Contingent Payments and Certification Quality

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Abstract

The Cuomo Plan, a New York State regulation introduced in 2008, prohibits issuers of residential mortgage-backed securities from making payments to rating agencies contingent on the assigned ratings. The Plan’s objective was to address potential conflicts of interest among rating agencies. To evaluate the policy behind the Plan, I construct a certification model which consists of the following features: (i) an issuer privately informed about her security’s quality can hire a rating agency to assign a rating; (ii) the agency can observe, at a cost, a private signal correlated with the quality of the security; (iii) an undeserved favorable rating reduces the agency’s future revenues. I show that the Plan has an effect on the informative content of the rating only if the quality signal is cheap. The Plan discourages the issuer of a low-quality security from requesting a rating. The rating agency has less incentive to obtain the quality signal because the low-quality issuer infrequently requests a rating. Overall, the Plan ensures that the rating is more informative.

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This is a major overhaul of the system ... it is a dramatic change.

This feels cosmetic to me, ... getting paid for just showing up doesn’t strike me as a good model or incentive structure.

1. INTRODUCTION

In 2011, the United States Senate produced a report on the causes of the recent financial crisis. An entire section of this report was devoted to the role of credit rating agencies, which were faulted for deliberately overlooking factors that induced lower ratings for structured financial products. The following excerpt is illustrative:

Despite the increasing number of ratings issued each year and record revenues as a result, neither Moody’s nor S&P hired sufficient staff or devoted sufficient resources to ensure that the initial rating process and the subsequent surveillance process produced accurate credit ratings.

In 2008, Andrew Cuomo, who at the time served as Attorney General of New York, reached an agreement with the three largest rating agencies to address these issues. The agreement was known as the Cuomo Plan. The Plan prohibited rating agencies from receiving compensation based on the issuer agreeing to publish an assigned rating. The Plan imposed a fee-for-service compensation scheme, requiring that issuers pay whenever they requested a rating. My model studies the effect of the policy on the informational content of ratings. The model has the following features:

- an issuer privately observes the quality (high or low) of her security,
- a monopolistic rating agency, decides whether to:

\[2\]Coburn Levin Report (2011), Section V. For a complementary account of the role of credit rating agencies in the recent financial crisis, see Benmelech and Dlugosz (2009).
\[4\]The agreement was signed by Standard & Poor’s, Moody’s and Fitch.
– observe a signal correlated with the quality of the security, and
– truthfully report this information.

In this model, the quality signal can be high or low, with the agency having the option to assign a high rating or refusing to rate the product at all. A high rating can be denied only if the agency observes a low signal. If the agency assigns a high rating to a low-quality security, it incurs a reputation cost, which is meant to capture the potential damage from a loss of credibility.\(^5\)

Moreover, before the policy is introduced, the rating agency can require an ex ante fee, to be paid to request a rating, and an ex post fee, to be paid only if a high rating is assigned.

The rating agency’s willingness to obtain a signal and to rate honestly will determine the issuer decision to request a rating. This potential effect is described by an analyst in her evaluation of the consequences of Moody’s procedures:

> I am worried that we are not able to give these complicated deals the attention they really deserve, and that they [Credit Suisse, the seller] are taking advantage of the light review...\(^6\)

In this work, I demonstrate that by prohibiting contingent ex post fees, the Cuomo Plan incentivizes the agency to obtain the quality signal and dissuades it from assigning a high rating upon observing a low signal. Nevertheless, in equilibrium, the rating agency obtains the costly signal less often as a result of the Plan. This is the case because a fee that is independent of the rating obtained reduces the incentive to submit a low-quality security for rating. The reason is straightforward: a low-quality security is less likely to receive a high rating than a high-quality one. The issuer prefers an ex ante to an ex post fee regardless of the quality of the security, but the low-quality issuer finds it particularly convenient to pay the fee ex post.

As the issuer’s reaction to the policy depends on the quality of the security, the expected quality of securities submitted for certification is thus affected by the policy. The average quality of a security submitted for rating increases as less low-quality securities are submitted, and as a

\(^5\)The argument that the value of ratings derives from the rating agency’s desire to maintain a good reputation is commonly accepted but not immune to critiques. Hunt (2008) and Partnoy (2001) propose an alternative explanation, based on the legal value attributed to the rates of Nationally Recognized Statistical Rating Organization (NRSRO) by the financial regulatory system.

\(^6\)Coburn Levin (2011), page 305.
consequence the agency has a weaker incentive to collect information. Overall, even if the agency is less informed as a result of the policy, buyers are more informed because a low-quality security is less likely to receive a high rating.

As might be expected, the Cuomo Plan has an impact on the rating process only if the reputational cost is not already sufficient to discipline the agency. When the reputational cost is high, it is optimal for the agency to require the entire fee ex ante to discourage issuers of low-quality securities from requesting a rating.

Less intuitive is the second characterization of the impact of the policy. In the context of low reputational cost, the effect of the Cuomo Plan depends on the cost of the quality signal:

- If the cost to obtain the signal is so high that in the absence of the policy the agency would not have any incentive to obtain a signal of quality and therefore would just blindly assign a high rate, then the policy has no effect.

- If, on the other hand, there is a low cost of effort, the Plan then reduces the probability of a low-quality security receiving a high rating, as described above.

Based on this assessment, it is clear therefore that while the Cuomo Plan is likely to reduce information asymmetries in markets in which rating agencies have weak reputational incentives, it is less likely to induce rating agencies to spend the resources necessary to evaluate complex financial products. To the extent that complex financial products are also characterized by the largest information asymmetries between issuers and investors, the Plan might be ineffective when informative ratings are most needed.

The remainder of the paper is structured as follows. Section 2 contains a review of the literature. Section 3 introduces the model. Section 4 discusses the equilibria. Section 5 provides a conclusion. All the proofs are contained in the Appendix.

2. Review of the Literature

The literature on the Cuomo Plan thus far (Bolton et. al (2012), Kovbasuuk (2010), Bouvard and Levy (2009)) does not consider how the procedures followed by rating agencies to assign
their ratings affect the decisions of sellers endowed with products of different quality. Bolton et al. (2012) show that non-contingent fees would dissuade rating agencies from collecting costly information on security quality. This conclusion, however, directly contradicts what my model suggests primarily because different assumptions are being made. In the model proposed by Bolton et al. (2012), buyers punish a rating agency for lying about the signal observed but not for assigning a rating based on a misleading signal. In my model, however, buyers are not able to distinguish the two cases.

According to Kovbasyuk (2011), the Plan can have opposite effects on the quality of ratings. This effect depends on whether or not the contract between issuer and rating agency can be observed by the investors. In my model, contracts are non-observable. Kovbasyuk (2011) shows that in this case the Plan reduces the incentive to assign high ratings to low-quality securities. Bouvard and Levy (2012) consider a rating market in which contingent fees are banned. They show that rating agencies, regardless, prefer to inflate their ratings. Rating inflation is motivated by the desire to attract low-quality issuers.

More generally, recent theoretical research on market credit ratings has taken on a broad focus. The role of reputation for honesty in disciplining rating agencies is considered in Strausz (2005), Mathis et al. (2009), and Frenkel (2010). Mariano (2012) argues that reputational motives do not necessarily ensure more reliable ratings: rating agencies improve their reputation for expertise by disregarding their private information and assigning ratings based on public information. Fahri et al. (2009), Skreta and Veldkamp (2009), Sangiorgi et al. (2009) focus on credit shopping, that is, the possibility of the issuers to cherry-pick the most favorable ratings. Unsolicited ratings are considered in Fulghieri et al. (2010). Pagano and Volpin (2009) and Fahri et al. (2009) study the transparency of rates and Damiano et al. (2008) considers the role of coordination among raters working for the same credit rating agency. White (2010) and Dranove and Jin (2010) provide comprehensive reviews on the subject.

The theoretical literature on credit ratings is matched by a limited number of empirical studies. A notable example is Ashcraft et al. (2011), who study whether credit ratings affect the market
price of rated products. They show that in a sample of residential mortgage-backed securities, issued in the years preceding the financial crisis, ratings did have an influence on prices.

3. The Model

The model I have developed is a game with four players: an issuer, a rating agency, and two buyers. The issuer owns a unit of a security of quality $q \in \{H, L\}$. The security is worth 1 to the buyers if $q = H$ and −1 if $q = L$. For the other agents the security has no value. The issuer is privately informed about $q$. The other agents know that $\Pr\{q = H\} = \alpha \in (0, 1)$.

The issuer cannot credibly communicate the quality of the security to the buyers, but she can request a rating. In order to request a rating, the issuer needs to commit to pay a schedule of non-negative fees set by the rating agency. The issuer pays an ex ante fee $\phi_I$ to request a rating, and an ex post fee $\phi_R$, only in case a high rating is assigned.

The rating agency can observe a signal $\theta = q$ with probability $e$ at a cost $c(e) = Ce$ where $C \geq 0$. The rating agency can either assign a high rating or no rating. The agency can refuse to issue a high rating only if it receives a private signal $\theta = L$. If the agency assigns a high rating to a security of quality $L$, it incurs a reputation cost $\rho > 0$. I model reputation as an exogenous cost, as in Bolton et al. (2012). The other players know that $\Pr\{q = H\} = \alpha \in (0, 1)$ and observe a public signal $\psi \in \{L, H\}$ distributed as follows:

$$\Pr\{\psi = H|q = H\} = 1 - \beta,$$

$$\Pr\{\psi = H|q = L\} = 0.$$

and observe a public signal $\psi \in \{L, H\}$ distributed as follows:

$$\Pr\{\psi = H|q = H\} = 1 - \beta,$$

$$\Pr\{\psi = H|q = L\} = 0.$$

Hence, the expected value of the security for a signal $\psi$, denoted as $V_\phi$, is:

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7If the agency observes the quality, this is equivalent to a very simple model of quality revelation in which the expert can either reveal $q \in \{L\}$ or $q \in \{L, H\}$. 

\[ E(q|\psi = H) := V_H = 1, \]
\[ E(q|\psi = L) := V_L = \frac{\alpha \beta - (1-\alpha)}{\alpha \beta + (1-\alpha)} < 1. \]

I assume that the security has a non-negative expected value for the buyers even if \( \psi = L \).

**Assumption 1.** \( V_L > 0 \).

The time-line of the game is depicted in Figure 1.

![Timeline](image)

**Figure 1.** Timeline

The equilibrium concept is sequential equilibrium. Buyers only observe the public signal and whether the security is certified. Their bids are given as follows:

\[ b_i : \{cert, no\ cert\} \times \{L, H\} \to \mathbb{R}_+ \cup \emptyset, \text{for } i \in \{1, 2\}. \]

Let \( 1_L = 1 \) iff \( q = L \). If a buyer gets the security, her payoff equals:

\[ U_{Bi}(b_i, q) = 1 - 2 1_L - b_i, \]

and if he does not get the security, \( U_{Bi}(b_i, q) = 0 \). In equilibrium, buyers bid the expected value of the security, as long as it is non-negative. If \( \psi = H \), the quality of the security is known, and the certificate has no informative content. If instead \( \psi = L \), the expected value of the security might depend on the presence of the certificate. For \( \psi = L \), I define the expected value of the security as \( \bar{v}_c \) or \( \bar{v}_{nc} \), depending on whether the certificate is issued or not, and I denote the corresponding bids as \( b_c \) and \( b_{nc} \), respectively.\(^8\) If \( v_c \) and \( v_{nc} \) are defined in equilibrium, then:\(^9\)

\[
\begin{cases}
    b_c = \bar{v}_c & \text{if } \bar{v}_c \geq 0, \\
    b_c = \emptyset & \text{otherwise},
\end{cases}
\]

\[
\begin{cases}
    b_{nc} = \bar{v}_{nc} & \text{if } \bar{v}_{nc} \geq 0, \\
    b_{nc} = \emptyset & \text{otherwise}.
\end{cases}
\]

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\(^8\)E.g. \( \bar{v}_c \equiv P \{q = H|cert, \psi = L\} - P \{q = L|cert, \psi = L\} \) and \( b_c \equiv b_i(cert, L) \).

\(^9\)I.e. whenever certification takes place respectively w.p. \( > 0 \) or \( < 1 \).
The issuer choice to request a certificate depends on the certification fees:

\[ r_q : \mathbb{R}_+^2 \to \{0, 1\} \text{ for } q \in \{H, L\}. \]

I assume, without loss of generality, that the security is sold to one of the buyers that make the highest bid. The payoff to the issuer is:

\[ U_S(r_q, \phi_I, \phi_R, b) = -r_q(\phi_I, \phi_R)(\phi_I + \mathbb{1}_{\text{cert}}\phi_R) + b, \]

where \( \mathbb{1}_{\text{cert}} = 1 \) iff certification takes place, and \( b \) equals the highest bid if a bid is made, and \( b = 0 \) otherwise.

The rating agency’s strategy is given by the fee structure \((\phi_I, \phi_R) \in \mathbb{R}_+^2\), a choice of effort \( e : \mathbb{R}_+^2 \to [0, 1] \), and a function \( c : \mathbb{R}_+^2 \to [0, 1] \), which defines the probability of certification when \( \theta = L \). The agency’s payoff when the issuer demands the certificate is:

\[ U_C(\phi_I, \phi_R, e) = \begin{cases} 
\phi_I - Ce + \phi_R - \mathbb{1}_L\rho & \text{if the certificate is issued,} \\
\phi_I - Ce & \text{otherwise.} 
\end{cases} \]

If the issuer does not request the certificate, her payoff equals 0.

4. THE EQUILIBRIA

The issuer demands the certificate only if it ensures higher bids for her security. As a consequence, there exists an equilibrium in which buyers are not willing to bid enough if the certificate is obtained, and therefore the issuer does not request the certificate.

Lemma 1. There exists an equilibrium in which certification does not take place.

In this section, I characterize the parameter values for which this equilibrium is not unique.

4.1 The General Case

The equilibria for \( C > 0 \) are presented here. A preliminary observation is that the rating agency incurs the cost, \( e \), of the private signal only if the information is a determinant of the decision to certify.
Lemma 2. For $C > 0$, if the rating agency chooses $e > 0$, the certificate is not assigned upon observing $\theta = L$.

Lemma 2 implies that the informative content of the certificate is determined by the rating agency’s choice to observe $\theta$. The decision depends on $C$ and $\rho$. The rating agency does not get the signal if the cost is too large compared to the loss from losing its reputation.

Proposition 3. If $C > (1 - \alpha)\rho$, the rating agency does not acquire the private signal. An equilibrium with certification exists iff $\rho \leq \beta V_L/(1 - \alpha)$ and in this equilibrium the certificate is not informative as the security is always certified, regardless of its quality.

It might seem surprising that an issuer demands a certificate which provides no information to the buyers, but the certification equilibrium can be sustained by low bids out of the equilibrium path. The rest of the section considers the alternative case, $C \leq (1 - \alpha)\rho$.

I characterize the strategies of issuer and certifier by considering the certification game, defined as the game between issuer and certifier, for a given pair of bids.

Proposition 4. In the certification game, certification takes place only if $\beta(b_c - b_{nc}) \geq C$. In any equilibrium with certification, the certifier announces a fee structure $(\phi_I, \phi_R)$ that satisfies: $\phi_I + \phi_R = \beta(b_c - b_{nc})$.

The contingent fee satisfies:

- if $\beta(b_c - b_{nc}) < \rho$, $\phi_R = 0$,
- if $\beta(b_c - b_{nc}) > \rho$, $\phi_R \geq \rho - C/\alpha$.

The certifier sets the sum of the two fees to extract the expected gain from certification of the high-quality issuer. The ex post fee determines the participation of the issuer of a low-quality security - $r_L$. For a large ex post fee, i.e., $\phi_R > \rho - C/(1 - \alpha)$, the rating agency does not collect any information ($e = 0$), certifies regardless of the quality; as a consequence $r_L = 1$. For a small

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10 This equilibrium is similar to the no-disclosure equilibrium described in Lizzeri (1999). The certifier does not reveal anything, but all the seller types request a certificate.
**Proposition 5.** In equilibrium, certification takes place only if \( C \leq \frac{\beta \rho}{2\beta + \rho} \), and three types of equilibria exist for different values of \( \rho \) and \( C \):

- An **indifferent certifier**.
- A **careful certifier**.
- A **careless certifier**.
1. If $\rho > f_2(C)$, the certifier is careful.
2. If $\rho \in [f_1(C), f_2(C)]$, the certifier is indifferent.
3. If $\rho < f_1(C)$, the certifier is careless.\(^{11}\)

Figure 2 shows the intervals described in Result 3.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{diagram.png}
\caption{Equilibria for different values of $\rho$ and $C$}
\end{figure}

$C > \beta \rho/(2\beta + \rho)$ is an extreme and not very relevant scenario: there exists no equilibrium with certification because the fees collected would not compensate the cost of getting the signal or losing the rating agency’s reputation. If $C \leq \beta \rho/(2\beta + \rho)$ instead, there are equilibria with certification, in which the informative content of the certificate depends on the size of $\rho$ and $C$.

If the reputation cost is low (area 3 in Fig. 2), in any equilibrium with certification the rating agency is careless. This implies that it is demanding a high share of the fee contingent on certification ($\phi_R \geq \rho - C/(1 - \alpha)$), thereby inducing the issuer of a low-quality security to request a

\(^{11}\) $f_1(x) \equiv \frac{1}{2} \left( \beta - \frac{x}{1-\beta} \left( \frac{\alpha \beta + 1 - \alpha}{\alpha (1-\alpha)} \right) + \sqrt{(\beta - \frac{x}{1-\beta} \left( \frac{\alpha \beta + 1 - \alpha}{\alpha (1-\alpha)} \right))^2 + \frac{4x \beta}{1-\beta} \left( \frac{\beta}{1-\alpha} - \frac{1}{\alpha} \right)} \right)$,

$f_2(x) \equiv \frac{\beta + \sqrt{\beta^2 - 8x \beta}}{2}$. 
certificate. The rating agency obtains the same payoff from certifying the security for any quality or from collecting a signal and certifying only the high-quality security. As a result, there exists a continuum of equilibria characterized by a different informative content of the signal - measured by $\tilde{v}_c$.

Equilibria in which the certificate is more informative - i.e., $\tilde{v}_c$ is larger - exists for larger $\rho$. In fact, for a low reputation cost, if buyers expect the certifier to provide reliable information, their willingness to pay for a certified security would induce the certifier to certify regardless of the quality.

For intermediate values of $\rho$ (area 2), i.e., $\rho = \beta(b_c - b_{nc})$, any choice of $\phi_R$ ensures the same payoff for the certifier. For higher values of $\rho$, the certifier demands the entire fee before assigning the certificate ($\phi_R = 0$) to minimize the participation of the issuer of a low-quality security. Figure 4 below summarizes these observations and shows how $\tilde{v}_c$ depends on the size of $\rho$.

![Figure 4. $\tilde{v}_c$ as a function of $\rho$.](image)

In any equilibrium $r_H = 1$ and $c = 0$, and, therefore, the ex ante total welfare is equal to:

$$W^1(r_L, e) = \alpha - (1 - \alpha)r_L(1 - e)(1 + \rho) - eC.$$  \hspace{1cm} (1)
The ex ante total welfare is the sum of the expected gain or loss of the buyer from obtaining the security, and the reputation, as well as the screening cost for the rating agency. Note that when there are equilibria characterized by different $\tilde{v}_c$ for a given level of $\rho$ (as happens for $\rho \in [C/(1 - \alpha), f_1(C)]$), equilibria with higher $\tilde{v}_c$ ensure a higher total welfare. Higher $\tilde{v}_c$ is, in fact, equivalent to a lower probability that a low-quality security is certified and, therefore, sold.

In equilibrium the marginal cost of effort equals the marginal private benefit of the rating agency, but is smaller than the social benefit, that is:

\[ C = (1 - \alpha)(\rho - \phi_R) < (1 - \alpha)(\rho + 1). \]

Therefore, in equilibrium, the rating agency chooses a level of effort lower that the socially optimal one.

**Effect of the Policy**

The result below defines the cap on $\phi_R$ that maximizes the informative content of the certificate.

**Proposition 6.** The optimal cap on the ex post fee is $\overline{\phi}_R = 0$. Introducing a cap has an effect on $\tilde{v}_c$ only if $\rho \leq f_2(C)$ and $C \leq (1 - \alpha)\rho$. In this case, a regulation imposing $\phi_R = \overline{\phi}_R = 0$ increases the informative content of the certificate and ensures $\tilde{v}_c = (\rho - 2C)/\rho$ and $\tilde{v}_{nc} = -1$.

If $C > (1 - \alpha)\rho$, the rating agency has no incentive to get the private signal for any $\phi_R$, and therefore the regulation has no effect. In particular, the equilibrium presented in Result 2 in which the certifier exerts no effort and publishes a certificate that has no informative content does not disappear. Notice that for a high cost $C$ inducing the certifier to obtain the private signal is not necessarily welfare enhancing. As an extreme case, if $C > (1 - \alpha)(\rho + 1)$, it is socially optimal for the certifier to exert no effort even if $r_L = 1$.

The policy has no effect either if $\rho > f_2(C)$. This should not surprise, as Proposition 5 shows that for large $\rho$ ($\rho \geq f_2(C)$), in any equilibrium with certification, the rating agency chooses $\phi_R = 0$ even in the absence of the regulation, in order to reduce the participation of the low-quality issuer.

If $\rho < f_2(C)$, imposing a cap ensures a $\tilde{v}_c$ higher than in any equilibrium described in the last subsection. The effect of the policy is the result of two components. Eliminating the contingent
fee increases the incentive of the rating agency to get the signal, for any given probability of participation of the low-quality issuer. At the same time, it reduces the issuer’s incentive to demand the certificate. The result is a new equilibrium with less effort on the part of the rating agency and a lower probability that the low-quality issuer requests the certificate, exactly as represented in Fig. 2. The two components have opposite effects on $\tilde{v}_c$, but the second component outweighs the first, and therefore the policy results in an overall increase of $\tilde{v}_c$.

The regulation strictly increases social welfare. This can be seen by rewriting (1) in terms of $\tilde{v}_c$ as shown below:

$$W^1(r_L, e) = \alpha - \alpha \frac{(1 - \tilde{v}_c(e, r_L))(1 + \rho)}{1 + \tilde{v}_c(e, r_L)} - eC. \quad (2)$$

The regulation increases welfare as it increases $\tilde{v}_c(e, r_L)$, and therefore increases the expected gain for the buyers and reduces the expected reputation loss while at the same time reducing the cost of effort $eC$.

### 4.2 Costless Private Signal

I consider here a rating agency that can find out the quality of the security at no cost: $C = 0$. In this case, the agency always assigns the certificate to a high-quality security, while for a low-quality one the decision depends on the reputation cost and the ex post fee.

**Proposition 7.** When $C = 0$, for $\rho > \beta$ in equilibrium $\tilde{v}_c = 1$, while for $\rho \leq \beta$ an interval of $\tilde{v}_c \leq 1$ can be sustained in equilibrium $\tilde{v}^*(x) \equiv$

$$\tilde{v}_c \in \begin{cases} [\frac{\rho}{\beta}, \tilde{v}^*(\rho)] & \text{if } \beta V_L \leq \rho < \beta, \\ [V_L, \tilde{v}^*(\rho)] & \text{if } \rho < \beta V_L. \end{cases}$$

where $\tilde{v}^*(\rho) := (2\alpha - 1 + (\alpha \beta + 1 - \alpha)x/\beta + \sqrt{[1 - 2\alpha - (\alpha \beta + 1 - \alpha)x/\beta]^2 - 4x(\alpha - (1 - \alpha)/\beta)})/2$.

A high reputation cost (i.e., $\rho > \beta$) induces the rating agency assign the certificate only if $\theta = H$. For a lower reputation cost, the low-quality security receives the certificate with positive
probability. A continuum of $\tilde{v}_c$ can be supported, and for a larger $\rho$ more informative equilibria survive, as $\frac{\partial \tilde{v}^*(x)}{\partial x} > 0$.

As in equilibrium $r_H = 1$ and wlg. $e = 1$, the ex ante total welfare is given by:

$$W^2(r_L, c) = \alpha - (1 - \alpha)(1 + \rho)cr_L.$$ 

Where $c$ is the probability that a certificate is issued for $\theta = L$.

**Effect of the Policy**

The lemma below describes the effect of the policy.

**Lemma 8.** If an upper bound $0 \leq \tilde{\phi}_R < \rho$ is imposed, in equilibrium $\tilde{v}_c = 1$.

The cap directly affects the incentive to assign the certificate upon observing $\theta = L$: the rating agency will deny a certificate if and only if the ex post fee is smaller than the reputation cost. Therefore, it is enough to impose a cap on the contingent fee that is low enough to ensure that only high-quality securities are certified. The policy has the expected impact on total welfare; it increases total welfare by preventing the sale of the low-quality security, and therefore ensures an ex ante total welfare: $W^2(r_L, c) = \alpha$.

This result holds for any $\rho > 0$ and is in line with the results of Bolton *et al.* (2012) and Kovbasyuk (2011).

5. **Conclusion**

A one-period model with a single rating agency is presented here. The agency is paid by the issuer and has the incentive to certify only securities of high-quality. The rating agency also has the incentive to exert effort to discover the quality of the security only to the extent that it needs to maintain a reputation for honesty among uninformed buyers of the security. I then analyze the effect of a regulation that requires payments from the issuer to the rating agency to be independent of the agency’s choice to issue the certificate. In this model, I allow a more general
policy of putting a non-negative cap on the amount of contingent payments. I show that the cap that ensures the highest informative content of the certification, or, in other words, minimizes the probability that a low-quality product receives the certificate, is the lowest possible cap: the complete elimination of contingent payments.

I also show that the policy has no effect when the cost of finding out the quality of the product is high, compared to the cost of losing reputation ($C > (1 - \alpha)\rho$). In that case, regardless of the policy, in the unique equilibrium in which certification takes place, the certificate is assigned regardless of the quality of the security and the policy does not reduce rating inflation.

When instead the cost of screening the product is relatively low ($C \leq (1 - \alpha)\rho$), the policy has an effect when the reputational incentive is also low. The policy increases the agency’s incentive to obtain a precise signal of the security’s quality - for a given probability that the issuer of a low-quality security requests the certificate. The policy also reduces the incentive of a low-quality issuer to request the certificate. In equilibrium, eliminating contingent payments at the same time reduces the probability that a low-quality product is brought to the rating agency and the agency’s effort. Overall, the first effect has a stronger impact on the probability that a low-quality security gets the certificate, and therefore, the policy reduces the inflation of ratings.

There are two dimensions along which the model could be further elaborated to get fruitful insights. First, allowing competition in the market for certification would have the effect of changing the profits of issuer and certifier(s) and, at the same time, affect the incentives of the certifier to exert effort, as well as the incentive for the issuer to demand one or more certificates, depending on the quality of her security. Second, introducing more types of securities could also prove interesting. It could be interesting to consider the issuer’s participation decision, conditional on the quality of her security, and the agency’s choice of screening, when there is a continuum of securities’ types.
Appendix

Proof of Lemma 1. There exists an equilibrium in which buyers bid: \( b_c \leq b_{nc} = V_L \) and the issuer chooses \( r_q = 0 \) for any \( q \) and any schedule of fees. In this equilibrium, certification does not take place \( \square \)

Proof of Lemma 2. For a schedule of fees \( \phi_I, \phi_R \) the agency chooses \( e(\phi_I, \phi_R) > 0 \) only if she expects the reputation cost from rating with no information to be higher than the cost of effort. The expected reputation cost depends on her beliefs over the participation choice of the issuer, denoted as \( r^e_H(\phi_I, \phi_R) \) and \( r^e_L(\phi_I, \phi_R) \). Choosing \( e(\phi_I, \phi_R) > 0 \) is optimal whenever the following condition holds:

\[
(1 - \alpha)r^e_L(\rho - \phi_R)/(\alpha r^e_H + (1 - \alpha)r^e_L) \geq C > 0 \rightarrow \rho - \phi_R > 0.
\]

Therefore, whenever \( e(\phi_I, \phi_R) > 0 \) is optimal, \( c(\phi_I, \phi_R) = 0 \) is the only seq. rational choice \( \square \)

Proof of Proposition 3. Assume:

\[ C > (1 - \alpha)\rho, \quad (3) \]

and consider the case \( \beta V_L \geq (1 - \alpha)\rho. \) In the unique set of equilibria with certification the agency announces \( \phi_I + \phi_R \equiv \phi = \beta(b_c - b_{nc}) \) and chooses \( e = 0 \): as the issuer chooses \( r_q = 1 \) for any \( q \) her expected payoff is:

\[ U_C = \beta(b_c - b_{nc}) - (1 - \alpha)\rho \equiv U. \]

where \( b_c = V_L \) and \( b_{nc} \in [0, b_c - \frac{1-\alpha}{\beta}\rho] \). For these strategies issuer and buyers have no profitable deviations and neither does the agency: if \( \phi > b_c - b_{nc} \), then \( U_C = 0 \), as \( r_H = r_L = 0 \), if instead \( \phi \in (\beta(b_c - b_{nc}), b_c - b_{nc}] \), then \( U_C = (1 - \alpha)(b_c - b_{nc} - \rho) < U \), by Assumption 1 as \( r_H = 0 \) and \( r_L = 1 \). Finally, if she announces \( \phi < \beta(b_c - b_{nc}) \), then \( U_C \leq \phi - (1 - \alpha)\rho < U \), as \( r_H = 1 \) and \( r_L \in (0, 1] \). For \( r_H = r_L = 1 \) \((3)\) implies that \( e = 0 \) is the unique seq. rat. choice and if the agency is expected to choose \( e = 0 \), \( r_L = 1 \) is the unique seq. rat. choice, while \( r_H = 1 \) is the unique choice in equilibrium: if \( r_H < 1 \) the equilibrium is not defined as the agency can get a payoff indefinitely close to \( U \) by announcing:
\[ \phi = \beta(b_c - b_{nc}) - \epsilon \text{ for } \epsilon \to +0. \]

Finally, in equilibrium it can not be the case that \( r_H \) and \( r_L \) are s.t. the agency finds it seq. rational to set \( e > 0 \), as in that case the agency obtains a payoff \( U_C < \overline{U} \) and an equilibrium is not defined by the argument presented in the last paragraph for the \( r_H < 1 \) case.

Therefore, for \( \beta V_L - (1 - \alpha)\rho \geq 0 \) in the unique equilibrium with certification \( \tilde{v}_c = V_L \) and \( \tilde{v}_{nc} \) is not defined. A corollary of this proof is that for \( \beta V_L < (1 - \alpha)\rho \) there is no equilibrium with certification: in equilibrium the agency announces \( \phi \geq b_c - b_{nc} \), and induces:

\[ r_H = r_L = 0 \text{ and } b_c < b_{nc}, \]

where \( b_{nc} = \tilde{v}_{nc} = V_L \), while \( \tilde{v}_c \) is not defined \( \square \)

**Proof of Proposition 4.** With an abuse of notation, I refer to \( r_H \in [0, 1] \) as a strategy in which the agency participates w. p. \( r_H \) for \( q = H \) (and similarly for \( r_L \)). Following \( \phi_I, \phi_R : \phi_I + \phi_R > \beta(b_c - b_{nc}) \), \( r_H = 0 \) and the maximum expected payoff for the agency equals \( U_1 \equiv \max\{0, (1 - \alpha)(b_c - b_{nc} - \rho)\} \). Let the agency announce:

\[ \phi_I, \phi_R : \phi_I + \phi_R \leq \beta(b_c - b_{nc}). \]  

(1)

Consider first the case \( \phi_R \leq \rho - C/(1 - \alpha) \).

If \( \phi_I + \phi_R < \beta(b_c - b_{nc}) \), the set of seq. rat. choices of the issuer are \( r_H = 1 \) and \( r_L = r(r_H, \phi_R) \), if \( \phi_I > 0 \), or \( r_L \in [r(r_H, \phi_R), 1] \) if \( \phi_I = 0 \). Where:

\[ r(x, y) \equiv \alpha xC/(1 - \alpha)(\rho - C - y) \]

The choice of effort is

\[ e = f(\phi_I, \phi_R, b_c - b_{nc}) \text{ if } r_L < 1, \text{ } e \in [0, f(\phi_I, \phi_R)] \text{ if } r_L = 1. \]

Where: \( f(x, y, z) \equiv 1 - \frac{x}{z-y} \). The agency expected payoff is:

\[ U_2(\phi_I, \phi_R) \equiv \alpha(\phi_I + \phi_R) + \frac{\alpha C(\phi_I + \phi_R - \rho)}{(\rho - C - \phi_R)} \]

if \( \phi_I > 0 \), and some \( x \leq U_2(\phi_I, \phi_R) \) if \( \phi_I = 0 \). Note that:

\[ \sup_{\phi_I < \beta(b_c - b_{nc}) - \phi_R} U_C = \overline{U}_2(\phi_R) \equiv U_2(\beta(b_c - b_{nc}) - \phi_R, \phi_R), \forall \phi_R. \]

If \( \phi_I + \phi_R = \beta(b_c - b_{nc}) \), for \( \phi_I > 0 \) there is a continuum of sets of seq. rat. choices characterized by: \( r_H \in [0, 1], r_L = r(r_H, \phi_R) \) and \( e = f(\phi_I, \phi_R, b_c - b_{nc}) \). The agency payoff is given by
$U_C = r_H \overline{U}_2(\phi_R)$. Instead for $\phi_I = 0$, $r_H \in [0, 1]$, $r_L = [r_H, \phi_R, 1]$ and $e = f(0, \phi_R, b_c - b_{nc}) = 1$ and the payoff satisfies $U_C \leq r_H \overline{U}_2(\phi_R)$.

Let instead $\phi_R > \rho - \frac{C}{1 - \alpha}$. If $\phi_I < \beta(b_c - b_{nc}) - \phi_R$, the set of seq. rat. choices are: $r_H = 1$, $r_L = 1$ and $e = 0$. The payoff is

$$U_3(\phi_I, \phi_R) \equiv \phi_I + \phi_R - (1 - \alpha)\rho.$$ 

Note that: $\sup_{\phi_I < \beta(b_c - b_{nc}) - \phi_R} U_3(\phi_I, \phi_R) = \overline{U}_3(\phi_R) \equiv U_3(\beta(b_c - b_{nc}) - \phi_R, \phi_R), \forall \phi_R.$

If instead $\phi_I = \beta(b_c - b_{nc}) - \phi_R$, there is a continuum of sets of seq. rat. choices characterized by $r_H \in [0, 1]$ and $r_L = \min \{1, r_H, \phi_R\}$. The effort choice is: $e = 0$ if $r_H, \phi_R > 1$, $e = f(\phi_I, \phi_R, b_c - b_{nc})$ if $r_H, \phi_R < 1$ and $e \in [0, f(\phi_I, \phi_R, b_c - b_{nc})]$ otherwise. For any given $\phi_R$ the largest expected payoff is obtained for $r_H = 1$ and equals $\overline{U}_3(\phi_R)$. If in a seq. equilibrium the agency announces with pos. prob. fees that satisfy (1), it must be the case that $\phi_I + \phi_R = \beta(b_c - b_{nc})$ and $r_H = 1$ for those fees, as in any other case the agency has a profitable deviation. Note that

$$\overline{U}_2(\rho - \frac{C}{1 - \alpha}) = \overline{U}_3(\phi_R), \forall \phi_R \geq \rho - \frac{C}{1 - \alpha},$$

and $\overline{U}_2(\rho - \frac{C}{1 - \alpha}) > U_1$

by Assumption 1. Moreover, $\overline{U}_2(\phi_R)$ is continuous, differentiable and $\overline{U}_2'(\phi_R) > (\ast)0$ iff $\beta(b_c - b_{nc}) > (\ast)\rho$.

Therefore in any equilibrium with certification $\phi_I = \beta(b_c - b_{nc}) - \phi_R$, and

- if $\beta(b_c - b_{nc}) > \rho$: $\phi_R \in [\rho - C/(1 - \alpha), \beta(b_c - b_{nc})]$,
- if $\beta(b_c - b_{nc}) = \rho$: $\phi_R \in [0, \beta(b_c - b_{nc})]$,
- if $\beta(b_c - b_{nc}) < \rho$:
  - $\phi_R = 0$ if $\overline{U}_2(0) \geq 0$ ($\leftrightarrow \beta(b_c - b_{nc}) \geq C$),
  - $\phi_R > b_c - b_{nc} \rightarrow r_H = r_L = 0$ and no certification otherwise.

**Proof of Proposition 5.** Define

$$v_c(x, y) \equiv \frac{\alpha \beta - (1 - \alpha) r(1, x)(1 - f(\beta y - x, x, y))}{\alpha (a + (1 - \alpha) r(1, x)(1 - f(\beta y - x, x)))},$$

where $f(.)$ and $r(.)$ are defined as in the Proof of Result 4. $v_c(\phi_R, b_c - b_{nc})$ is the expected value of a certified security when $\psi = L$, $r_H = 1$, $r_L = r(1, \phi_R)$, $\phi_I + \phi_R = \beta(b_c - b_{nc})$ and $e = f(\phi_I, \phi_R, b_c - b_{nc})$. Note that for

$$\phi_R \in [0, \beta(b_c - b_{nc})] \cap [0, \rho - \frac{C}{1 - \alpha}],$$
$v_c(\phi_R, b_c - b_{nc})$ is continuous and differentiable. From Result 4, for $\beta(b_c - b_{nc}) \in [C, \rho)$, $\phi_I = \beta(b_c - b_{nc})$, $\phi_I = 0$ and therefore

$$\tilde{v}_c = v_c(0, b_c - b_{nc}) = (\rho - 2C)/C,$$

and $\tilde{v}_{nc} = -1$. An equilibrium with $\beta(b_c - b_{nc}) \in [C, \rho)$ exists iff

$$(C, \rho) \in \text{set}_1 \equiv \{(x_1, x_2) \in \mathbb{R}_+^2 : (\beta(x_2 - 2x_1)/x_2) \in [x_1, x_2]\}.$$

This condition is equivalent to:

1) $\beta(\rho - 2C/\rho) < \rho \leftrightarrow \rho \notin [(\beta - \sqrt{\beta^2 - 8C\beta})/2, f_2(C)],$

2) $\beta(\rho - 2C/\rho) \geq C \leftrightarrow C \leq \beta\rho/(2\beta + \rho)$.

Where $f_2(C) \equiv (\beta + \sqrt{\beta^2 - 8C\beta})/2$. As $\partial v_c(x, \rho/\beta)/\partial x < 0$, an equilibrium in which $\beta(b_c - b_{nc}) = \rho$ exists iff:

$$(C, \rho) \in \text{set}_2 \equiv \{(x_1, x_2) \in \mathbb{R}_+^2 : \beta(x_2 - 2x_1/x_2) \geq x_2\}

or equivalently $\rho \notin [(\beta - \sqrt{\beta^2 - 8C\beta})/2, f_2(C)]$. In particular, $\tilde{v}_c = \max \{\rho/\beta, V_L\}$, and $\tilde{v}_{nc} = -1$ whenever $\tilde{v}_c > V_L$ (for $\tilde{v}_c = V_L$, $\tilde{v}_{nc}$ is not defined). If $\rho/\beta \in (v_c(\rho - C/(1 - \alpha), \rho/\beta), v_c(0, \rho/\beta)]$, in equilibrium the agency announces $\phi_R : v_c(\phi_R, \rho/\beta), e = f(\rho - \phi_R, \rho/\beta)$ and $r_L = r(1, \phi_R)$.

If instead $\rho/\beta \in (V_L, v_c(\rho - C/(1 - \alpha), \rho/\beta)]$, in equilibrium $\phi_R = \rho - C/(1 - \alpha)$, $r_L = 1$ and $e$ satisfies

$$\frac{(\alpha\beta - (1 - \alpha)(1 - e))}{(\alpha\beta + (1 - \alpha)(1 - e))} = \rho/\beta.$$

If $\rho/\beta \leq V_L$, then $\phi_R = \rho - C/(1 - \alpha)$, $e = 0$ and $b_{nc} = V_L - \rho/\beta$.

From Proposition 4, for $\beta(b_c - b_{nc}) > \rho$ the agency sets $\phi_I = \beta(b_c - b_{nc}) - \phi_R$ and $\phi_I \geq \rho - C/(1 - \alpha)$.

If $\phi_R > \rho - C/(1 - \alpha)$, then $\tilde{v}_c = V_L$ and $\tilde{v}_{nc}$ is not defined. When instead $\phi_R = \rho - C/(1 - \alpha)$, either $\tilde{v}_c = V_L$ and $\tilde{v}_{nc}$ is not defined, or $\tilde{v}_c \in (V_L, v_c(\rho - C/(1 - \alpha), b_c - b_{nc})]$ and $\tilde{v}_{nc} = -1$. Wlg I consider only equilibria in which $\phi_R = \rho - C/(1 - \alpha)$. Note that $\partial v_c(x, y)/\partial y < 0$, $\forall x, y$, and $1 > v_c(\rho - C/(1 - \alpha), 1)$.

Therefore, iff $\rho/\beta < v_c(\rho - C/(1 - \alpha), \rho/\beta)$ there exists a unique $x \in (V_L, 1) \cap (\rho/\beta, 1)$ that satisfies $x = v_c(\rho - C/(1 - \alpha), x)$. Denoting $\tilde{v}_2$ this unique value, then $\tilde{v}_2 = \tilde{v}^*(\rho - C/(1 - \alpha))$, where:

$$\tilde{v}^*(x) \equiv (2\alpha - 1 + (\alpha\beta + 1 - \alpha)x/\beta + \sqrt{[1 - 2\alpha - (\alpha\beta + 1 - \alpha)x/\beta]^2 - 4x(\alpha - (1 - \alpha)/\beta)})/2.$$

\[\text{\ref{2} } \partial v_c(x, y)/\partial y < 0 \text{ if } v_c(x, y) \geq 0 \text{ which is ensured } \forall x, y \text{ by Assumption 1.}\]
\( \rho/\beta < v_c(\rho - C/(1 - \alpha), \rho/\beta) \) is a necessary and sufficient condition for an equilibrium in which \( \beta(b_c - \beta + b_{nc}) > \rho \) to exist. Under Assumptions 1 and 2, the condition is equivalent to: \( \rho < f_1(C) \), where

\[
f_1(C) \equiv (\beta - \frac{C}{1 - \beta} \frac{\alpha + 1 - \alpha}{\alpha(1 - \alpha)}) + \sqrt{\left(\beta - \frac{C}{1 - \beta} \frac{\alpha + 1 - \alpha}{\alpha(1 - \alpha)}\right)^2 + \frac{4C \beta}{1 - \beta} \left(\frac{\beta}{1 - \alpha} - \frac{1}{\alpha}\right)}/2.
\]

Let set_3 \( \equiv \{(x_1, x_2) \in \mathbb{R}_+^2 : x_2 < f_1(x_1)\} \). If \( (C, \rho) \in \text{set}_3 \), for any \( z \in [V_L, \tilde{v}_2] \cap (\rho/\beta, \tilde{v}_2] \) there exists an equilibrium ensuring \( \tilde{v}_c = z \). If \( \tilde{v}_c > V_L, \tilde{v}_{nc} = -1 \), otherwise \( \tilde{v}_{nc} \) is not defined.

Finally, I show that set_3 \( \subset \text{set}_2 \), and therefore for any set of parameters for which an equilibrium with \( \beta(b_c - \beta + b_{nc}) > \rho \) exists there is no equilibrium with \( \beta(b_c - \beta + b_{nc}) < \rho \). Assume an equilibrium with \( \beta v_c > \rho \) exists. This implies \( \tilde{v}_2 > \rho/\beta \). Note also that:

\[
(\rho - 2C)/\rho = v_c(0, \rho/\beta) > v_c(\rho - C/(1 - \alpha), \rho/\beta) > v_c(\rho - C/(1 - \alpha), \tilde{v}_2) = \tilde{v}_2 > \rho.
\]

So \( \beta\tilde{v}_2 > \rho \) implies \( \beta(\rho - 2C)/\rho > \rho \) or equivalently \( \text{set}_3 \subset \text{set}_2 \) \( \square \)

**Proof of Proposition 6.** If \( C > (1 - \alpha)\rho \), in any equilibrium in which \( r_H > 0 \) and/or \( r_L > 0 \), the agency chooses \( e = 0 \) regardless of \( \phi_R \). Therefore for any cap \( \overline{\phi}_R \geq 0 \) the equilibria are identical to the one described in Result 5.

Let instead \( C \leq (1 - \alpha)\rho \). If \( C > \beta \rho/(2\beta + \rho) \) from the Proof of Result 4 and the Proof of Result 5 certification does not take place. Moreover for any set of bids, the fees that ensure the highest payoff for the agency while ensuring certification with positive probability are \( \phi_I = \beta(b_c - \beta + b_{nc}) \) and \( \phi_R = 0 \). An equilibrium with those fees does not exists as it would entail a negative expected payoff of the agency. Therefore imposing a cap has no effect.

If instead \( C > \beta \rho/(2\beta + \rho) \), in equilibrium if \( \phi_R > \rho - C/(1 - \alpha) \), \( \tilde{v}_c = V_L \) and \( \tilde{v}_{nc} = -1 \). If \( \phi_R \leq \rho - C/(1 - \alpha) \) the most informative equilibrium (which is also the unique for \( \phi_R < \rho - C/(1 - \alpha) \)) ensures \( \tilde{v}_{nc} = -1 \) and \( \tilde{v}_c = v_c(\phi_R, \tilde{v}_c) \) (for the definition of \( v_c(\ldots) \) see the Proof of Result 5).

Therefore \( \tilde{v}_c \) is a continuous and differentiable function of \( \phi_R \). If \( \beta\tilde{v}_c \geq \rho - C \), then \( \partial\tilde{v}_c/\partial\phi_R \leq 0 \). If instead \( \beta\tilde{v}_c < \rho - C \):

\[
\partial\tilde{v}_c/\partial\phi_R > 0 \iff \phi_R > \beta\tilde{v}_c - \sqrt{(1 - \beta)(\rho - \beta - \beta\tilde{v}_c)}\tilde{v}_c
\]

In equilibrium the agency sets \( \phi_R > 0 \) only if \( \beta(b_c - \beta + b_{nc}) \geq \rho \rightarrow \beta\tilde{v}_c \geq \rho \rightarrow \beta\tilde{v}_c \geq \rho - C \).
Therefore imposing a cap \( \tilde{\phi}_R = 0 \) maximizes \( \tilde{v}_c \) in equilibrium. And from Result 5 it ensures 
\[
\tilde{v}_c = (\rho - 2C)/\rho \quad \square
\]

**Proof of Proposition 7.** As \( C = 0 \) it is wlg to consider only equilibria with \( e = 1 \). Let \( \rho > \beta \). On the equilibrium path the agency sets: \( \phi^*_I, \phi^*_R : \phi^*_I + \phi^*_R = \beta \), chooses \( c = 0 \) and the issuer chooses: \( r_H = 1, r_L = 0 \) and the buyers bid \( b_c = 1, b_{nc} = 0 \). The issuer has no profitable deviations and the expected payoff of the agency is \( \pi = \alpha \beta \). The agency cannot obtain a higher payoff by announcing different fees: setting \( \phi_I, \phi_R : \phi_I + \phi_R < \beta \) ensures \( \pi \leq \alpha(\phi_I + \phi_R) < \alpha \beta \), while choosing \( \phi_I, \phi_R : \phi_I + \phi_R > \beta \) ensures, by Assumption 1:
\[
\pi = \max \{0, (1 - \alpha)(1 - \rho)\} < (1 - \alpha)(1 - \beta) < \alpha \beta.
\]
This is the unique equilibrium as for \( \rho > \beta \) the agency cannot obtain more than \( \pi = \alpha \beta \) in equilibrium. Moreover, this is the unique equilibrium ensuring \( \pi = \alpha \beta \), and any set of strategies that ensures \( \pi < \alpha \beta \) cannot be an equilibrium: the agency can obtain profits \( \pi' > \pi \) by setting: \( \phi_I + \phi_R = \beta - \epsilon \), for \( \epsilon = (\alpha \beta - \pi)/2\alpha \).

Let \( \rho \leq \beta \). An equilibrium with \( b_c - b_{nc} < \rho/\beta \) does not exist. For those bids the agency sets \( \phi_I + \phi_R = \beta(b_c - b_{nc}) \) and certifies only high-quality securities in which case \( \tilde{v}_c = 1 \) and \( \tilde{v}_{nc} = -1 \rightarrow \tilde{v}_c \neq b_c \).

For \( b_c - b_{nc} \geq \rho/\beta \), the unique strategies that can be part of an equilibrium are: for the agency to set \( \phi_I = \beta(b_c - b_{nc}) - \phi_R \) and \( \phi_R \geq \rho \) and choose
\[
eq \epsilon \in [(\beta(b_c - b_{nc}) - \rho)/(b_c - b_{nc} - \rho), 1],
\]
if \( \phi_R = \rho \) and \( c = 1 \), if \( \phi_R > \rho \) and for the issuer to choose \( r_H = r_L = 1 \) for those fees. In equilibrium, \( \tilde{v}_c = f(c) \), where
\[
f(x) \equiv (\alpha \beta - (1 - \alpha)x)/(\alpha \beta + (1 - \alpha)x),
\]
and therefore \( \tilde{v}_c \in [V_L, f(\beta(b_c - b_{nc}) - \rho)/(b_c - b_{nc} - \rho))] \).

In equilibrium, it must be the case that \( \tilde{v}_c = b_c \) and whenever \( \tilde{v}_c > f(1), b_{nc} = 0 \), therefore
\[
\tilde{v}_c \leq f((\beta \tilde{v}_c - \rho)/(\tilde{v}_c - \rho)),
\]
which is equivalent to \( \tilde{v}_c \leq \tilde{v}^*(\rho) \). \( \tilde{v}^*(x) \) is defined as in the Proof of Result 5. Any \( \tilde{v}_c \in [\max \{V_L, \rho/\beta\}, \tilde{v}^*(\rho)] \) can be sustained in equilibrium, if the agency chooses the appropriate \( c \in [0, (\beta(b_c - b_{nc}) - \phi_R)/(b_c - b_{nc} - \phi_R)] \quad \square \)
Proof of Lemma 8. \( \phi_R \leq \bar{\phi}_R < \rho \rightarrow \) in equilibrium \( c = 0, \phi_I + \phi_R = \beta, r_H = 1 \) and \( r_L = 0 \). Therefore, \( \tilde{v}_c = 1 \) and \( \tilde{v}_{nc} = -1 \) \( \Box \)


