Alleviating poverty: A proposal to mitigate the economic cost of disease

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Abstract

We develop an overlapping generations model to examine how illness affects a household's long-term economic condition through its immediate implication on the household's decision to invest in their child's education. A prolonged period of illness or premature death of an earning member reduce household income, which in turn adversely affects a household's ability to invest in child education. Low investment in child education contributes to low household income in the future. Thus illness can generate a low-income low-education trap. We show how a policy of providing alternate education in skilled care-giving to those who were forced to forgo their formal education to take care of ill family members, could help economically devastated families to escape the trap.

1 Introduction

The World Health Organization (WHO) recognizes disease as one of the primary challenges to reducing poverty.[1] An immediate implication of illness is a loss of earnings and increased expenditure for medical care (Howarth et al. [1991]; Booysen [2003]). In addition, the long-term impact of disease on economic development can be serious. The education and well-being of children in disease-affected

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[1]See The WHO Commission on Macroeconomics and Health (CMH) report *Investing in Health for Economic Development* [2001]; Sachs [2005].

households can suffer profoundly. Studies (Miguel [2005], Mutangadura [2000], Topuzis [1994]) show that children in disease-affected families are less likely to remain in school. Children may be needed at home to take care of the patient when families cannot bear the costs of professional nursing or families may be unable to afford school fees. Lack of education results in low future income for households. In this fashion illness can trap families in poverty. The severity of this trap multiplies if households are deprived of healthy living conditions and are therefore at greater risk for disease. These relationships can explain the strong correlation between poverty, poor health, and lack of education across nations (see UNDP Report [2004], Gan and Gong [2007]).

The implication of illness on future generations' education and wealth are less explored in development studies.[2] In this paper, we investigate how illness adversely affects education and income. In an overlapping-generations model, we allow income and investment in education to evolve simultaneously. This is done by introducing investment in education as a strategic decision variable in our model. In each period, a household (the current generation) makes a single decision: whether or not to invest in child's (the future generation) education. Education yields a high return in the form of higher earnings in the next period.[3] In our model, the opportunity cost of education has two components—a financial cost and foregone earnings. Our analysis shows that as long as the return from education does not increase as income decreases, a household's optimal strategy will be to invest in education if and only if the income in the current period is sufficiently high. Given this household behavior, we analyze the economic effect of illness.

We assume that illness causes a loss in potential earnings in the short run. This assumption is well documented by many recent observers. In a study conducted in South Africa, Booysen [2003] finds that households that experienced illness or death in the recent past were more than twice as likely to be poor than non-affected households. Howarth et al. [1991], and Menon et al. [1998], find similar evidence among AIDS-affected families in Zambia and HIV-affected families in Rakai, Uganda, respectively.[4]

We show that the short-term loss in income may lead households into a low-income and low-education trap when the loss affects their decision to invest in education. As a result of the short-term financial burden, relatively less affluent families may reverse their decision to send children to school

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[2]Traditional development theories explain the high correlation between health and education in one of three ways (Grossman [2000]). First, schooling results in better health either by increasing information about the true effects of the inputs on health (allocative efficiency as described in Kenkel [2000]) or because a more educated person obtains greater health output from given amounts of endogenous inputs (productive efficiency as described in Grossman and Kaestner [1997]). Second, the direction of causality may run from better health to more schooling (Auster, Leveson and Sarachek [1969]). The third theory argues that the presence of other variables - such as physical and mental ability and parental characteristics - affect both health and schooling in the same direction, thereby leading to a high correlation between health condition and education. None of the above theories incorporate the indirect effect of poor health on the household's decision to invest in a child's education.

[3]The World Development Report [1993] published by the World Bank finds that four years of primary education increases a farmer's annual productivity by 9 percent on average and workers who do better at school earn more. Studies in Ghana, Kenya, Pakistan, and Tanzania indicate that workers who scored 10 percent above the sample on various cognitive tests have a wage advantage ranging from 13 to 22 percent. On a related note, a study by Blattman and Annan [2007] on the effect of child soldiering in Uganda shows that a one-year loss of schooling leads to nearly a third lower earnings.

[4]In a study conducted in India, Basu, Gupta and Krishna [1997] report other social reasons behind the transition from wealth to poverty for families in which a male earning member died due to disease. In many families, female members do not join the workforce after the death of a spouse because the society considers it inappropriate for a woman to work outside the home.

if the main earning member suffers a prolonged period of illness or premature death. This missed opportunity contributes to a low expected income in the next period and therefore also adversely affects the household's decision to invest in education in subsequent generations. This low-income and low-education trap occurs mostly in resource-constrained economies. Further, since poor families often live in poor conditions and therefore have greater chances of disease, poor households are more likely to be trapped.

The findings from our model on the impact of disease on school dropouts and on the long-term poverty trap are confirmed by empirical studies. In a case study on AIDS-affected families throughout Zambia, Haworth et al. [1991] report that almost one quarter of children there have been forced to leave school. In another longitudinal study conducted in Rakai, Uganda, between 1989 and 1992, Menon et al. [1998] report a dropout rate of 40 percent in three primary schools over a period of four years. A World Bank study reports that in the United Republic of Tanzania, school attendance by students 15-20 years old was cut in half among households that had lost an adult female due to illness.[5] The impact of illness on long-term poverty is also documented in Booysen [2003]. Reporting on a study in South Africa, the author reveals that households that experienced illness or death in the recent past were more likely to experience long-term poverty.

Our model has implications for economic policies. In particular, we propose a novel policy to help individuals escape this poverty trap. This policy involves helping younger caregivers, who were forced to forgo their formal education for the sake of caring for their ill family members, to acquire an alternate education as trained, certified caregivers. This would allow them to bring their acquired skills to the market for future employment as paid caregivers. It would create an opportunity for economically devastated families of dying patients to escape the trap of poverty and at the same time increase the level of skilled care-giving available in the community. We demonstrate, using our model, how this policy could work. This policy may be especially suited in areas with particular features. For instance, it may be especially suited for areas where ill patients are cared for by their relatives due to unavailability of professional caregiving, and where serious illness is prevalent, and at the same time sufficient wealth exists in a portion of the households so that paid work as a caregiver can be found.[6] Professional care may be unavailable due to a shortage of professional health care workers, systemic deficiencies in the regional health care and insurance systems, hospitals being overwhelmed by the HIV/AIDS epidemic, or other features.

Our paper complements several works in the literature. Studies on health and education are summarized by Grossman [2000] and Grossman and Kaestner [1997]. In a recent study, Gan and Gong [2007] investigate the interdependence between health and education in a dynamic model. Related studies on investment in human capital and earnings are well compiled in Mincer [1974]. Our model and analysis differ substantially from those mentioned here because our focus is on the indirect effect of illness on income through the decision by households to invest in education. Barham et al. [1995] develop a dynamic model of income and education. Though the focus of their analysis is not on the economic effect of illness, our findings on household behavior are consistent with their observation that children of poor families are caught in a poverty trap because of an inability to finance their education. The importance of health care policies to reduce poverty has caught the attention of many economists

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[5]See also Mutangadura [2000], Kasawa [1993].

[6]Nearly 80 percent of sub-Saharan terminally ill patients are cared for by their spouses or their school-aged children, who very often lack the skills necessary to provide nursing adequately.

in recent years (Miguel [2005], Sala-i-martin [2005]). Dreze and Sen [2004] and Sachs [2005] point out disease and lack of efficient health care policies as causes of prolonged economic underdevelopment in sub-Saharan Africa and India respectively.

Our work also provides an explanation for the existence of a poverty trap. There are several theories of poverty traps in the literature. See, for instance, the articles in Bowles, Durlauf and Hoff [2006] and the references therein. Our argument is similar to the theory of 'threshold effect'.[7,8] According to the theory of threshold effect, there may exist a critical threshold of investment — in wealth or in human capital, at an individual level or at a macro level — that must be reached to achieve the ideal level of productivity. If the investment falls short of the critical threshold, the economy may be trapped into a low productivity-investment regime. The literature on the threshold effect of health capital (where health is considered as a human capital) has largely focused on the direct implication of health capital on schooling.[9] Unlike these models, we focus on the indirect effect of health capital of a particular generation on the education received by the following generations. Further, unlike earlier studies, we develop a microeconomic model of household decision making that feature heterogeneity of education and thus earnings.[10]

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 analyzes the dynamics of education and income. Section 4 analyzes the impact of disease on the socioeconomic relation between income and education. Section 5 discusses the policy implication and. Section 6 concludes.

2 The model

We begin by developing a model to study the relation between income and investment in education. In section 4, we develop our model to incorporate the effect of illness on the above relation.

2.1 Overlapping generations

Consider the decision-making problem of a single household in an overlapping-generations model. Time periods are denoted by $t = 0, 1, 2....$ In every period a new member of the household is born. Each member lives for two periods only - childhood (the first period of his life) and adulthood (the second period of his life). Therefore, in each period, there are two members in a household - a child and an adult member.

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[7] See Azariadis [2006] for a detailed discussion on the threshold argument.

[8] There are two other lines of argument for the persistence of poverty. In the first, dysfunctional social and political institutions are identified as a reason for creating stagnation in underdeveloped economies. See, for example, Bowles [2006], Engerman and Sokoloff [2006], Hoff and Sen [2006], Mehlum, Moene and Torvik [2006]. And the second category of explanations emphasizes the group effect where an individual's decision of investment is influenced by the action of his peer group. The consequence of such behavior may result in socially undesirable low-attainment equilibrium trap among subgroups in the population. See, for example, Austen-Smith and Fryer [2005], Durlauf [2006].

[9] See Grossman [2000] and the references therein.

[10] Caucutt and Kumar [2003] find a similar result on the relation between education and income. However, the focus of their analysis is not on studying the effect of illness on the dynamics of education and income.

2.2 During childhood

During an individual's childhood, the household decides whether to invest in educating the child. To keep our model simple and tractable, we assume there are only two levels of investment - costly investment or no investment. If the household invests in education, it incurs a cost, given by $s'' > 0$. This cost can be thought of as fees of schooling.[11] If the household does not invest in education, we assume that the member may join the workforce in the first period of his life, and earns $-s' > 0$. Let $s_t \in \{s', s''\}$ denote the investment in education for an individual born at period $t = 0, 1, \ldots$.

2.3 During adulthood

An individual born at period t enters adulthood at period $t+1$. During adulthood, he earns an income, which depends on his education and income of the adult member from the previous generation. Let m_{t+1} denote the income earned by the adult member at period $t + 1$. We assume that income is a positive real number. Furthermore, for given m_t and s_t, we assume that m_{t+1} is determined by a conditional probability density function $p(\cdot|m_t, s_t)$.

For any $n \geq 0$, let $q(n|m_t, s_t)$ denote the probability that an adult member earns at least n at period $t + 1$, given his education s_t and income of the adult member of the previous generation m_t. Hence,

$$q(n|m_t, s_t) = \int_n^\infty p(r|m_t, s_t)\, dr$$

We make the following assumptions about how education of an individual affects his income and how income of the previous generation is related to income of the current generation.

Assumption 1 *For given $n \geq 0$ and $m \geq 0$, $q(n|m, s'') > q(n|m, s')$.*

Assumption 2 *For given $n \geq 0$ and $s \in \{s', s''\}$, $q(n|m, s)$ is increasing in m.*

Assumption 1 implies that education is likely to result in higher income. This is a realistic assumption since education increases an individual's ability as well as his professional opportunities. On the other hand, the connection between the current generation's income and the previous generation's income is indirect in nature for our model. The previous generation's income affects the living condition and the health condition of the child, which in turn, affect his ability of earning when he enters adulthood. Assumption 2 implies that if two children from different households receive the same level of education, it is more likely that the child from the wealthier household would earn at least as much the child from the poorer household earns.

The following assumption states how effect of education changes as income varies.

Assumption 3 *For given $n \geq 0$ and $m^+ > m^- \geq 0$, $q(n|m^+, s'') - q(n|m^+, s') \geq q(n|m^-, s'') - q(n|m^-, s')$*

Assumption 3 states that the effect of education does not decreases as the income increases. For a given positive real number $n \geq 0$ and an income $m \geq 0$, the expression $q(n|m, s'') - q(n|m, s')$ measures the increase in probability that the child born in a household with income m in the current

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[11] In a broader sense, the cost may also include the cost of time spent by the family for child development.

period, would earn at least n in the following period, if the household invests in education. Therefore, $q\left(n|m,s''\right)-q\left(n|m,s'\right)$ reflects the value of education to a household with current income m. Consider two households with the adult members earning m^+ and m^- respectively and suppose $m^+ > m^-$. Assumption 3 implies that the value of education to the first household is not less than the value of education to the second household.[12] This assumption is critical in proving our main result on the existence of low-income trap. At the end of section 3, we further discuss the importance of this assumption in connection to the existence of low-income trap.

Let w_t denote the wealth of the household at period $t = 0, 1, \ldots$ and let it be defined as the income m_t earned by the adult member at period t minus the cost of education s_t of the child born at period t.

$$w_t = m_t - s_t \tag{2.1}$$

At any period $t = 0, 1, \ldots$, the household observes the income m_t and chooses the level of investment in education. A *decision rule* $d_t : R^+ \to \{s', s''\}$ prescribes a level of investment in education for a given positive level of income. A policy π specifies the infinite sequence of investment decision rules followed by the household in every period, i.e.,

$$\pi = (d_0, d_1, \ldots).$$

A policy is called *stationary* if it uses the same decision rule in every period . Therefore, under a stationary policy, the household will choose the level of investment in education only based on the current period income. Hence, under a stationary policy, the decision rule in any period will be of the form

$$d : R^+ \to \{s', s''\}$$

where $d\left(m\right)$ is the level of investment in education when the adult member in the household earns m in the current period. In our analysis, we will focus on the class of stationary policies. It can be shown that the optimal policy will be stationary in our model (proof given in Appendix 2), and therefore, the optimal stationary policy is also the optimal policy in general.

Suppose that the household receives an utility from its total wealth in every period. Let the temporal utility function is given by $u\left(\right)$. We assume that u is increasing and concave[13]. Further, we assume that utility is bounded, and we let B be such that $|u\left(w\right)| < B$ for all $w \geq 0$. The assumption of bounded utility is made to ensure the existence of optimal policy.

To determine the optimal stationary policy, we first need to decide on an optimality criterion. In our analysis, we use the total expected discounted utility as our criterion. Notice that a stationary policy $\pi = (d, d, \ldots)$ and an initial level of income m_0 is sufficient to describe the sequence of income and investment in education at every period $((m_0, s_0), (m_1, s_1), \ldots)$.[14] Therefore, given a policy π and an initial level of income $m_0 = m$, the total expected discounted utility is given by

$$V_\pi\left(m\right) = \mathcal{E}\left[\sum_{t=0}^{\infty} \beta^t \left(u\left(m_t - s_t\right)\right) \mid m_0 = m\right] \tag{2.2}$$

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[12] This assumption is technically known as *supermodularity condition*.

[13] This is a standard assumption in the economics literature.

[14] Given m_0, s_0 is given by $d\left(m_0\right)$. The next period income m_1 will be determined by the probability distribution $p\left(m|m_0, s_0\right)$. The level of investment in education in period 1, s_1, is given by $d\left(m_1\right)$. Proceeding this way, one can construct the infinite sequence of income and investment in educataion $((m_0, s_0), (m_1, s_1), \ldots)$.

where $\mathcal{E}$ represents the conditional expectation, given that policy π is followed by the household. β denotes the discount factor and we assume that $\beta \in (0,1)^{[15]}$. We assume the same discount factor for every period[16]. Note that (2.2) is well defined since utility is bounded and $\beta < 1$, which implies that $|V_\pi(m)| < B/(1-\beta)$.

We look for the optimal stationary decision rule $d^*(\cdot)$ such that $\pi^* = (d^*, d^*, \ldots)$ maximizes the total expected discounted utility.

$$V_{\pi^*}(m) = \max_\pi V_\pi(m) \text{ for all } m \geq 0.$$

3 Low-income and low-education trap

The household faces a trade-off when determining how optimally to invest in the child's education. The benefit of investing in education is that the household is expected to have a high income in the next period. The extra income would also lead the household to invest in education in the next period, and thereby achieve an increase in the expected income in the subsequent period. But a potentially important cost is that the household has to curtail its current period utility by incurring the cost of education and by loosing the child's potential earning. The relative cost of investing in education is higher at a low level of income compared to a high level of income. On the other hand, by Assumption 3, the value of education does not decrease as income increases. Therefore, there will be a clean division of the range of income into low and high values separated by a threshold such that investing in education is optimal for high values of income and not investing in education is optimal for low values of income. Hence, we have the following proposition:

Proposition 1 *Suppose Assumptions 1, 2 and 3 hold. There exists a threshold income m^d such that the household will spend on education in any period t if the adult member of the household earns at least m^d in period t.*

Proof. In appendix. ∎

We call the threshold level of income m^d below which the household does not invest in education as the *dropout threshold*. The optimal decision rule by a household is given by

$$
\begin{aligned}
d^*(m) &= s'' \text{ if } m > m^d \\
&= s' \text{ if } m < m^d \\
&\in \{s', s''\} \text{ if } m = m^d
\end{aligned}
\tag{3.1}
$$

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[15]If $\beta = 0$, the household does not value its future stream of wealth. It only maximizes its current period wealth and as a result, will not invest in education. Therefore, it will get stuck into a low-wealth situation where the young member does not receive any education, and earns a low level of income every period. We rule out this possibility by assuming that $\beta > 0$, or in other words, the household cares about its future stream of wealth and therefore cares about educating the young member (indirectly). Further, the assumption that $\beta < 1$ is motivated by the fact that a reward to be earned in the future is less valuable than one earned today.

[16]The assumption of the same discount factor across time periods can be supported on several grounds. First, as parents instill their values to the children, a generation's consideration for the future generations should not vary largely across time. Second, as we are analysing the decision making process for a specific household, more often than not, the generations would be influenced by the same culture.

From Proposition 1, we see that if income of a household in any period falls below the dropout threshold, it will stop investing in education. Then it is more likely that the household will have low income in the subsequent period. If the income at the next period also falls below the dropout threshold, the household continues to stop investing in education and its income is further likely to decrease. Hence, Proposition 1 exhibits a low-income and low-education trap.

To better understand how a household may enter the low-income trap, it is useful to do the analysis with the expected income. Let $\mathcal{E}\left(m_{t+1} \mid m_t, s_t\right)$ denote the expected income at period $t+1$, given the current period income, m_t and investment in education, s_t.

$$\mathcal{E}\left(m_{t+1} \mid m_t, s_t\right) = \int_0^\infty r p\left(r \mid m_t, s_t\right) dr.$$

Notice that Assumptions 1, 2 and 3 respectively imply

1. $\mathcal{E}\left(m_{t+1} \mid m_t, s''\right) > \mathcal{E}\left(m_{t+1} \mid m_t, s'\right)$

2. $\mathcal{E}\left(m_{t+1} \mid m_t, s_t\right)$ is increasing in m_t for any given s_t.

3. $\mathcal{E}\left(m_{t+1} \mid m_t, s''\right) - \mathcal{E}\left(m_{t+1} \mid m_t, s'\right)$ is non-decreasing in m_t.

To make our argument simple and tractable, let us assume that for a given $s \in \{s', s''\}$, $\mathcal{E}\left(m_{t+1} \mid m_t, s\right)$ is concave in the current period income m_t and equals m_t at a finite value.[17] The concavity assumption of $\mathcal{E}\left(m_{t+1} \mid m_t, s\right)$ implies that as the current period income increases, its effect on the next period income diminishes. This assumption is realistic since, in our model, the current period income affects the next period income only through its effect on the household's living condition and its member's health condition. For a sufficiently wealthy household, a marginal increase in income would cause little change in their living condition. On the other hand, to a poor household, the same level of increase in income can improve the living condition to a large extent. Therefore, it is reasonable to assume that the effect of the current period income on the next period income decreases as income increases.

Assumption 4 *For any given $s \in \{s', s''\}$, $\mathcal{E}\left(m_{t+1} \mid m_t, s\right)$ is concave in m_t and equals m_t at a finite value.*

Let $\tilde{m}\left(s\right)$ denote the level of income at which the next period expected income equals the current period income when the level of investment in education is given by s. Hence,

$$\mathcal{E}\left(m_{t+1} \mid m_t = \tilde{m}\left(s\right), s\right) = \tilde{m}\left(s\right) \text{ for } s \in \{s', s''\}.$$

We call $\tilde{m}\left(s\right)$ as the steady expected income, given the level of investment in education s.

In Figure 1, we plot the next-period expected income as a function of the current period income for two levels of investment, s' and s''. From Figure 1, we see that if the household invests in education at the current period, the next-period expected income will be less (more) than the current period income if the current period income is above (below) $\tilde{m}\left(s''\right)$. Similarly, if the household does not

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[17]This assumption is not necessary for exhibiting the possibility of low-income trap, but it helps understanding diagrammatically why a low-income trap may exist due to the trade-off between receiving high expected utility in the current period by not investing in education and receiving high expected utility in future by investing in education.
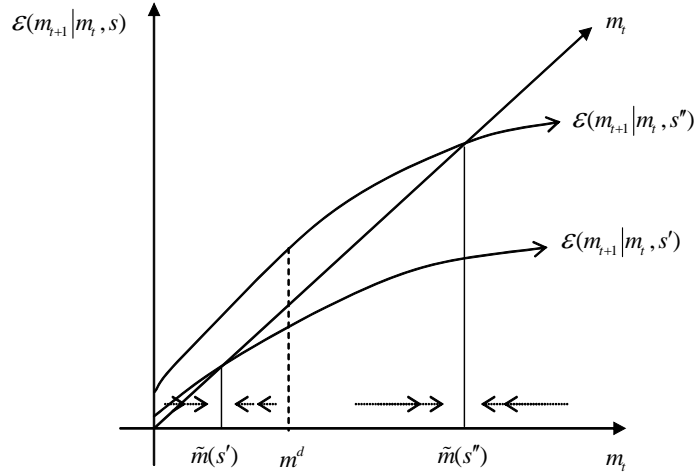
Figure 1: Expected income paths

invest in education at the current period, the next-period expected income will be less (more) than the current period income if the current period income is above (below) $\tilde{m}(s')$.

There are three possibilities: either the dropout threshold m^d is above $\tilde{m}(s'')$ or it is below $\tilde{m}(s')$ or it lies in between $\tilde{m}(s')$ and $\tilde{m}(s'')$.

Case 1: $m^d > \tilde{m}(s'')$: If the current period income is above m^d, a household invests in education. Since the current period income is also above $\tilde{m}(s'')$ (as $m^d > \tilde{m}(s'')$), the next-period expected income decreases. If the next-period income falls below m^*, the household stops investing in education, and its expected income still decreases as long as the current income is above $\tilde{m}(s')$. Over time, the expected income converges to $\tilde{m}(s')$. Thus, there is only one steady level of income, which is $\tilde{m}(s')$.

Case 2: $m^d < \tilde{m}(s')$: Using an equivalent argument, it can be shown that the expected income converges to $\tilde{m}(s'')$ when $m^d < \tilde{m}(s')$.

Case 3: $\tilde{m}(s') \leq m^d \leq \tilde{m}(s'')$: Finally, consider the situation where the dropout threshold m^d lies in between the two steady levels of expected income $\tilde{m}(s')$ and $\tilde{m}(s'')$. We illustrate this case in Figure 1. There are two converging level of incomes in the long run. If the current period income is below m^d, then the household follows the expected income path with no education (the lower curve in Figure 1) and its expected income converges to $\tilde{m}(s')$. On the other hand, if the current period income is above m^d, then the household follows the expected income path with education (the higher curve in Figure 1) and its expected income converges to $\tilde{m}(s'')$.

A comment about the distribution of income is worth noting. Figure 1 depicts the expected income path rather than actual income path. When the expected future income is below the current income, the actual future income could well be above the current income. The income distribution however, is typically an asymmetric distribution with a heavy tail towards the lower range of income. For this kind of distribution, the actual value of income is more likely to be below the expected value of income.

The above analysis suggests that the position of the dropout threshold m^d with respect to the steady levels of expected income, $\tilde{m}(s')$ and $\tilde{m}(s'')$, determines where a household's future expected income converges. From our discussion above, we see that if the dropout threshold m^d is above $\tilde{m}(s')$

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(i.e., the situation depicted in Case 1 and Case 3), a household's long term expected income may converge to the lower steady level expected income with no education, $\tilde{m}(s')$. For policy studies, it is important to estimate where the dropout threshold lies. Notice that at the dropout threshold, a household is indifferent between investing in education and not investing in education. A household faces a trade off while determining whether to invest in education. If the cost of education is high or if the return from education is low, a household has less incentive to invest in education. The dropout threshold in that case will be high.

When can we expect to have a high cost of education or low value of education in reality? The cost of education does not only reflects the education fees, but also the foregone earnings by the children. In poor economies, individuals typically start earning at a low age, which reflects the fact that the opportunity cost of education is very high. On the other hand, the return from education depends on employment opportunities. More often than not, poor economies are also the ones with burdens of unemployment. The return from education is low in those economies, and the dropout threshold tends to be high.

Before we move to the next section, a comment about Assumption 4 is worth mentioning. If we relax the concavity assumption, the expected future income may equal the current period income at more than one point. Therefore, there could be more than one converging steady level of expected income for every given level of investment in education. However, by Assumption 1, the minimum of the converging steady levels of income with no education will always be less than any converging steady level of income with education. As long as the dropout threshold is above the minimum of the converging steady levels of income with no education, a household may be trapped into lower steady level of expected income with no education if its current income is below the dropout threshold.

4 The effect of illness on decision to invest in education

The central idea in our model is that if the earning member suffers due to illness, the effective income of a household goes down, which in turn affects the household's investment in education. In our model, an individual receives education at the first phase of his life and the income he earns at the next period represents his total lifetime earning. A premature death or a prolonged period of illness can reduce income by a high margin. We model the direct effect of illness by a reduction in the current period income. The purpose of this section is to illustrate how the short term decrease in income due to illness may affect a household's long term expected income.

We incorporate the effect of illness in our model in the following way: We allow for a possibility that the adult member of the household may suffer due to illness. Illness is a probabilistic event, and therefore, income in each period is uncertain and contingent on the event, i.e. whether the adult member is suffering due to illness. For simplicity we assume that there is a fixed probability of illness, given by $p > 0$, in every period. Let $\delta > 0$ denote the decrease in the current period income if the adult member suffers due to illness. If the adult member does not suffer due to illness, he earns an income m_t, call it *no-illness income*, which is determined by the same conditional probability distribution $p(\cdot \mid m_{t-1}, s_{t-1})$, given the no-illness income m_{t-1} in the previous period and the investment in education s_{t-1} in the previous period. If the adult member suffers due to illness, his income will be the no-illness income minus δ. We call it *illness-affected income*.[18] We call the actual income that

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[18] Here we assume that the next period no-illness income depends on last period no-illness income, not on the illness-

the household receives before making the decision of investing in education *effective income*. The effective income can be either no-illness income or illness-affected income, depending on whether the adult member suffers due to illness or not.

Since we are interested in finding out the effect of illness on a household's investment decision on education, we now modify the sequence of events in our model as follows. Only at the beginning of every period, a household finds out if the adult member is suffering due to illness. After finding out the effective level of income, the household decides whether to invest in education. By considering this sequence of events, we ensure that household's decision to invest in education is contingent on whether the adult member is suffering due to illness. As before, the household maximizes the total expected discounted utility.

It is easy to see that in the current modified model, a household's optimal strategy would, as in the previous unmodified version, be to invest in education if its current period income is above a certain threshold. Compared to the previous unmodified version, the only difference is that the effect of illness introduces a new income distribution. This new income distribution is the product of two probability distributions: the original income distribution $p\left(\cdot \mid m_{t-1}, s_{t-1}\right)$ and a binary distribution, which reduces income by a margin of δ with probability p, or does nothing with probability $1 - p$. As long as the original income distribution satisfies Assumptions 1, 2 and 3, it can be shown that the new income distribution also satisfies these three assumptions. Therefore, under Assumptions 1, 2 and 3, we have a result similar to Proposition 1 except that the household conditions its decision on the first period effective income instead of the first period no-illness income.

Proposition 2 *Suppose Assumptions 1, 2 and 3 hold. There exists a threshold income such that the household will spend on education in any period t if the effective income in period t is above that threshold.*

Proof. Given that the probability distribution function of the effective income satisfies Assumptions 1, 2 and 3, the argument to prove Proposition 2 is exactly similar to the one used in proving Proposition 1. Here we show that the probability distribution of the effective income satisfies Assumptions 1, 2 and 3 if the probability distribution of no-illness income satisfies Assumptions 1, 2 and 3.

For some positive real number $n \geq 0$, the probability that the effective income will be above n, given that no-illness income is m and the level of investment in education is s at the last period, is

$$(1 - p)\, q\left(n \mid m, s\right) + pq\left(n + \delta \mid m, s\right). \tag{4.1}$$

Let us denote this probability by $\tilde{q}\left(n \mid m, s\right)$. It is easily verifiable that $\tilde{q}\left(n \mid m, s\right)$ satisfies Assump-

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affected income, even if the earning member may suffer from illness in the last period. Illness can affect the next period income in two ways. First, it influences the household's decision to invest in schooling, and thereby affects the next period income. Or, it reduces the current period income and thereby affects the next period income. We are interested to measure the first kind of effect. Therefore, to nullify the second effect, we assume that the next period no-illness income only depends on the last period no-illness income.

tions 1 and 2 if $q(n \mid m, s)$ satisfies Assumptions 1 and 2. Furthermore, for given $m^+ > m^- \geq 0$,

$$
\begin{aligned}
& \tilde{q}(n \mid m^+, s'') - \tilde{q}(n \mid m^+, s') \\
= & (1-p)q(n \mid m^+, s'') + pq(n+\delta \mid m^+, s'') - (1-p)q(n \mid m^+, s') - pq(n+\delta \mid m^+, s') \\
\geq & (1-p)[q(n \mid m^-, s'') - q(n \mid m^-, s')] \\
& + p[q(n+\delta \mid m^-, s'') - q(n+\delta \mid m^-, s')] \text{ (by Assumption 3)} \\
= & \tilde{q}(n \mid m^-, s'') - \tilde{q}(n \mid m^-, s').
\end{aligned}
$$

Hence, $\tilde{q}(n \mid m, s)$ satisfies Assumption 3. ■

Notice that the next period expected income in our current illness-incorporated model is given by

$$
\mathcal{E}(m_{t+1} \mid m_t, s_t) = \int_0^\infty rp(r \mid m_t, s_t)\, dr - \delta p. \tag{4.2}
$$

The first term in (4.2), $\int_0^\infty rp(r \mid m_t, s_t)\, dr$, is the expected no-illness income and the second term in (4.2), δp, is the expected decrease in income due to illness. We will use the same notation as used in the previous section and let m^d, $\tilde{m}(s')$ and $\tilde{m}(s'')$ to denote the dropout threshold, steady expected income with no education and steady expected income with education respectively. We know that if the dropout threshold, m^d, is above $\tilde{m}(s'')$ or below $\tilde{m}(s')$, there will be only one converging steady level of expected income. On the other hand, if the dropout threshold m^d is in between $\tilde{m}(s')$ and $\tilde{m}(s'')$, there will be two converging steady levels of expected income, and the current period income is critical in determining where a household's future expected income converges. For analytical purposes, we concentrate on the case when the dropout threshold m^d is in between $\tilde{m}(s')$ and $\tilde{m}(s'')$.

To see the effect of illness on the decision to invest in education, consider a household whose current period no-illness income is above m^d. If the reduction in income due to illness is sufficiently high so that the effective income falls below m^d, the household will not invest in education. However, because of this decision, the household's next period no-illness income is expected to be lower than what it would have been if it has invested in child education. If the next period no-illness income is also below the dropout threshold m^d, the household continues not to invest in education and its expected future income further decreases. Thus, illness can trap households into a low expected income and less education state in the long run.

Here is how we can compute the exact fraction of the households that would be vulnerable to the effect of illness. Due to a single occurrence of illness, a household with income in between m^d and $m^d + \delta$ reverses its decision from investing in education to not investing in education if the adult member of the household suffers due to illness. A fraction of these households will also find the next period income below the dropout threshold, given that they do not invest in education. Let m^n denote the level of no-illness income at the current period such that the next period expected income will be exactly the same as the dropout threshold m^d if the household does not invest in education. In particular, m^n solves the equation (See Figure 2)

$$
\mathcal{E}(m_{t+1} \mid m_t = m^n, s') = m^d.
$$

If a household has its current period no-illness below m^n and does not invest in education, it is expected that the household next period income will be below the dropout threshold. Therefore, households

with income between m^d and $\min\{m^d + \delta, m^n\}$ are the ones that will reverse their decision from investing in education when the adult member suffers due to illness and will have expected next period income also below the dropout threshold. This result is summarized below.
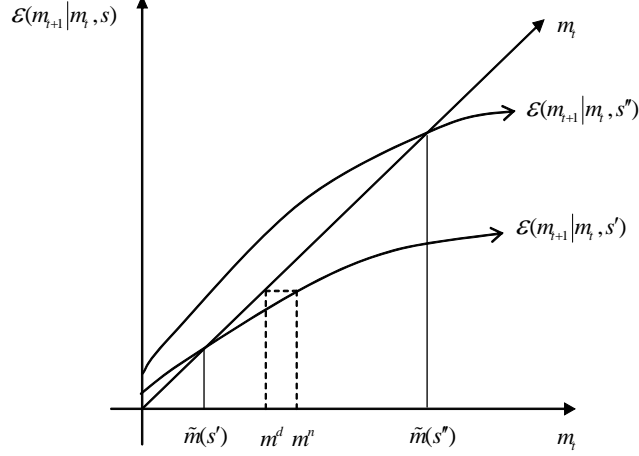

Figure 2: Expected income paths and income thresholds

Proposition 3 *Suppose Assumptions 1, 2, 3 and 4 hold. Assume $\tilde{m}(s') \leq m^d \leq \tilde{m}(s'')$. A single occurrence of illness influences a household's decision to invest in education if its current period income in between m^d and $m^d + \delta$. A fraction of these households, in particular with income in between m^d and $\min\{m^d + \delta, m^n\}$ may be trapped into a state of low expected income and low education by a single occurrence of illness.*

4.1 The impact of illness in successive generations

In the above analysis, we assume that the next-period no-illness income depends on education and the current period no-illness income. By assuming this, we rule out the possibility the decrease in the current period income due to illness may affect the next period income in ways other than the household's decision to invest in education. Though this assumption helps us to track the effect of illness on income through a household's decision to invest in education, it nullifies the impact that repeated occurrences of illness can have. If we relax this assumption and consider the possibility that a decrease in the current period income can also have an adverse effect on the next period income (since the current period income affects the living and health condition of the members of the household, and thus can affect the next period income), repeated occurrences of illness can have further devastating effect. If m^n is strictly less than $m^d + \delta$, then households with income in between m^n and $m^d + \delta$ will not be affected by a single occurrence of illness. However, successive occurrences may lead them to a state of low expected income and low education. For example, consider a household with income $m^d + \delta$. If $m^n < m^d + \delta$, the household's next period income will be above m^d, but less than $m^d + \delta$.

13

So the household will invest in education at the following period if there is no further occurrence of illness. Otherwise, it may not invest in education and its income in the subsequent period will further decrease. Therefore, successive occurrences could lead the household's income to fall below m^d ultimately, even if it does not do so in a single period.

The existence of low expected income and low education trap depends on the level of dropout threshold. As we argued before, the value of dropout threshold critically depends on the relative cost of education as well as the return from education. Poor economies are typically associated with high opportunity cost of education as well as low return from education due to unemployment, low wages, unorganized employment structure etc. Therefore, the possibility of this low-income low education trap is more prevalent in poor economies than in rich economies.

5 Policy analysis

The danger of ending up in low income and low education trap has implications for economic policies. In this section, we discuss the impact of a public policy that could improve the economic condition of the illness-affected households.

In most situations, patients are cared primarily by the family members. There exists a potential for health care policies to ease the burden of these economically devastated families. One way in which this could work would be to train and certify caregivers of terminally ill patients in nursing. These certified caregivers could then be available to other patients and families in need of skilled care-giving. This would create an opportunity for economically devastated families of dying patients to escape the trap of poverty, while at the same time increasing the level of skilled care-giving available in the community.

When there is an excess demand for the skilled caregivers, implementation of our proposed policy will increase the next period income of an illness-affected household that is not investing in education (in particular, when $s = s'$). Let k denote the level of increase in the next period income of an illness-affected household given that the household is not investing in education. We assume that $k > 0$. We call this policy P_1.

Before we discuss our results, we will introduce a few notations, which are useful when considering how to measure the policy effect. Let m^{nn} denote the level of no-illness income at the current period such that the next period expected income will be $m^d - k$, if the household does not invest in education. In particular, m^{nn} solves the equation

$$\mathcal{E}\left(m_{t+1} \mid m_t = m^{nn}, s'\right) = m^d - k. \tag{5.1}$$

Observe that $m^{nn} < m^n$, because by Assumption 2, the next period expected income is increasing in the current period income. Introduction of policy P_1 will only change the income distribution of a household that is affected by illness. Further, this increase in income is only for a single period, the period just after an adult member suffers due to illness. For households that not affected by illness, there is no change in their income distribution. Therefore, there are effectively two classes of households in our model, one affected by illness and the other not affected by illness. These two classes of households will have different strategic incentives when deciding on whether to invest in child education. Hence, these two classes of households will face two different dropout thresholds. It is useful

to introduce a separate notation to denote the dropout threshold for illness-affected households. Let m^{dd} denote the dropout threshold for the illness affected households.

The following observations describe the policy effect. The first observation states that the dropout threshold for illness affected household is higher than the dropout threshold for households not affected by illness. The intuition behind this result is the following: If policy P_1 is implemented, the return to an illness-affected household from not investing in education is more compared to the return from not investing in education to a household not affected by illness. Therefore, at the dropout threshold for households not affected by illness, m^d, an illness affected household will find it strictly preferable not to invest in education. This implies that the new dropout threshold for illness-affected households is greater than m^d.

Observation 1 *Suppose Assumptions 1, 2 and 3 hold. Dropout threshold for households affected by illness will be higher than the dropout threshold for households not affected by illness, i.e., $m^{dd} > m^d$.*

Since the policy will only affect the short term income of the illness-affected households, observe that the long term steady levels of income should not change due to the introduction of this policy.

Observation 2 *Suppose Assumptions 1, 2, 3 and 4 hold. Long term steady levels of income $\tilde{m}(s')$ and $\tilde{m}(s'')$ do not change due to introduction of P_1.*

To see the effect of illness under policy P_1, first consider the case of a household which is not affected by illness. For such a household, the dropout threshold remains at m^d. If m^d is in between $\tilde{m}(s')$ and $\tilde{m}(s'')$, there are two steady levels of expected income. If the current income is below (or above) m^d, the household's expected income converges to $\tilde{m}(s')$ (or $\tilde{m}(s')$). Next consider the case of an illness-affected household. For such a household, the dropout threshold increases to m^{dd}. If the current period income is above $m^{dd} + \delta$ so that the effective income remains above m^{dd}, the household will in any case invest in child education. For households with income below $m^{dd} + \delta$, there will be no investment in child education when the adult member suffers due to illness. A fraction of these household, in particular households with income below m^{nn}, will have the next period expected income below the dropout threshold m^d. This is because m^{nn}, by definition, is the level of current period income for which the next period expected income will be $m^d - k$ and policy P_1 increases the next period income by k. Notice that we consider the next period dropout threshold as m^d since we are interested to find out the impact of a single occurrence of illness. Hence, under policy P_1, the exact fraction of households whose decision to invest in education would be affected by a single occurrence of illness are those whose current period income is in between m^d and $\min\{m^{dd} + \delta, m^{nn}\}$. The above finding is summarized in the following proposition.

Proposition 4 *Suppose Assumptions 1, 2, 3 and 4 hold. Assume $\tilde{m}(s') \leq m^d < m^{dd} \leq \tilde{m}(s'')$. If P_1 is introduced, household with income in between m^d and $\min\{m^{dd} + \delta, m^{nn}\}$ will be trapped into a state of low income and low education by a single occurrence of illness.*

Figure 3: Income thresholds under policy P_1

Comparing this result with Proposition 3, we will argue that policy P_1 has potential for improving the economic condition of the illness-affected households. We will consider the three following cases separately.

Case 1: $m^{nn} \leq m^d$.

From Proposition 4, we see that policy P_1 completely mitigates the effect of illness since $\min\{m^{dd} + \delta, m^{nn}\} \leq m^{nn} < m^d$. In this situation, a household whose decision to invest in education is reversed due to illness, finds its next period expected income sufficiently high under policy P_1 so that the following generation is expected to invest in education. This case is illustrated in Figure 3.

Case 2: $m^d < m^{nn} \leq m^d + \delta$.

Since $m^d < m^{dd}$, we have $m^{nn} < m^{dd} + \delta$. From Proposition 4, we find that household with income in between m^d and m^{nn} would be affected by a single occurrence of illness. If policy P_1 is not introduced, from Proposition 3, we see that a single occurrence of illness affects the long run economic condition of households with income in between m^d and $m^d + \delta$. Therefore, policy P_1 only partially mitigates the effect of illness in this case.

Case 3: $m^{nn} > m^d + \delta$.

Since $m^d < m^{dd}$, we have $m^d + \delta < \min\{m^{dd} + \delta, m^{nn}\}$. Also, note that $m^d + \delta < m^{nn} < m^n$. Therefore, from Proposition 3, we find that households with income in between m^d and $m^d + \delta$ would be affected in the long run due to illness if policy P_1 is not introduced. On the other hand, when policy P_1 is in effect, households with income in between m^d and $\min\{m^{dd} + \delta, m^{nn}\}$ would be affected in the long run due to illness. As $m^d + \delta < \min\{m^{dd} + \delta, m^{nn}\}$, policy P_1 has a negative impact in mitigating the low-income trap caused by illness.

The above discussion suggests that policy P_1 will be effective in reducing the adverse effect of illness if $m^{nn} \leq m^d + \delta$. When will m^{nn} be below $m^d + \delta$? This happens if either the loss in household income due to illness, δ, is high or the magnitude of increase in income under policy P_1, k, is high (from (5.1), we see that m^{nn} is low for high values of k).

When can we expect δ or k to be high in reality? The parameter δ, in our set up, measures the short-term economic burden of illness. In case of prolonged period of illness, or premature death of the

earning member, δ is expected to be high. On the other hand, k measures the magnitude of increase in expected income of an illness-affected household if it is under the policy P_1. The value of k depends on factors such as demand for skilled care-giving, availability of well developed market for caregivers or other factors like income from alternate socially desirable professions. To understand this point, notice that k measures the relative increase in income under this policy compared to an alternate profession that a caregiver, who had to leave the educational setting to provide care, could get. A preliminary study (Emanuel et. al. [2007]) on patients' informal caregivers (ICs) in Uganda finds that the financial burden for a terminally ill family member made IC's feel pressured to stop attending school (51%). Under our proposed policy, caregiver training can be considered as an alternate form of human capital investment. To measure the full potential of the proposed policy, we need to estimate the demand for skilled care-giving as well the cost of creating a market for the trained caregivers. Emanuel et. al. [2007] finds that a large fractions (77%) of the surveyed illness-affectecd households are willing to hire an outside caregiver on their own. The main benefits of hiring someone to help care for their relatives were seen as increasing the quality of care (86%), having more time for themselves and their family (63%), having less stress on their family (60%), and giving more time for employment (37%). The same study also finds that a significant proportion (47%) of ICs get involved in socially undesirable professions, for example,.prostitution and smuggling. In that respect, the proposed policy could also positively affect the society's well-being. Palliative care research in Africa finds that there is a limited supply of professional caregiving, which is also a reason why a majority of the terminally ill patients are cared for by their school-aged children[19].

Also, It is worth noting that there is an administrative cost involved in implementing this policy. This cost includes the cost of training the caregivers and the cost of connecting them to the market. An advantage of our proposed policy is that the cost to train ICs is relatively low since they have already acquired some skill in caregiving due to their experience in treating their closed relatives.[20] Therefore, a formal professional training would increase the quality of caregiving at a low marginal cost (Reitschuler-Cross and Emanuel [2007]). Our analysis suggests that there is a potential benefit associated with the introduction of this policy. Whether this benefit outweighs the cost of the policy is an empirical question. Estimating the relative cost and benefit of this policy is an important area for future research.

6 Implications

This paper proposes an analytical framework to study the interactions among income, education and health. The analysis above has emphasized the threshold effect as the reason for creating a low income — low education trap. We study how illness leads a household into this trap. The main mechanism is intuitive: Illness creates a short-term poverty. It lasts a long term effect if the short-term poverty push the household below the critical income threshold. In such a scenario, a household decision to invest in education gets negatively affected. We recommend a possible policy intervention to mitigate the negative effect of illness. It has a potential to improve a household's well-being in situations where

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[19]See Kikule [2003].

[20]It is worth noting that home hospice and palliative care services also provides training to the family caregiver so that they can care for the patient. In this sense, the medical system has already invested marketable expertise in this population. See Reitschuler-Cross and Emanuel [2007].

the short-term burden of illness is severe or the level of increase in household income due to the policy is sufficiently high.

An immediate next step of this research agenda is to determine whether our proposed policy can provide benefits at an aggregate level. As argued above, the dropout threshold changes as an effect of the policy. To understand the aggregate effect of the policy on the society's well-being, we should estimate the potential benefit and cost involved in implementing the policy. To this end, we will seek to gather socioeconomic data from regions where the policy can provide potential benefit. Such data include income, cost of education, burden of serious illness, health care infrastructure, and social customs and acceptance of change. Preliminary data from a feasibility study of such a policy in Uganda revealed a large financial burden among caregiving relatives of terminally ill patients, and 97% of these caregivers expressed an interest in becoming trained as professional caregivers (Emanuel et. al. [2007]). Formal empirical studies on the macro-effect of the recommended policy are needed provide fruitful insights.

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7 Appendix

Proof of Proposition 1:
Proof. Let $V(m)$ denote the maximum total expected utility if the initial level of income, m_0 is given as m. Therefore

$$V(m) = \max_{\pi} V_{\pi}(m) \text{ where } \pi \text{ denotes any arbitrary feasible policy.} \tag{7.1}$$

Claim 1: V satisfies the following optimality equation

$$V(m) = \max_{s \in \{s', s''\}} \left[u(m - s) + \beta \int_0^{\infty} p(r|m, s) V(r) \, dr \right] \tag{7.2}$$

Proof of Claim 1: Let $\pi_{s'}$ be any arbitrary policy in the set of policies that choose s' at period 0. Then,

$$V_{\pi_{s'}}(m) = u(m - s') + \beta \int_0^{\infty} p(r|m, s') W_{\pi_{s'}}(r) \, dr$$

where $W_{\pi_{s'}}(r)$ denotes the total expected discounted utility from period 1 onward, given that policy $\pi_{s'}$ is being followed and the period -1 income is r. Notice that if the period 1 income is r, the

20

situation at period 1 is exactly the same as the situation at period 0 with initial income as r, with the only exception that all gains should now be multiplied by the discount factor β. Hence, we must have

$$W_{\pi_{s'}}(r) \leq \beta V(r)$$

since $V(r)$, by definition, is the maximum total discounted expected utility if the initial income is m.

Thus,

$$V_{\pi_{s'}}(m) \leq u(m - s') + \beta \int_0^\infty p(r|m, s') V(r) \, dr. \tag{7.3}$$

Similarly, for any arbitrary policy $\pi_{s''}$ in the set of policies that choose s'' at period 0, we have

$$V_{\pi_{s''}}(m) \leq u(m - s'') + \beta \int_0^\infty p(r|m, s'') V(r) \, dr. \tag{7.4}$$

As $\pi_{s'}$ and $\pi_{s''}$ are chosen arbitrarily, (7.3) and (7.4) together imply

$$V_\pi(m) \leq \max_{s \in \{s', s''\}} \left[u(m - s) + \beta \int_0^\infty p(r|m, s) V(r) \, dr \right] \text{ for any arbitrary policy } \pi.$$

Therefore,

$$V(m) \leq \max_{s \in \{s', s''\}} \left[u(m - s) + \beta \int_0^\infty p(r|m, s) V(r) \, dr \right]. \tag{7.5}$$

Next, to prove the other way, suppose s^* denotes the level of investment that maximizes the right-hand side of (7.2). Consider a policy π that chooses s^* at period 0 and from period 1 onward, the policy π coincides with some policy π_r if the period -1 income is r such that

$$V_{\pi_r}(r) \geq V(r) - \varepsilon \tag{7.6}$$

for some arbitrary small real number $\varepsilon > 0$. Notice that such a policy π_r is always feasible since $V(r)$ is the maximum total discounted expected utility with initial income r. From (7.6), we see that

$$\begin{aligned}
V_\pi(m) &= u(m - s^*) + \beta \int_0^\infty p(r|m, s^*) V_{\pi_r}(r) \, dr \\
&\geq u(m - s^*) + \beta \int_0^\infty p(r|m, s^*) V(r) \, dr - \beta \varepsilon.
\end{aligned}$$

As $V(m) \geq V_\pi(m)$,

$$\begin{aligned}
V(m) &\geq u(m - s^*) + \beta \int_0^\infty p(r|m, s^*) V(r) \, dr - \beta \varepsilon \\
&= \max_{s \in \{s', s''\}} \left[u(m - s) + \beta \int_0^\infty p(r|m, s) V(r) \, dr \right] - \beta \varepsilon.
\end{aligned}$$

Since ε is arbitrary, we have

$$V(m) \geq \max_{s \in \{s', s''\}} \left[u(m - s) + \beta \int_0^\infty p(r|m, s) V(r) \, dr \right]. \tag{7.7}$$

(7.6) and (7.7) together imply Claim 1.

Claim 2: V is the unique bounded solution of the optimality equation (7.2).

Proof of Claim 2. V is bounded since the utility function u is bounded. To prove the uniqueness, suppose $W(m), m \geq 0$, be another bounded function that satisfies the optimality equation (7.2). Let $s_m \in \{s', s''\}$ be the level of investment such that

$$W(m) = u(m - s_m) + \beta \int_0^\infty p(r|m, s_m) W(r) \, dr.$$

Then,

$$
\begin{aligned}
W(m) - V(m) &= u(m - s_m) + \beta \int_0^\infty p(r|m, s_m) W(r) \, dr \\
&\quad - \max_{s \in \{s', s''\}} \left[u(m - s) + \beta \int_0^\infty p(r|m, s) V(r) \, dr \right] \\
&\leq \beta \int_0^\infty p(r|m, s_m) (W(r) - V(r)) \, dr \\
&\leq \beta \int_0^\infty p(r|m, s_m) \left\{ \sup_{r \geq 0} |W(r) - V(r)| \right\} dr \\
&= \beta \sup_{r \geq 0} |W(r) - V(r)|.
\end{aligned}
$$

By reversing the role of V and W, we can similarly show that

$$V(m) - W(m) \leq \beta \sup_{r \geq 0} |V(r) - W(r)|.$$

Therefore,

$$|V(m) - W(m)| \leq \beta \sup_{r \geq 0} |V(r) - W(r)|,$$

and so

$$\sup_{m \geq 0} |V(m) - W(m)| \leq \beta \sup_{r \geq 0} |V(r) - W(r)|.$$

Since $\beta < 1$, we must have

$$\sup_{m \geq 0} |V(m) - W(m)| = 0.$$

Thus, Claim 2 is proved.

Claim 3: $V(m)$ is an increasing function of m.

Proof of Claim 3: By definition, $V(m)$ is the maximum total expected utility if the initial income is m. Let π^* be the optimal stationary policy that maximizes total expected utility. Notice that

$$V_{\pi^*}(m) = \mathcal{E} \left[\sum_{t=0}^\infty \beta^t (u(m_t - s_t)) \mid m_0 = m \right] \tag{7.8}$$

where $\mathcal{E}$ represents the conditional expectation, given that policy π^* is followed by the household.

Fix $m_1 < m_2$. Consider a household with initial income m_2 and suppose it follows exactly the same level of investment taken by a household that has initial income m_1 and follows the optimal strategy π^*. Let us call this strategy π'. Given that s_t is the same for both households, the household with initial income m_2 receives a higher utility at the first period compared to the household with initial income m_1. Moreover, by Assumption 2, the household with initial income m_2 is expected to receives a higher income at period 1 since it starts with a higher period 0 income. Therefore, the household with initial income m_2 is expected to receive higher utility at period 1 compared to the

household with initial income m_0 Following similar argument, it can be shown that the household with initial income m_2 is expected to receive higher utility at every period it takes exactly the same level of investment in education as the household with initial income m_1 does. Hence,

$$V_{\pi'}(m_2) \geq V_{\pi^*}(m_1).$$

Since π^* is the optimal stationary policy that maximizes total expected utility, we have

$$V_{\pi^*}(m_2) \geq V_{\pi'}(m_2),$$

and so

$$V_{\pi^*}(m_2) \geq V_{\pi^*}(m_1),$$

or

$$V(m_2) \geq V(m_1).$$

Hence, Claim 3 is proved.

Claim 4: There exists a unique stationary policy $\pi^* = (d^*, d^*, \ldots)$ that maximizes the total discounted expected utility, i.e., $V(m) = V_{\pi^*}(m)$. This optimal stationary policy is given by

$$d^*(m) = \arg\max_{s \in \{s', s''\}} \left[u(m - s) + \beta \int_0^\infty p(r|m, s) V(r) \, dr \right]. \qquad (7.9)$$

Proof of Claim 4: By Claim 2, we already know that V is the unique bounded solution of the optimality equation. To prove the current claim, it only remains to show that by following the stationary policy with the decision rule as described in (7.9), a household can actually get the maximum expected discounted utility.

By construction,

$$V(m) = u(m - d^*(m)) + \beta \int_0^\infty p(r|m, d^*(m)) V(r) \, dr.$$

Hence, V can be considered as the expected return of a two-stage problem where at the first stage a household follows $d^*(m)$ and at the following stage it receives a terminal return given as V. But the terminal return V has the same value as using d^* for another period and then receiving the terminal return V. Therefore, V can be considered as the expected return of a three-stage problem in which a household follows d^* at the first two stages and then receive the terminal return V. Continuing this line of argument, we get that

$$
\begin{aligned}
V(m) = {}& \mathcal{E}\,(n - \text{stage return by following } d^* \text{ at the first } n \text{ periods} \mid m_0 = m) \qquad (7.10) \\
& + \beta^n \mathcal{E}\,(V(m_{n+1}) \mid m_0 = m \text{ and } d^* \text{ is followed at the first } n \text{ periods}).
\end{aligned}
$$

Since V is bounded and $\beta < 1$, the second term in (7.10) goes to zero as n goes to infinity. As $\pi^* = (d^*, d^*, \ldots)$, the first term converges to $V_{\pi^*}(m)$ as n goes to infinity. Thus, letting $n \to \infty$, we get

$$V(m) = V_{\pi^*}(m).$$

Hence, Claim 4 is proved.

Claim 5: The optimal policy will be *monotone* in the level of investment in education. In particular, there will be a threshold income m^d such that households with income above m^d will spend on education where as households with income below m^d will choose not to invest in education.

Proof of Claim 5: By Claim 4, we know that the optimal stationary policy is given by (7.9). To prove Claim 5, we will show the following: if a household with an income m finds it optimal to invest in education, any household with income above m must find it optimal to invest in education. On the other hand, if a household with an income m finds it optimal not to invest in education, any household with income below m finds it optimal not to invest in education. To prove the the first statement, let us assume that it is optimal to invest in education when the income is m. Therefore,

$$u\left(m-s''\right)+\beta\int_0^\infty p\left(r|m,s''\right)V\left(r\right)dr \geq u\left(m-s'\right)+\beta\int_0^\infty p\left(r|m,s'\right)V\left(r\right)dr$$

or,

$$-\left\{u\left(m-s'\right)-u\left(m-s''\right)\right\}+\beta\int_0^\infty\left\{p\left(r|m,s''\right)-p\left(r|m,s'\right)\right\}V\left(r\right)dr\geq 0 \qquad (7.11)$$

Fix $m_1 > m$. Since u is increasing and concave, $u\left(m-s'\right)-u\left(m-s''\right)$ is positive and decreasing in m (note that $s' < 0 < s''$). Therefore,

$$u\left(m-s'\right)-u\left(m-s''\right)\geq u\left(m_1-s'\right)-u\left(m_1-s''\right)$$

or,

$$-\left\{u\left(m_1-s'\right)-u\left(m_1-s''\right)\right\}\geq-\left\{u\left(m-s'\right)-u\left(m-s''\right)\right\}.$$

Next, we will argue that $\int_0^\infty\left\{p\left(r|m,s''\right)-p\left(r|m,s'\right)\right\}V\left(r\right)dr$ is increasing in m. To prove the above statement, we first assume that $V\left(r\right)$ is continuously differentiable almost everywhere (that is, except for finitely many points). We then show that the optimal policy will be monotone, and under such a policy, $V\left(r\right)$ is indeed continuously differentiable almost everywhere. Suppose V is continuously differentiable excepts for finitely many points $d_1, d_2, ..., d_{k_r}$ for some $k \geq 0$. Define $d_0 = 0$. We then have

$$\int_0^\infty\left\{p\left(r|m_1,s''\right)-p\left(r|m_1,s'\right)\right\}V\left(r\right)dr-\int_0^\infty\left\{p\left(r|m,s''\right)-p\left(r|m,s'\right)\right\}V\left(r\right)dr \qquad (7.12)$$

$$=\int_0^\infty\left[\left\{p\left(r|m_1,s''\right)-p\left(r|m_1,s'\right)\right\}-\left\{p\left(r|m,s''\right)-p\left(r|m,s'\right)\right\}\right]V\left(r\right)dr$$

$$=\int_0^\infty\left[\begin{array}{c}\left\{p\left(r|m_1,s''\right)-p\left(r|m_1,s'\right)\right\}\\-\left\{p\left(r|m,s''\right)-p\left(r|m,s'\right)\right\}\end{array}\right]\left(\begin{array}{c}\sum_{i=1}^{\max\{j:d_j<r\}}\int_{d_{i-1}}^{d_i}V'\left(n\right)dn\\+\int_{d_i,i=\max\{j:d_j<r\}}^r V'\left(n\right)dn+V\left(0\right)\end{array}\right)dr$$

$$=\int_0^\infty V'\left(n\right)\left[\int_n^\infty\left\{\begin{array}{c}\left\{p\left(r|m_1,s''\right)-p\left(r|m_1,s'\right)\right\}\\-\left\{p\left(r|m,s''\right)-p\left(r|m,s'\right)\right\}\end{array}\right\}dr\right]dn$$

$$+V\left(0\right)\int_0^\infty\left\{\begin{array}{c}\left\{p\left(r|m_1,s''\right)-p\left(r|m_1,s'\right)\right\}\\-\left\{p\left(r|m,s''\right)-p\left(r|m,s'\right)\right\}\end{array}\right\}dr$$

By Assumption 3,

$$\int_n^\infty\left\{\left\{p\left(r|m_1,s''\right)-p\left(r|m_1,s'\right)\right\}-\left\{p\left(r|m,s''\right)-p\left(r|m,s'\right)\right\}\right\}dr =$$

$$q\left(n|m_1,s''\right)-q\left(n|m_1,s'\right)-q\left(n|m,s''\right)-q\left(n|m,s'\right) \geq 0 \text{ for every } n\geq 0.$$

Furthermore, $V'\left(n\right)\geq 0$ by Claim 3 and $V\left(0\right)\geq 0$ since $u\geq 0$. Hence, both terms in (7.12) are positive, which implies that $\int_0^\infty\left\{p\left(r|m,s''\right)-p\left(r|m,s'\right)\right\}V\left(r\right)dr$ is increasing in m. Therefore, the left-hand side of (7.11) is increasing in m. Hence, if it is optimal to invest in education when the initial

income is m, it is also optimal to invest in education when the initial income is $m_1 > m$. Similarly, it can be shown that if it is optimal not to invest in education when the initial income is m, it is also optimal not to invest in education if the initial income is below m.

To complete the proof of Claim 5, it remains to show that our assumption that $V(r)$ is continuously differentiable almost everywhere, is a valid assumption. Consider an interval of income over which a household's choice of investment in education remains constant. Then,

$$V(m) = u(m - s) + \beta \int_0^\infty p(r|m, s) V(r) \, dr. \tag{7.13}$$

Since u is continuously differentiable and $p(\cdot \mid m, s)$ is continuously differentiable probability density function, the solution V of equation (7.13) is also continuously differentiable. Since the optimal decision rule is monotone, there will be at most two intervals of income at which a household's choice of investment remains constant. Hence, $V(r)$ will be continuously differentiable except for at most one point, the threshold income m^*. Hence Claim 5 is proved.

Claims 1,2,3,4 and 5 together imply Proposition 1. ∎