$, with $y = \emptyset$ meaning that the project is not sold, and therefore not financed. The rating and the outcome of the project are observed by

\textsuperscript{7}The nature of the information about the project ensures that the agency must assign rating $r = h$ if $s = h$.

\textsuperscript{8}The assumption that a committed agency gets an informative signal at no cost ensures that a committed agency always obtains a non-negative payoff from rating. If the initial reputation for commitment is strong enough, then the equilibrium strategies do not change even if a committed agency incurred a cost $\kappa$ to obtain the informative signal.

\textsuperscript{9}Without loss of generality, I assume that a project is not sold if its expected value is equal to zero.
all firms and investors. Therefore, firm and investors active in period $t+1$ make their decisions knowing the entire history of ratings and outcomes. This history is denoted:

$$H_{t+1} = \{(r_1, y_1), \ldots, (r_t, y_t)\}.$$ 

Firms and investors use this history to form beliefs about the agency’s type. The agency’s reputation for commitment at the beginning of period $t$ is denoted $\mu_t$.

I focus on Perfect Bayesian Equilibria (PBE) in stationary Markov strategies.\footnote{A PBE is defined by strategies and beliefs. In equilibrium, strategies are individually rational on and off the equilibrium path and beliefs are based on Bayes’ Rule whenever possible.} The state variable is the agency’s reputation $\mu$. Equilibria in stationary Markov strategies require players to choose the same actions following any two histories $H_t, H'_{t+n}$ whenever $\mu_t$ resulting from $H_t$ is identical to $\mu_{t+n}$ resulting from $H'_{t+n}$. The agency chooses $\{\phi, \gamma\} : [0,1] \rightarrow \mathbb{R}_+ \times [0,1]$, and, if strategic, also $e : [0,1] \rightarrow [0,1]$. Both $\{\phi, \gamma\}$ and $e$ are functions of $\mu$. Each firm decides whether or not to hire the agency. The strategy of each investor consists of a bid for any rating. As I consider stationary strategies, I drop the time subscript wherever possible.

The payoffs are as follows. If in period $t$ at least one bid is larger than zero, the payoff of the investor who buys (and finances) the project is $y_t - b_t - \lambda$, where $b_t$ denotes his bid. The payoffs of the investors who do not buy the project are equal to 0. The payoff of firm $t$ amounts to $b_t - 1^H_t (\gamma_t + 1^h_t (1 - \gamma_t)) \phi_t$, where $1^H_t = 1$ if the firm hires the agency ($1^H_t = 0$ otherwise) while $1^h_t = 1$ if $r_t = h$ ($1^h_t = 0$ otherwise). The payoff of the agency is the sum of each period’s profit discounted at rate $\delta \in (0,1)$.

At the beginning of every period, the expected continuation payoff of a strategic agency is only a function of its reputation, and can be described with a value function $V(\mu)$.

In the remainder of the analysis, the term ‘equilibrium’ will refer to PBE in stationary Markov strategies that satisfy the following restriction.

**Restriction 1.** Whenever $\mu > 0$, the active firm hires the agency.
The restriction, in the spirit of Mathis, McAndrews and Rochet (2009), rules out uninteresting equilibria in which active firms do not hire the agency because investors hold arbitrary out-of-equilibrium-path beliefs about the agency’s type whenever a rating is assigned. The assumption also rules out equilibria in which the agency signals its type by choosing a fee that discourages firms from hiring the agency.

In the next section, I characterize the equilibrium in the market described so far, as well as in markets in which the fee structure is regulated.

3. Equilibria

In the first part of this section, I analyze equilibrium properties that hold regardless of whether the fee structure is regulated. I begin with a characterization of the equilibrium bids. In every period, active investors bid taking into account the rating assigned, or the lack thereof. If \( r=h \), the expected net return of a project is

\[
v(\mu, e) := \frac{\lambda}{\lambda + (1 - \lambda)(1 - \mu)(1 - e)} - \lambda.
\]

The expected net return is zero if \( r=\emptyset \), and \(-\lambda\) if \( r=l \). A project is sold (and financed) only if its expected net return is larger than zero. Whenever a project is sold, the highest bid is equal to \( v(\mu, e) \).

As long as the agency’s reputation for commitment is larger than zero, \( h \)-rated projects are always sold. If the agency is believed to be strategic with certainty, investors buy an \( h \)-rated project only if they believe the agency to have exerted effort. Nevertheless, if the agency is believed to be strategic with certainty, it has no reason to exert effort and therefore an \( h \) rating is not sufficient to induce investors to buy a project. As a result, firms are not willing to pay anything for such a rating.

**Lemma 1.** In any equilibrium, if \( \mu=0 \) then \( e=0 \). As a result, \( V(0)=0 \).

The agency’s reputation depends on its ratings and the projects’ outcomes. The reputation evolves as follows:
\[
\mu_{t+1} = \psi(r_t, y_t | \mu_t, e(\mu_t)) := \begin{cases} 
\mu_t & \text{if } r_t = h \text{ and } y_t = 1 \quad (I) \\
0 & \text{if } r_t = h \text{ and } y_t = 0 \quad (II) \\
\mu^l(e(\mu_t)) & \text{if } r_t = l \quad (III) \\
\mu_t & \text{if } r_t = \emptyset \quad (IV)
\end{cases}
\] (3.2)

where \( \mu^l(e(\mu_t)) := \frac{\mu}{\mu + (1-\mu)e(\mu_t)} \) and \( e(\mu_t) \) refers to the equilibrium effort probability. If \( q_t = H \), the signal observed and the rating assigned do not depend on the agency’s effort and the reputation of the agency remains unchanged (I). If instead \( q_t = L \), either the agency fails to collect the hard information and its type is inferred with certainty (II), or the agency collects and discloses the hard information, in which case its reputation (weakly) increases (III). Firms and investors hold passive beliefs about the agency’s type when they do not observe any rating (IV). For the cases in which Bayes’ Rule does not hold, I show in Appendix B that (3.2) imposes only restrictions that hold without loss of generality.

The value function of a strategic agency can be represented recursively with a Bellman equation. In equilibrium, the value function has the following form.

\[
V(\mu) = \max_e \left\{ (\gamma(\mu) + (1 - (1-\lambda)e)(1 - \gamma(\mu)))\phi(\mu) - e \kappa + \delta [\lambda V(\mu) + (1-\lambda) e V(\mu^l(e(\mu)))] \right\}. \quad (3.3)
\]

Where \( \phi(\mu) \) and \( \gamma(\mu) \) refer to the equilibrium fee. I now consider a market in which the agency can choose any fee structure.

### 3.1 Unregulated Market

The agency uses the fee to extract all expected surplus from the active firm. The next lemma characterize the fee choice.

**Lemma 2.** The agency sets a fee equal to the active firm’s willingness to pay. In particular, a strategic agency demands a contingent fee \( \gamma(\mu) = 0 \) and \( \phi(\mu) = v(\mu, e) \) as long as \( e(\mu) < 1 \) and \( \mu < 1 \). In any other case, any fee structure can be part of an equilibrium strategy.
If the agency is expected to shirk with positive probability, then, a strategic agency demands a contingent fee. If instead the agency is expected to collect the informative signal with probability 1, or if the agency is committed, then any fee structure is optimal and might be chosen by the strategic agency, as long as the fee amount extracts all the expected surplus of the active firm. This in turn implies the existence of multiple equilibria, all identical in terms of effort choice and payoffs. In the rest of the section I focus on the equilibrium in which the agency, regardless of its type, always chooses contingent fees.

Lemma 2 provides a novel insight into the markets for ratings. The lemma shows that contingent fees can be chosen whenever investors do not observe the rating fees, and the commitment of the agency to honest rating is privately known. As discussed in Section 1, Faure-Grimaud, Peyrache and Quesada (2009) provide an alternative rationale for this phenomenon. Their argument rests on the assumption that firms have private information about the quality of the projects. Our arguments are not mutually exclusive, but rest on different sources of asymmetric information.

The next proposition characterizes the choice of a strategic agency to acquire information as a function of \( \mu \) and \( \delta \).

**Proposition 1.** Suppose that the agency always sets contingent fees. Then a (generically) unique equilibrium exists. In equilibrium, as long as \( \mu > 0 \):

1) for \( \delta \in [\delta, 1) \), a strategic agency exerts effort \( (e = 1) \)

2) for \( \delta \in (\delta, \delta) \), a unique \( \bar{\mu} \in (0, 1) \) exists such that a strategic agency mixes between effort and shirking if \( \mu < \bar{\mu} \ (e \in (0, 1)) \); if instead \( \mu \geq \bar{\mu} \), the agency shirks \( (e = 0) \),

3) for \( \delta \in (0, \delta\] \), a strategic agency shirks,

where: \( \delta := \frac{(1-\lambda)^2+\kappa}{(1-\lambda)(1-\lambda)+\lambda \kappa} \) and \( \bar{\delta} := \frac{\kappa}{\kappa \lambda + (1-\lambda) \sigma^2} \).

As shown in Figure (2), if \( \delta \) is sufficiently large, a strategic agency with reputation \( \mu > 0 \) exerts effort. For intermediate \( \delta \), a strategic agency exerts effort with positive probability as long as its reputation is weak. If the reputation is strong enough, the strategic type 'cashes
in': it shirks and collects positive fees until it rates a project of low quality and loses all its reputation for commitment. If $\delta$ is small, a strategic agency shirks and always assigns a high rating.

The thresholds $\bar{\delta}$ and $\hat{\delta}$ increase monotonically with the cost $\kappa$ (see Figure 6). The case of a costless signal is the closest to Mathis, McAndrews and Rochet (2009). As in their equilibrium, $\bar{\delta}$ is larger than 0 even if $\kappa=0$. For a sufficiently small $\delta$, a strategic agency shirks regardless of the cost $\kappa$.

3.2 Regulated Markets

I consider here markets in which the fee structure is imposed, once and for all, by a regulator. The proposal by Andrew Cuomo mentioned in Section 1 corresponds to a regulation that requires up-front fees. In regulated markets, the agency can nevertheless choose in each period the fee amount, denoted $\phi^\gamma$. The superscript refers to a market in which the regulator requires $\gamma_t=\gamma$ in every period.

The agency, regardless of its type, maximizes its profit by setting a fee amount that extracts all the expected surplus from the active firm (as long as the active firm hires the agency for such a fee). If the agency has reputation $\mu$, it demands $\phi^? = \phi^? (\mu)$, where $\phi^? (\mu)$ satisfies:

$$-\gamma \phi^? (\mu) + (\lambda + (1-\lambda)(1-\phi^? (\mu))(\nu(\mu, \phi^? (\mu)) - (1-\gamma) \phi^? (\mu)) = 0. \quad \forall \mu, \forall \phi^? (\mu). \quad (3.4)$$

Equilibria in which the active firm does not hire the agency if $\phi^? = \phi^? (\mu)$, and the agency sets $\phi^? < \phi^? (\mu)$ also exist. These equilibria require rather ad-hoc out-of-equilibrium-path beliefs by the active firm. The following restriction on out-of-equilibrium-path beliefs is sufficient to ensure that in equilibrium $\phi^? = \phi^? (\mu)$.

**Restriction 2.** Upon observing an out-of-equilibrium-path $\phi^?$, every active firm believes that both types of agency are equally likely to have deviated.
The regulations considered here do not directly affect the quality of the ratings. These regulations operate indirectly, through their influence on a strategic agency’s effort choice. The next proposition characterizes the equilibrium effort.

**Proposition 2.** Suppose a regulation $\gamma$ is in place. A (generically) unique equilibrium that satisfies Restriction 2 exists. In equilibrium, as long as $\mu > 0$:

1) if $\delta \in [\delta^\gamma, 1)$, a strategic agency exerts effort,
2) if $\delta \in (\delta^\gamma, \delta^-)$, a unique $\bar{\mu} \in (0, 1)$ exists such that a strategic agency mixes between effort and shirking if $\mu < \bar{\mu}$; if instead $\mu \geq \bar{\mu}$, then the agency shirks,
3) if $\delta \in (0, \delta^-)$, a strategic agency shirks,

where: $\delta^\gamma := \frac{(1-\gamma)(1-\lambda)^2+\kappa(\gamma+(1-\gamma)\lambda)}{(1+(1-\gamma)\lambda)(1-\lambda)^2+\kappa(\gamma+(1-\gamma)\lambda)}$ and $\delta^- := \frac{(\gamma+(1-\gamma)\lambda)\kappa}{(1-\lambda)^2+\kappa(\gamma+(1-\gamma)\lambda)}$.

Figure 3 shows the effort of a strategic agency under a regulation that imposes $\gamma = 1/2$. For each $\gamma$, thresholds $\delta^\gamma$ and $\delta^-$ are comparable, respectively, to $\delta$ and $\delta^-$. Thresholds $\delta^\gamma$ and $\delta^-$ get closer to each other as the regulator increases $\gamma$:

$$\frac{\partial \delta^\gamma}{\partial \gamma} > 0 > \frac{\partial \delta^-}{\partial \gamma}.$$ 

If the regulation imposes up-front fees, the optimal strategy of the agency is somewhat unusual. As shown in Figure 4, the thresholds $\delta^1$ and $\delta^- 1$ coincide (I refer to the value of these two threshold as $\hat{\delta}$). An agency paid up-front fees never mixes between effort and shirking, and, as long as $\mu > 0$, $e^1$ does not depend on $\mu$. In order to shed light on this unusual feature, I will describe how reputation affects the incentive to shirk under different fee structures.

If fees are contingent, the incentive to shirk is positively correlated with the reputation for commitment. This is the case as the agency is paid only if it assigns a high rating, and the amount of the fee increases with reputation. For intermediate $\delta$, the high fee associated with strong reputation more than compensates the inevitable loss of future revenues that the agency incurs when it assigns an undeserved $h$ rating. As a result, for intermediate $\delta$, a

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11 In similar models (e.g. Mathis, McAndrews and Rochet (2009) and Benabou and Laroque (1992)) there is always an interval of parameter values for which a strategic type becomes less likely to mimic the committed type as the reputation for commitment improves.
strategic agency exerts effort (with positive probability) if \( \mu < \overline{\mu} \), but shirks with probability 1 if \( \mu > \overline{\mu} \).

If instead fees are up-front, the incentive to shirk is negatively correlated with \( \mu \), as a strategic agency with strong reputation has ‘a lot to lose’. For \( \delta < \hat{\delta} \), a strategic agency would not be willing to incur the cost \( \kappa \) even if \( \mu = 1 \). For any reputation \( \mu < 1 \), the incentive to exert effort is even smaller, and a strategic agency shirks. For \( \delta > \hat{\delta} \), whenever the agency enjoys a strong reputation, investors and firms expect a strategic agency to exert effort, the rating is valuable, and the agency’s revenues are high. As a result, a strong reputation is extremely valuable. Suppose for a moment that when a strategic agency has a weak reputation it is expected to shirk. In this case, the expected future revenues following a decision to exert effort would be larger than the payoff from shirking. As a result, shirking cannot be part of the equilibrium and \( e = 1 \) as long as \( \mu > 0 \).

4. Policy Analysis

A rating is correct if the agency exerts effort and it reports honestly. Assumption 1 ensures that the value for investors of a correct rating is larger than a strategic agency’s cost of acquiring information. As a result, in each period ex ante total surplus is positively correlated with the probability of effort. The function \( e(.) \) is indeed the unique determinant of ex ante total surplus, and in order to evaluate a regulation, it is sufficient to consider its effect on \( e(.) \). The next proposition shows that the entire set of regulations can be easily evaluated.

**Proposition 3.** For any set of parameter values, \( \delta < \hat{\delta} < \delta \). If \( \delta \leq \delta \) or \( \delta \geq \delta \), regulations have no effect on total surplus. If \( \delta \in (\hat{\delta}, \delta) \), every regulation that imposes \( \gamma > 0 \) reduces total surplus. If \( \delta \in [\hat{\delta}, \delta) \), a regulation requiring contingent fees (\( \gamma = 1 \)) increases total surplus and ensures (weakly) larger total surplus than any other regulation.
For extreme values of $\delta$, a strategic agency with reputation $\mu > 0$ chooses either $e = 0$ (if $\delta \leq \tilde{\delta}$) or $e = 1$ (if $\delta \geq \tilde{\delta}$) regardless of the fee structure, and regulations have no effect on expected total surplus. If instead $\delta \in (\tilde{\delta}, \hat{\delta})$, the regulator should let the agency decide the fee structure. Any regulation requiring $\gamma > 0$ decreases total surplus as it results in $e^\gamma \leq e$ where the inequality holds strictly if $\mu \in (0, \mu)$. Finally, if $\delta \in [\hat{\delta}, \delta)$, the regulator should impose up-front fees. In this parameter region, a regulation imposing $\gamma = 1$ increases total surplus as $e^1 > e$ for any $\mu > 0$, and it is never worse than other regulation: $e^1 \geq e^\gamma$ for any $\gamma$ and any $\mu$, where, for each $\gamma < 1$ the inequality holds strictly if $\delta \in (\hat{\delta}, \tilde{\delta})$.

Proposition 3 contains a rather surprising result: contingent fees can ensure higher total surplus than regulated fees. In order to explain this result, I compare up-front and contingent fees. ‘Intermediate’ regulations ($\gamma \in (0, 1)$) are in any case never the unique optimal policy. I proceed in two steps. First, I discuss why the regulator might prefer contingent fees to up-front ones. Then, I consider why this is the case for small (but not too small) $\delta$.

A regulation that imposes up-front fees has two effects on the incentive to exert effort. The first effect is static. Unlike contingent fees, up-front fees do not require that the agency assigns a high rating to be collected. As a result, up-front fees have a static effect that curbs the incentive to shirk. The second effect is dynamic. Regardless of whether fees are up-front or contingent, when the agency exerts effort, it does so in order to improve its reputation for commitment, because a stronger reputation ensures larger expected revenues in the subsequent periods. In any given period, the expected revenue of a strategic agency is more responsive to reputation changes under contingent than under up-front fees, as shown in Figure 5. Up-front fees make reputation less valuable and therefore have a dynamic effect that increases the incentive to shirk.

The effect of contingent fees on the value of reputation deserves further explanation. Regardless of whether fees are up-front or contingent, the agency sets the fee amount to extract all the expected surplus from the active firm. This implies that for any $\mu$, as long as $e(\mu) = e^1(\mu)$, the active firm expects to pay the same amount whether fees are up-front or contingent. Nevertheless, the two fee structures ensure different revenues for a strategic agency. If fees are
paid up-front, a strategic agency earns the same revenue as a committed one. If instead fees are contingent, whenever $e>0$, a strategic agency is more likely to obtain the fee than a committed agency, and the difference in expected revenue between the two types of agency widens as $\mu$ increases. As a result, whenever the reputation of the agency improves, the revenue of a strategic agency increases at a faster rate under contingent fees than under up-front ones.

Static and dynamic effect work in opposite directions and a regulation requiring up-front fees decreases total surplus whenever its dynamic effect is stronger than its static one. Moreover, the dynamic effect is absent whenever the strategic agency is expected to set $e=1$, as whenever the strategic type mimics the committed one, the reputation of the agency remains unchanged over time. As a result, up-front fees result in a larger interval of discount factors for which $e=1$ is an equilibrium strategy. For small discount factors instead, investors conjecture that a strategic agency might shirk, the rating fee is relatively small, and the static effect is weak. When the strategic type is likely to shirk, $l$ ratings considerably improve $\mu$ and therefore the dynamic effect is strong. As a result, contingent fees dominate up-front fees for low $\delta$.

Proposition 3 shows that the relative merits of regulated fees depend on $\delta$ and the thresholds $\delta$, $\hat{\delta}$ and $\hat{\delta}$. These thresholds, in turn, depend on the cost of the informative signal, as shown in Figure 6. When $\kappa=0$, my setting does not differ from Mathis, McAndrews and Rochet (2009): as long as fees are contingent, a strategic agency might shirk ($\delta>0$ even if $\kappa=0$).

In contrast, up-front fees ensure that the agency, regardless of its discount factor, always collects the informative signal: if $\kappa=0$ then $\hat{\delta}=0$. Nevertheless, as $\kappa$ increases, the interval of discount factors $[\hat{\delta},\delta)$, for which contingent fees ensure higher total surplus than up-front fees, becomes larger.

These comparative statics show how the trade-off discussed above hinges crucially on the presence of a cost to obtain informative signals. In the aftermath of the recent financial crisis, researchers and policymakers have highlighted agencies’ lack of investment in information.

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12Mathis, McAndrews and Rochet (2009), footnote 8 contains a similar comparative static, for the case of $\kappa=0$. 
acquisition as one of the main causes of incorrect ratings of structured financial products (e.g. US Senate (2011)).

5. EXTENSION: SUPERVISION OF THE RATING PROCESS

In this section, I assume that the regulator supervises the agency and effectively enforces information acquisition. Whenever this is the case, the potential trade-off between up-front and contingent fees ceases to hold, as up-front fees eliminate all incentives to shirk or lie. Supervision of the rating process is indeed on regulators’ agenda. For example, the Dodd Frank Act introduced new regulation for rating agencies (US Senate (2010)). The Act allows the Securities and Exchange Commission (SEC) to:

... temporarily suspend or permanently revoke the registration of a nationally recognized statistical rating organization with respect to a particular class or subclass of securities, if the Commission (SEC) finds ... that the nationally recognized statistical rating organization does not have adequate financial and managerial resources to consistently produce credit ratings with integrity. (US Senate (2010): 499).

Suppose that the regulator observes the effort choice of the agency and requires the agency to always exert effort. If the agency shirks once, it cannot collect the fee in the current period, nor assign a rating, and is forbidden from operating in the future. In this setting, I compare a market in which the agency can set any fees with a market in which the regulator requires the agency to collect up-front fees.

In a setting in which effort is not observable, I argued that is without loss of generality to assume that a strategic agency reports honestly. This is not true in the current setting. In this section, I explicitly consider the strategic agency’s decision to disclose unfavorable hard information.
If the fee structure is not regulated, contingent fees are the equilibrium choice of a strategic agency, and a discount factor sufficiently larger than zero is necessary to ensure that the strategic agency discloses unfavorable information.

**Proposition 4.** Suppose a regulator supervises effort, and the fee structure is not regulated. An equilibrium in which the agency exerts effort and reports honestly in every period exists if and only if \( \delta \geq \delta^s := \frac{1-\lambda}{1-\lambda^2-\kappa} \).

Assumption 1 ensures that \( \delta^s > 0 \). If the discount factor is small (\( \delta < \delta^s \)), a strategic agency demands contingent fees. In this parameter region, a strategic agency either exerts effort but with positive probability fails to disclose the unfavorable hard information or else it shirks. If the agency shirks, it is required by the regulator to abandon the market.

In contrast, under up-front fees, effort and honest reporting can be sustained for any discount factor.

**Proposition 5.** Suppose a regulator supervises effort, and regulation \( \gamma = 1 \) is in place. For any \( \delta > 0 \), the agency exerts effort and reports honestly in every period.

As the agency’s revenue does not depend on the rating assigned, a strategic agency has no incentive to deviate from honest reporting. Up-front fees eliminate all incentives to shirk or lie and ensure the first best outcome. This result, together with Proposition 4, shows that up-front fees and supervision of the rating process are *complements*. Supervision of the rating process ensures that a regulation requiring up-front fees is never counterproductive, while at the same time up-front fees ensure that supervision will result in honest ratings even if the agency weighs heavily current revenues.

In a static setting, Bolton, Freixas and Shapiro (2012) show that if the agency’s effort is observable and contractible, up-front fees induce more effort than contingent ones.\(^{13}\) My contribution consists in showing that in a *dynamic* setting, whenever effort is enforced, the undesirable dynamic effect of up-front fees is always weaker than their desirable static effect.

\(^{13}\)See footnote 35 in Bolton, Freixas and Shapiro (2012).
I model the market for ratings as a game in which short-lived firms can hire a long-lived monopolistic rating agency to rate their projects. Firms and investors do not observe whether the agency acquires costly information about the projects it rates. I use the model to study the effect of different ways to compensate a rating agency. I compare a market, in which the agency demands fees contingent on the rating assigned, with regulated markets, in which the agency must charge a fraction or the entire fee regardless of the rating assigned. I show that these regulations can be counterproductive, and reduce the quality of the ratings. In particular, whenever the agency’s discount factor is sufficiently small, the agency is less likely to exert effort as a result of these regulations.

Regulations that prohibit contingent fees are counterproductive exactly in the markets in which reputation incentives are weakest. On a positive note, when matched with regulatory supervision of the rating process, up-front fees ensure that the agency always collects the informative signal.

The model could be extended to consider the effect of competition among rating agencies and to allow for repeated interaction between firms and the rating agency. The previous work on the effect of market structure on the quality of ratings includes, among others, Strausz (2005); Faure-Grimaud, Peyrache and Quesada (2009); Bolton, Freixas and Shapiro (2012); Doherty, Kartasheva and Phillips (2012); Bouvard and Levy (2013); Hirth (2014); and Bizzotto (2014). This literature shows how specific features of the market for ratings determine whether competition is feasible and desirable. Along these lines, it would be interesting to evaluate the effect of the entry of new raters when different compensation schemes are in place.

In Frenkel (2015), whenever firms hire the same agency multiple times, the agency has an incentive to inflate ratings. My framework could be extended to study how compensation schemes compare in markets in which firms repeatedly hire the same rating agency.
Proof of Lemma 1.
Let the reputation in period \( t \) be \( \mu_t=0 \). Assume there is an equilibrium in which firm \( t \) hires the agency, and the agency is expected, by the active firm and the investors, to choose \( e_t>0 \). In such a candidate equilibrium, \( \mu_{t+1}=0 \) regardless of \( r_t \). A strategic agency maximizes its payoff by choosing \( e_t=0 \), which is a contradiction. Therefore, in any equilibrium the active firm hires the agency only if \( \phi_t=0 \). As a result, the agency does not earn any revenue and \( V(0)=0 \).

Proof of Lemma 2.

Step 1. Active firms hire the agency if \( \{\phi, \gamma\} = \{v(\mu, e) - \epsilon, 0\} \) where \( \epsilon > 0 \).
This is the case as for \( \{\phi, \gamma\} = \{v(\mu, e) - \epsilon, 0\} \) a firm prefers to hire regardless of its belief about the agency’s type or the agency’s effort choice.

Step 2. As long as \( \mu > 0 \), the agency sets a fee such that the firm hires the agency.
By step 1, as long as \( \mu > 0 \) the agency earns a positive payoff. Therefore a strategy that requires the agency to always set a fee for which the active firms do not hire the agency cannot be optimal. Similarly, a strategy in which for \( n \) periods the agency sets fees for which active firms do not hire is not optimal. By deviating and following the prescription of the original strategy for period \( n+1 \) in period \( n \), the agency ensures a higher expected payoff.

Step 3. If \( e=1 \), any \( \gamma \) can be chosen in equilibrium, as long as \( \phi = \frac{\lambda(1-\lambda)}{(\gamma + \lambda(1-\gamma))} \).
Suppose \( e=1 \). By Step 1, every fee demanded with positive probability ensures a payoff at least as large as \( \{\phi, \gamma\} = \{v(\mu, 1), 0\} \) does. For any \( \gamma \in [0,1] \), \( \phi \) such that \( \phi = \frac{\lambda(1-\lambda)}{(\gamma + \lambda(1-\gamma))} \) does ensure such a payoff, while for \( \phi > \frac{\lambda(1-\lambda)}{(\gamma + \lambda(1-\gamma))} \) the agency is not hired. For any \( \gamma \in [0,1] \), if the agency sets \( \{\phi, \gamma\} \in F^1 := \{\{\phi, \gamma\} | \phi = \frac{\lambda(1-\lambda)}{(\gamma + \lambda(1-\gamma))} \} \), the active firm is indifferent to hire the agency, but in equilibrium it does hire. As a result, the fee satisfies \( \{\phi, \gamma\} \in F^1 \).

Step 4. If \( e<1 \) then a strategic agency sets \( \{\phi, \gamma\} = \{v(\mu, e), 0\} \).
Let \( e(\phi, \gamma) \) denote the effort when the fee is \( \{\phi, \gamma\} \). Suppose there exists a \( \{\phi', \gamma'\} \) demanded with positive probability by a strategic agency, such that \( \gamma' > 0 \) and \( e(\phi', \gamma') < 1 \).
Suppose that all \( \{\phi', \gamma'\} \) demanded with positive probability by a strategic type such that \( \gamma' > 0 \), and \( e(\phi', \gamma') < 1 \) are demanded only by that type. Each \( \{\phi', \gamma'\} \) must ensure at least the same payoff as setting \( \{\phi, \gamma\} = \{v(\mu, e), 0\} \) and choosing \( e(\phi', \gamma') \). Therefore:

\[
\gamma' \phi' + (1 - \gamma')(\lambda + (1 - \lambda)(1 - e(\phi', \gamma')))(\phi' \geq (\lambda + (1 - \lambda)(1 - e(\phi', \gamma'))))v(\mu, e(\phi', \gamma')).
\]

In such a separating equilibrium, a committed type’s current payoff cannot exceed \( \lambda v(\mu, e(\phi', \gamma')) \).

But, by deviating to any \( \{\phi', \gamma'\} \), a committed agency gains, as it ensures:

\[
\lambda(v(\mu, e(\phi', \gamma')) - \alpha) + (1 - \lambda)(1 - e(\phi', \gamma'))(v(\mu, e(\phi', \gamma')) - (1 - \gamma')\phi') > \lambda v(\mu, e(\phi', \gamma')).
\]

The inequality holds as \( \gamma' > 0 \) implies \( (1 - \gamma')\phi' < v(\mu, e(\phi', \gamma')) \). If a strategic agency demands a fee such that \( \gamma' > 0 \), and \( e(\phi', \gamma') < 1 \), then a fee \( \{\phi'', \gamma''\} \) demanded with positive probability by both types such that \( \gamma'' > 0 \) and \( e(\phi'', \gamma'') < 1 \) must exist.

Suppose such a fee exists. The firm hires the agency only if \( \phi'' \leq \phi''' \) where:

\[
\gamma'' \phi''' + (\lambda + (1 - \lambda)(1 - \mu''')(1 - e(\phi'', \gamma''')))(1 - \mu(\phi'')) = \lambda + (1 - \lambda)(1 - \mu''')(1 - e(\phi'', \gamma''')),
\]

and \( \mu'' \) is the firm’s belief upon observing fee \( \{\phi'', \gamma''\} \). Such a fee cannot be part of an equilibrium as a strategic agency profits from deviating to \( \{\phi, \gamma\} = \{v(\mu, e(\phi', \gamma')) - \epsilon, 0\} \), for \( \epsilon \)

small enough and choosing \( \epsilon = \epsilon(\phi', \gamma') \)

**Step 5.** If \( \epsilon < 1 \) then a committed agency sets a fee such that \( \phi = \frac{\lambda v(\mu, e)}{\gamma + (1 - \gamma)\lambda} \).

By step 4, if \( \epsilon < 1 \), a strategic agency sets \( \{\phi, \gamma\} = \{v(\mu, e), 0\} \). Assume for a moment that there is an equilibrium in which the committed type chooses a different fee. In such an equilibrium, for any \( \gamma \), the highest fee amount that the committed agency can charge is \( \phi = \frac{\lambda v(\mu, e)}{\gamma + (1 - \gamma)\lambda} \). For such a fee amount, any \( \gamma \in [0, 1] \) ensures the same payoff to the committed type.

**Proof of Proposition 1.**

In order to characterize \( e(\mu) \), it is without loss of generality to consider only equilibria in which \( \{\phi, \gamma\} = \{v(\mu, e), 0\} \). This is the case as (a) Lemma 2 ensures that \( \gamma = 0 \) is always optimal and (b) if a strategic agency has no profitable deviation from \( e = 1 \) for \( \{\phi, \gamma\} = \{1 - \lambda, 0\} \), then it has no profitable deviations for any \( \{\phi, \gamma\} \) such that \( \gamma > 0 \) and \( \phi < 1 - \lambda \).
The one-stage deviation principle ensures that there are no profitable deviations from $e$ if and only if:

$$
(e'-e) \left( \phi(\mu,e)+\frac{\kappa}{1-\lambda} \right) \geq \delta (e'-e) \left( V(\mu^l(e)) \right) \quad \forall e'.
$$

(6.1)

**Step 1. Existence of equilibrium for $\delta \geq \delta_\ast$.**

Let $e(\mu)=1 \forall \mu > 0$. Then $\forall \mu > 0$: $V(\mu)=\frac{\lambda(1-\lambda)-\kappa}{1-\delta}$, and if $\delta \geq \delta_\ast$, then Condition (6.1) holds.

**Step 2. (Generic) uniqueness of equilibrium for $\delta \geq \delta_\ast$.**

Let $\delta \geq \delta_\ast$. First, for $\delta > \delta_\ast e(1)=1$, while if $\delta = \delta_\ast$ any $e(1)$ ensures the same payoff and can be part of an equilibrium (on the equilibrium path, $\mu < 1$ if the agency is strategic). Suppose there exists an equilibrium in which $e(\mu) < 1$ for some $\mu \in (0,1)$. $\delta \geq \delta_\ast$ implies that $V(1)=\frac{\lambda(1-\lambda)-\kappa}{1-\delta} > v(\mu,0)+\frac{\kappa}{1-\lambda}$, and therefore $e(\mu) > 0$. Then (6.1) requires: $v(\mu,e)+\frac{\kappa}{1-\lambda}=\delta V(\mu^l(e))$. The equality is satisfied only if $e(\mu^l) < 1$, so it can be written as $v(\mu,e)+\frac{\kappa}{1-\lambda}=\delta v(\mu^l(e),e(\mu^l(e)))$. Therefore such an equilibrium requires the existence of a sequence: $x_{n+1}=\frac{(1-\delta)}{1-\lambda} + \frac{1-\lambda}{\lambda} x_n$, where $x_i \in [0,1-\lambda] \forall i \in \{1,2,\ldots\}$. $\delta \geq \delta_\ast$ implies $\frac{(1-\delta)}{(1-\lambda)} < \frac{(1-\lambda)}{\lambda}$ and $\frac{1-\lambda}{\lambda} < \frac{(1-\lambda)^2}{(1-\lambda)+\kappa}$. This implies that if $x_1 < 1-\lambda$, then $\lim_{n \to \infty} x_n < 0$, which is a contradiction.

**Step 3. Existence and uniqueness of equilibrium for $\delta < \delta_\ast$.**

**Property 3.** If $\delta < \delta_\ast$, then in equilibrium $e(\mu) \leq \bar{e} < 1 \forall \mu$ where $\bar{e}$ is defined by $\frac{\kappa}{1-\lambda}+v(0,\bar{e})=\delta f(1,1)$. Property 3 is implied by (6.1).

If $\delta = \delta_\ast$, then $\bar{e} = 0$. Therefore, by Property 3, if $\delta \leq \delta_\ast$ then $e(\mu)=0 \forall \mu \in (0,1)$. If instead $\delta \in [\delta_\ast, \delta_\ast)$, then a unique $\mu \in (0,1)$ exists such that (6.1) holds as an equality for $\mu = \mu$ and $e=0$. For $\mu \geq \mu$, Property 3 requires $e(\mu)=0$. Therefore the set of candidate equilibrium value functions is:

$$
W:=\left\{ w: (0,1] \to \mathbb{R} | w(\mu)=\frac{v(\mu,e)}{1-\delta \lambda}, e(\mu) \leq \bar{e} \forall \mu, \text{ and } e(\mu)=0 \text{ if } \mu \geq \mu \right\}.
$$

**Property 4.** A unique $w$ exists.

Let $m_1$ be implicitly defined by $m_1^l(\bar{e})=\mu$. $m_1$ exists, is unique, and $m_1 \in (0,\mu)$. For any $\mu \in [m_1,\bar{\mu}]$, $\mu^l(e) \geq \mu, \forall e \leq \bar{e}$. Moreover, as $w(x)$ is uniquely defined and increasing for $x \in [\bar{\mu},1]$,
then for any \( \mu \in [m_1, \bar{\mu}] \) a unique \( e(\mu) \in [0, \bar{\mu}] \) exists that satisfies (6.1). Therefore \( w(x) \) is uniquely defined for \( x \in [m_1, 1] \).

Let \( \mu_a, \mu_b \in [m_1, \bar{\mu}] \) and \( \mu_a > \mu_b \). Suppose \( v(\mu_a, e(\mu_a)) \leq v(\mu_b, e(\mu_b)) \), which implies \( e(\mu_a) < e(\mu_b) \) and \( \mu_a' (e(\mu_a)) > \mu_b' (e(\mu_b)) \). But \( \mu_b' (e(\mu_b)) \leq \bar{\mu} \), so \( w(\mu_a' (e(\mu_a))) > w(\mu_b' (e(\mu_b))) \), which implies that (6.1) cannot be satisfied by \( e(\mu_a) \) for \( \mu = \mu_a \) and \( e(\mu_b) \) for \( \mu = \mu_b \) at the same time, which is a contradiction. Therefore \( w \) is increasing in the interval \([m_1, 1]\).

The same steps used to characterize \( w \) in \([m_1, 1]\), can be used to show that \( w(\mu) \) is unique and increasing if \( \mu \in [m_n, 1] \) for some \( m_n \), where the sequence \( m_i \) for \( i = 1, 2, \ldots \) is recursively defined: \( m_{i+1} \) is implicitly defined by \( m_{i+1}' (\hat{\epsilon}) = m_i \). Moreover, as \( \tilde{\epsilon} < 1 \), \( \lim_{i \to \infty} m_i = 0 \), \( w \) is uniquely defined and increasing \( \forall \mu \in (0, 1] \). This proves Property 4 and Step 3.

**Proof of Proposition 2.**

The proof follows the same steps as the proof of Proposition 1. The only differences are that in a regulated market: \( \phi^\gamma \) satisfies (3.4), and the one-stage deviation principle ensures that there are no profitable deviations from \( e \) if and only if:

\[
(e' - e) \left( (1 - \gamma) \phi^\gamma + \frac{\kappa}{1 - \lambda} \right) \geq \delta (e' - e) V^\gamma (\mu_i'(e)) \quad \forall e' \in [0, 1].
\]

**Proof of Proposition 3.**

Note that \( \delta < \delta' \leq \delta' < \delta' \forall \gamma > 0 \). As a result, if \( \delta \leq \delta \) then \( e(\mu) = e^\gamma (\mu) = 0 \) \( \forall \gamma > 0 \) and \( \forall \mu \in [0, 1] \), while if \( \delta \geq \delta \) then \( e(\mu) = e^\gamma (\mu) = 0 \) \( \forall \gamma > 0 \) and \( \forall \mu \in [0, 1] \). Regulations, therefore, have no effect on effort or total surplus if \( \delta \leq \delta \) or \( \delta \geq \delta \).

I now consider the case of \( \delta \in (\tilde{\delta}, \delta) \).

**Property 1.** Suppose \( \delta < \delta \). Let \( \mu^\gamma : = \mu^\gamma (e^\gamma (\mu)) \). For any \( \gamma > 0 \), \( e(\mu^\gamma) = e^\gamma (\mu^\gamma) \), implies \( e(\mu) > e^\gamma (\mu) \) \( \forall \mu \). Suppose this is not the case, then \( \exists \mu \) such that \( e(\mu^\gamma) = e^\gamma (\mu^\gamma) \), or equivalently:

\[
\frac{\delta d(\mu^\gamma)}{(1 - \delta \lambda) (\gamma + (1 - \gamma) d(\mu^\gamma)) \left( \frac{\lambda}{d(\mu^\gamma, e(\mu^\gamma))} - \lambda \right)} = \frac{\kappa}{1 - \lambda} + \frac{(1 - \gamma) d(\mu)}{(\gamma + (1 - \gamma) d(\mu)) \left( \frac{\lambda}{d(\mu)} - \lambda \right)}, \tag{6.2}
\]

where \( d(\mu) : = \lambda + (1 - \lambda)(1 - \mu)(1 - e^\gamma (\mu)) \), and \( e(\mu) \leq e^\gamma (\mu) \), or equivalently:
\[
\frac{\delta}{1-\delta \lambda} \left( \frac{\lambda}{d(\mu^{l\gamma})} - \lambda \right) \leq \frac{\kappa}{1-\lambda} + \left( \frac{\lambda}{d(\mu)} - \lambda \right).
\]  

(6.3)

Using (6.2) to substitute for \( \left( \frac{\lambda}{d(\mu)} - \lambda \right) \) in (6.3):

\[
\frac{\kappa(1-\delta \lambda)}{\delta(1-\lambda)\lambda(1-\lambda)} \leq \left( 1 - (1-\mu^{l\gamma})(1-e(\mu^{l\gamma})) \right) \left( \frac{1+\gamma(1-\mu^{l\gamma})}{\gamma(1-\gamma)d(\mu^{l\gamma})} \right).
\]

(6.4)

Note that, as \( V^{\gamma} \) is increasing in \( \mu \), then \( \mu^{l\gamma} \geq \mu \) implies \( d(\mu^{l\gamma}) \leq d(\mu) \) and \( \frac{1+\gamma(1-\mu^{l\gamma})}{\gamma(1-\gamma)d(\mu^{l\gamma})} < 1 \). As a consequence the right hand side of (6.4) is (weakly) smaller than 1, and therefore (6.4) holds only if \( \frac{\kappa(1-\delta \lambda)}{\delta(1-\lambda)\lambda(1-\lambda)} \leq 1 \), or equivalently \( \delta \geq \hat{\delta} \). This contradiction proves Property 1.

Property 1 implies that, if \( \delta \in (\delta \gamma) \) then \( \forall \gamma > 0, \forall \mu > \hat{\mu} \) and, for any \( \mu \leq \hat{\mu} \), \( e(\mu) > e^{\gamma}(\mu) \). This in turn implies that, for \( \delta \in (\delta \gamma) \), every regulation reduces total surplus.

For \( \delta \in (\delta \gamma) \), the proof is immediate: if \( \delta \geq \hat{\delta} \), then \( e^{\gamma} = 1 \). Moreover \( \hat{\delta} < \delta \) and \( \hat{\delta} < \delta \gamma \) for any \( \gamma < 1 \). So \( \exists \delta \in (\delta \gamma) \) for which \( e(\mu) < 1 \) and/or \( e^{\gamma}(\mu) < 1 \) for some \( \mu \).

Proof of Proposition 4.

Step 1: Existence of equilibrium with \( e=1 \) and honest reporting for \( \delta > \delta^* \).

The strategic agency’s reporting strategy is denoted as a function \( a:[0,1] \rightarrow [0,1] \), where \( a(x) = Pr(r=l|s=l, \mu=x) \). Consider the following equilibrium strategy of the agency. The agency sets \( \phi(\mu)=v(\mu,1) \) and \( \gamma(\mu) = 0 \ \forall \mu \). A strategic agency chooses \( e(\mu)=a(\mu)=1 \ \forall \mu > 0 \), and \( e(0)=a(0)=0 \). For this strategy: (a) the highest bid is equal to \( 1-\lambda \) as long as \( \mu > 0 \), (b) if \( \mu > 0 \), the active firm hires the agency, and (c) \( V(0)=0 \).

Suppose \( s=l \). If \( r=l \) a strategic agency ensures continuation payoff \( x:=\delta \frac{(1-\lambda) - \kappa}{1-\delta} \). If instead \( r=h \), then a strategic agency ensures continuation payoff \( 1-\lambda \). A deviation to \( r=h \) is not profitable if and only if \( \delta \geq \delta^* \).

By exerting effort, a strategic agency ensures continuation payoff \( x > 0 \). By shirking, the strategic agency ensures a continuation payoff of 0. Therefore \( \forall \mu > 0 \) there is no incentive to choose \( e(\mu) < 1 \).
For any $\phi > v(\mu, 1)$, the active firm would not hire the agency. As the fee extracts all the surplus from the active firm, the agency has no incentive to deviate to a different fee.

Step 2: Non-existence of equilibrium with $e=1$ and honest reporting for $\delta > \delta^s$. Suppose now that $\delta < \delta^s$. Consider a candidate equilibrium strategy which prescribes $e(\mu) = a(\mu) = 1$ for some $\mu > 0$. Then if in period $t$, $\mu_t = \mu$ and the strategic agency follows the equilibrium strategy, then highest possible continuation payoff is $x$. If instead the strategic agency sets $\gamma = 0$, $\phi = 1 - \lambda - \epsilon$ for some small $\epsilon > 0$ and chooses $e = 1$ and $a = 0$, it ensures a payoff indefinitely close to $1 - \lambda - \kappa$.

This is the case because as long as $\gamma = 0$ and $\phi < v(\mu, e)$ the active firm hires the agency regardless of its belief. $\delta < \delta^s$ implies $1 - \lambda - \kappa > x$ and the agency profits from the deviation.

Proof of Proposition 5.

Consider the following equilibrium strategies: every period $\phi = \lambda(1 - \lambda)$. The strategic type chooses $e^1 = a^1 = 1$ as long as $\mu > 0$, $e^1(0) = 0$ and $a^1(0) = 0$. Every firm hires the agency. If $\mu > 0$, the agency does not gain from deviating from honest reporting and Assumption 1 ensures that the agency does not gain from shirking (if the agency shirks it cannot collect the fee and it must leave the market). Additionally, neither type of agency gains from demanding a different fee. Restrictions 1 and 2 ensure that this equilibrium is generically unique. As $\mu^0 > 0$, in equilibrium that agency exerts effort and reports honestly in every period.

Appendix B

Expression (3.2) imposes three restrictions on out-of-equilibrium-path beliefs:

(a) $\psi(h, 0|1, e) = 0 \forall e$,

(b) $\psi(\emptyset, 0|0, e) = 0 \forall e$,

(c) $\psi(0, 1|\mu, e) = \psi(0, 0|\mu, e) = \mu \forall \mu, e$.

Lemma 1b. It is without loss of generality to focus on equilibria in which beliefs satisfy restrictions (a), (b) and (c).

Proof of Lemma 1b.
a) If the agency is strategic, on the equilibrium path the reputation of the agency always satisfies $\mu < 1$. Suppose there is an equilibrium in which $\psi(h,0|1,e) > 0$. In this equilibrium, a strategic agency does not gain from deviating from the equilibrium strategy and ensuring reputation $\mu = 1$. Let the value function for $\mu = 1$ in this equilibrium be $V(1)$. Let $V^0(1)$ be the value function for $\mu = 1$ in a candidate equilibrium identical to the original one in every respect, with the exception that $\psi(h,0|1,e) = 0 \forall e$. As according to Lemma 1 $V(0) = 0$, it must be the case that $V(1) \geq V^0(1)$. Therefore, if a strategic agency does not gain from deviating from the original equilibrium strategy and ensuring reputation $\mu_1$ when the value function for $\mu = 1$ is equal to $V(1)$, then the agency would not want to deviate to ensure reputation $\mu = 1$ in the candidate equilibrium, as $V^0(1) < V(1)$. Therefore, the candidate equilibrium is indeed an equilibrium.

b) As stated in Lemma 1, $e(0) > 0$, is not part of an equilibrium. It is, therefore, without loss of generality to assume $\psi(\emptyset,0|0,e) = 0 \forall e$.

c) Assume an equilibrium exists in which beliefs do not satisfy (c). In this equilibrium, it must be the case that the agency’s expected continuation payoff is non-negative in each period, as long as $\mu > 0$. If this was not the case, the agency would ”leave the market” by setting forever on fees higher than the firms’ willingness to pay. This equilibrium strategy would, in turn, violate Restriction 1. Consider a candidate equilibrium identical to the original, with one exception: in the candidate equilibrium $\psi(\emptyset,1|\mu,e) = \psi(\emptyset,0|\mu,e) = \mu$. In this candidate equilibrium, if $\mu > 0$, the agency, regardless of its type, never prefers to set a fee higher than the firm’s willingness to pay. This is the case as, if the agency chooses this fee, its reputation is left unchanged and its sequence of non-negative expected payoffs are postponed by one period. This in turn implies that the candidate equilibrium is indeed an equilibrium and it ensures the same payoffs and outcomes as the original equilibrium.
References


Figures

Figure 1. Time line.

Figure 2. Effort probability $e$ in an unregulated market.
Figure 3. Effort probability $e$ under regulation $\gamma = 0.5$.

Figure 4. Effort probability $e$ under regulation $\gamma = 1$.

Figure 5. One-period revenue of a strategic agency in case of up-front and contingent fees (for $e = e^1$).
Figure 6. Thresholds discount factors as a function of $\kappa$. 

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure6}
\end{figure}