Pledge-and-Review Bargaining

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Abstract

This paper analyzes a bargaining game that is new to the literature, but that is inspired by real-world international negotiations. With so-called pledge-and-review bargaining, as stipulated in the Paris climate agreement, each party submits an intended nationally determined contribution (INDC), quantifying a cut in its own emission level. Thereafter, the set of pledges must be unanimously ratified. The procedure is repeated periodically, as newly developed technology makes earlier pledges obsolete. I first develop a dynamic model of the pledge-and-review game before deriving four main results: (1) If there is some uncertainty on the set of pledges that is acceptable (for example because of unknown discount factors), each equilibrium pledge is approximated by the asymmetric Nash Bargaining Solution. The weights placed on the others’ payoffs reflect the underlying uncertainty. The weights vary from pledge to pledge, and the set of equilibrium pledges is inefficient. (2) When this result is embedded in a climate change model, I show that each party contributes too little to the public good, the incentive to develop new technology is weak, and the optimal commitment period length is (therefore) long. (3) This result is overturned when each party can decide whether or not to participate: The undemanding pledge-and-review process motivates a larger number of parties to participate. This increases aggregate contributions and technology investments, and the optimal commitment period length is (therefore) short. (4) When the parties can choose between alternative bargaining games in advance, the (broad but shallow) pledge-and-review game is preferred too often (seldom) by technology leaders (laggards), relative to what is socially efficient. The results are consistent with several crucial differences between the climate agreements signed in Kyoto (1997) and Paris (2015) and they rationalize the development from the former to the latter.

Key words: Dynamic games, bargaining games, foundation for the Nash Bargaining Solution, climate change, the Paris Agreement, technology
1 Introduction

- The pledge-and-review strategy is completely inadequate.

Christian Gollier and Jean Tirole
The Economist (guest blog)
June 1st, 2015

Pledge-and-review bargaining refers to the structure of the negotiation process adopted in Paris, December, 2015. Each party in the negotiation process was first asked to submit an "intended nationally determined contribution" (INDC). After the INDCs had been announced by all parties, the parties were expected to ratify the treaty. The INDCs should specify cuts in the emissions of greenhouse gases being effective from 2020 to 2025 (or to 2030), and every five years the parties shall review and make new pledges for another five-year period.

This negotiation structure is remarkably different from the one used under the Kyoto Protocol of 1997. There, a "top-down" approach was used to ensure the parties made legally binding commitments to cut emissions by (on average) five percent compared to the 1990-levels. By comparison, pledge-and-review has been referred to as a "bottom-up" approach since countries will themselves determine how much to cut nationally, without making these cuts conditional on other countries' emission cuts. No wonder, then, that economic theorists question the effectiveness of the pledge-and-review bargaining game.

Interestingly, the Paris agreement differs from the Kyoto Protocol in several other ways, as well. First, while the second commitment period under the Kyoto Protocol was eight years (2012-2020), each commitment period under Paris will be only five years. Second, while only 35 countries faced binding emission cuts under Kyoto, the Paris agreement has been signed by nearly every country in the world. At the same time, both types of agreements share the emphasis on emission cuts rather than specifying national investments in environmentally friendly technology, for example, although the importance of developing such new technology has been emphasized in every recent treaty text.

The purpose of this paper is to propose a framework for studying pledge-and-review bargaining and to shed light on the development from the Kyoto-style agreement to Paris.

The next section describes a bargaining game that is new to the theoretical literature, although it is based on actual (Paris-style) negotiations. With perfect information, the outcome of the described bargaining game will coincide with the non-cooperative (or "business-as-usual") outcome, where every party simply contributes so as to maximize their own utility. With sufficient uncertainty regarding the other parties' willingness to decline and delay ratification, I show that each party's equilibrium contribution level coincides with the quantity that maximizes the asymmetric Nash product, where the weights on other parties' payoffs reflect the extent of uncertainty and how shocks are correlated. (The weights also reflect differences in discount rates in an intuitive way.) The weights on other parties' payoffs is less than 1/2 for single-peaked and symmetric shock distributions, and they are (close to) zero when the variance of the shocks is small. These small weights on others' payoffs makes pledge and review quite different from the (symmetric) Nash Bargaining Solution often used to describe Kyoto-style negotiations.

Naturally, emission cuts are smaller and (thus) investments in new technology less when the weights are small, for any fixed number of parties. Since technology then develops slowly, it is not important to revise the cuts very frequently, and the optimal commitment period is longer. All these conclusions are reversed, however, when the decision to participate in the agreement is endogenized. Since not much is expected from the participating countries (when the weights on others' payoffs are small), it is not that costly to participate and the equilibrium coalition size is larger. The larger number of parties makes the cuts more ambitious, investments larger, and the optimal commitment period shorter. The larger equilibrium coalition makes the pledge-and-review bargaining process less inefficient than if the participants negotiated according to the (symmetric) Nash Bargaining Solution, and it is thus the preferred choice if countries are homogenous. If countries have heterogeneous investment costs, the pledge-and-review process (with small weights) is the equilibrium choice of bargaining game when, and only when, the investment cost of the "technology laggards" is sufficiently small. The theory is thus...
consistent with the several differences between Kyoto and Paris, and why there has been a transition from the former to the latter type of bargaining process.

The pledge-and-review bargaining game has not been analyzed in the theoretical literature, as far as I know. By showing that this bargaining game implements an asymmetric (or "generalized") Nash Bargaining Solution (NBS) for each party’s contribution, I contribute to the ‘Nash Program’, aimed at finding noncooperative games implementing cooperative solution concepts. The Nash demand game, first described by Nash (1953), intended to implement the Nash Bargaining Solution, axiomatized by Nash (1950). There is a large subsequent literature investigating the extent to which the Nash demand game implements the Nash Bargaining Solution (Binmore, 1992; Abreu and Gul, 2000; Kambe, 2000), and some contributions also allow for uncertainty, as I do here (Binmore, 1987; Carlsson, 1991; Andersson et al., 2017).¹

The alternating offer bargaining game by Rubinstein (1982) also implements the Nash Bargaining Solution, as shown by Binmore et al. (1986), and asymmetric discount rates rationalizes the asymmetric Nash Bargaining Solution (axiomatized by Harsanyi and Selten, 1972; Kalai, 1977; Roth, 1979). Although there can be multiple equilibria with more than two players (Sutton, 1986; Osborne and Rubinstein, 1990), the weights in the asymmetric NBS may then also depend on recognition probabilities (Miyakawa, 2008; Britz et al. 2010; Laruelle and Valenciano, 2008), in addition to the discount rates (Kawamori, 2014).²

The next section contributes to this literature by showing that also 'pledge-and-review' bargaining implements the asymmetric NBS for each party’s contribution. However, the weights are shown to vary from one party’s contribution level to another’s, so the set of contributions is not Pareto optimal. The weights will reflect differences in the discount rates (as in some of the papers already mentioned), but also the extent of uncertainty in shocks and the correlation in shocks across the parties.

2 A Model of Pledge-and-Review Bargaining

This section describes a novel bargaining game, not yet analyzed in the literature, and characterizes its outcome. The section may be read independently from the [to be added] other sections, as the model here may have several other applications than the climate negotiations motivating the subsequent sections.³

¹ Also the Nash bargaining solution with endogenous threats has been given noncooperative foundations in dynamic games (Abreu and Pearce, 2007 and 2015).

² There are also papers showing how the NBS is implemented exactly in other ways, either by a specific game (Howard, 1992) or in a matching context (Cho and Matsui, 2013).

³ For example, the bargaining game could be appropriate when a number of business partners are negotiating a package deal, and each partner has expertise on and is making the proposal on one single aspect of the package (such as quality, price, delivery time, etc).

⁴ If the real discount rate is \( \tilde{\rho}_{i,t} \), the discount factor is \( e^{-\tilde{\rho}_{i,t}\Delta} = \delta_{i,t}^{\Delta} \), so \( \rho_{i,t} = \left(1 - e^{-\tilde{\rho}_{i,t}\Delta}\right)/\Delta \), which approaches \( \tilde{\rho}_{i,t} \) when \( \Delta \rightarrow 0 \). I thus refer to \( \rho_{i,t} \) as the discount rate even though the identity holds only in the limit.
I will restrict attention to stationary subgame-perfect equilibria (SPEs). If information were perfect, it is easy to see that \( x^* \) could be a stationary SPE only if \( x_i^* \) were equal to the noncooperative level \( x_i^* = \arg \max_{x_i} U_i(x_i, x_{-i}^*) \), if \( U_j(x^*) > 0 \forall j \). For any other equilibrium candidate, \( i \) could always suggest an \( x_i \) slightly different from \( x_i^* \) without violating (1) if just \( \rho_{i,t} > 0 \forall j \). Therefore, with the assumptions added above, the "trivial equilibrium" \( x^* = 0 \) always exists. This observation confirms the pessimism associated with pledge and review, as described in the Introduction.

In reality, party \( i \) is unlikely to know precisely the condition under which an offer will be accepted. One way of modelling this uncertainty is to assume that the discount rates for the next period are not known (to anyone) at the time the offers are made. After all, a country’s impatience when it comes to ratifying a treaty may depend on a range of temporary domestic policy or economic issues. To capture this, write \( \rho_{i,t} = \theta_{i,t} \rho_1 \), where \( \rho_1 \) is \( i \)'s expected discount rate while \( \theta_{i,t} \) is a shock with mean 1. The shocks are jointly distributed with pdf \( f(\theta_{1,t}, \ldots, \theta_{n,t}) \) on support \( \prod_{i} [0, \overline{\theta}] \), i.i.d. at each time \( t \), and the marginal distribution of \( \theta_{i,t} \) is \( f_i(\theta_{i,t}) = \int_{\theta_{j,t}} f(\theta_{1,t}, \ldots, \theta_{n,t}) \), where \( \Theta_{-i} = \prod_{j \neq i} [0, \overline{\theta}] \). The \( \theta_{i,t} \)'s are realized and observed by everyone after the offers but before acceptance decisions are made.\(^5\)

After learning \( \theta_{i,t} \), \( i \) accepts \( x \) if and only if:

\[
U_i(x) \geq (1 - \theta_{i,t} \rho_1 \Delta) U_i(x^*) \Rightarrow \theta_{i,t} \geq \frac{U_i(x^*) - U_i(x)}{\rho_1 \Delta U_i(x^*)}.
\] (2)

When \( \theta_{i,t} \) is drawn from a continuous distribution, the probability that \( i \) accepts will be continuous in \( x_i \). As the following result will show, this continuity can motivate larger contributions: \( x^* \) can be sustained as a "nontrivial" stationary SPE if the marginal benefit for \( i \) by reducing \( x_i \) slightly is outweighed by the risk that at least one party might be sufficiently patient to decline the offer and wait for \( x^* \).

**Theorem 1.** If \( x^* \) is a nontrivial stationary SPE in which \( U_i(x^*) > 0 \forall i \), then, for every \( i \in N \):

\[
x_i^* \leq \arg \max_{x_i} \prod_{j \in N} (U_j(x_i, x_{-i}^*))^{w^j_i}, \quad \text{where} \quad \frac{w^j_i}{w^j_i} = \frac{\rho_i}{\rho_j} f_j(0) E(\theta_{j,t} | \theta_{j,t} = 0), \quad \forall j \neq i.
\] (3)

The colored area in Figure 1 illustrates the set of equilibria when \( n = 2 \) and \( u_i = x_j - c x_j^2 / 2 \). There are multiple equilibria, since the inequality in (3) can be strong: The reasoning above does not limit how small the equilibrium \( x_i^* \)'s can be, as there is no point for \( i \) to contribute more than \( x_i^* \), whatever the equilibrium \( x^* \) is. (The stationary equilibrium \( x^* \) will always be accepted when parties are impatient.)

The inequality in (3) must hold with equality if we introduce some small chance that even \( x^* \) will be declined: this will be the consequence of introducing small trembles either in the parties’ actions—or in the support for the \( \theta_{i,t} \)'s:

(A1) Assume when the intended offers are given by \( x \), \( x + e_i^k \) is realized, where \( e_i^k \) is a vector of \( n \) shocks, each i.i.d. over time with mean zero and \( E(e_i^k)^2 \rightarrow 0 \) as \( k \rightarrow 0 \).

(A2) Assume that the support of \( \theta_{j,t} \) is \( [\theta^k_{j,t}, \overline{\theta}_j] \) (instead of \( [0, \overline{\theta}_j] \)) where \( \theta^k_{j,t} = 0 \) and \( \theta^k_{j,t} \uparrow 0 \forall j \) as \( k \rightarrow 0 \).\(^6\)

**Theorem 2.** If, under either (A1) or (A2), \( x^*(k) \) is a nontrivial stationary SPE, then (3) holds with equality for \( x_i^* = \lim_{k \rightarrow 0} x_i^*(k), \) for every \( i \in N \).

\(^5\)This is not unreasonable: (i) Technically, instead of letting \( \Delta > 0 \) be the delay between rejections and new offers, \( \Delta \) can be the delay between offers and acceptance decisions, if we assume that new offers can be made as soon as earlier offers are rejected. (ii) Since there is (then) a lag between offers and acceptance decisions, it is natural that policy makers in the meanwhile learn about how urgent it is for them to conclude the negotiations, or about the attention they instead have to give to other policy and economic issues.

\(^6\)The interpretation of a negative discount rate may be that, in some circumstances, a party prefers to delay signing agreements due to other urgent economic/policy issues that requires the decision makers’ attention. It is required that the lower boundaries, the \( \theta^k_{j,t} \)'s, approach zero in the limit (as \( k \rightarrow 0 \)), since otherwise there will be delay on the equilibrium path.
Figure 1: There are multiple equilibrium contribution levels, but they are all smaller than the efficient levels.

As a comparison, note that if $x$ were given by an asymmetric Nash Bargaining Solution, then $x$ could be described as:

$$
\begin{align*}
  x^N_i &= \arg\max_{x_i} \prod_{j \in N} (U_j(x_i, x^N_{-i}))^{w_j} = \arg\max_{x_i} \sum_j w_j \frac{U_j(x_i, x^N_{-i})}{U_j(x^N_i)},
\end{align*}
$$

so each $x^N_i$ maximizes the asymmetric Nash product, and thus a weighted sum of utilities, where the weights $w_j$’s are exogenously given. In this case, the set of $x^N_j$’s will be Pareto optimal.

Also when (3) binds in the pledge-and-review bargaining game, the outcome for $x^*_i$ maximizes a Nash product, but, in general, the weights vary with $i$ and thus the set of $x^*_i$’s is not Pareto optimal. In particular, if every $w_j^i > 1$, it is possible to make every party better off by increasing all the contributions relative to $x^*$.

The theorems also endogenize the weights and show how they depend on three thing. First, the weights on $j$’s utility is larger if $j$ is expected to be patient relative to $i$. This is natural (and in line with other bargaining papers, as discussed in the Introduction): When $j$ is patient, $j$ is more tempted to reject an offer that is worse than what one can expect in the next period, and thus $i$ finds it too risky to reduce $x_i$, especially when $i$ is quite impatient and dislikes delay.

Second, the weight on $j$’s payoff is larger when there is a lot of uncertainty regarding $j$’s shock. Of importance is especially the (marginal) likelihood that $j$’s discount rate is close to 0, so that even a small reduction from $x^*_i$ involves some risk that $j$ will decline.

Third, if the shocks are correlated, then the weight on $j$’s payoff is larger when there is a lot of uncertainty regarding $j$’s shock. Of importance is especially the (marginal) likelihood that $j$’s discount rate is close to 0, so that even a small reduction from $x^*_i$ involves some risk that $j$ will decline.

The theorems provide several important corollaries:

1. If $f_j(\cdot)$ is single-peaked and symmetric, then clearly $f_j(0) \leq 1/2$.\(^7\) In this situation, the weights

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\(^7\)To see this, note that if $f_j(0) > 1/2$, then, when $f_j(\cdot)$ is single-peaked and symmetric around the mean of one,
on other parties is less than 1/2 of the weight on \( i \) when \( x_i \) is proposed, if discount rates are equal and shocks not negatively correlated.

(2) If uncertainty vanishes, such that the pdf \( f_j(\cdot) \) concentrates around its mean, \( f_j(0) \to 0 \) and \( x_i^* \) must approach the level in the trivial equilibrium as when there is no bargaining.

(3) In the completely symmetric case (with symmetric discount rates and utility functions), Theorem 2 predicts that the nontrivial \( x_i^* \)'s are simply given by:

\[
x_i^* = \arg \max_{x_i} U_i(x_i, x_{-i}^*) + w \sum_{j \neq i} U_j(x_i, x_{-i}^*) ,
\]

where \( w = f_j(0)E(\theta_{i,t} | \theta_{j,t} = 0) \forall i, j \).

These observations are in stark contrast to the symmetric Nash Bargaining Solution, predicting that the \( x_i \)'s would follow from (4) with \( w = 1 \), as illustrated by \( x^2 \) and \( x^3 \) in Figure 1 for the example in which \( n = 2 \) and \( u_i = x_j - cx_i^2/2 \).

For this example, it is also easy to check that all equilibria satisfying (3) with strict inequalities are Pareto dominated by the equilibrium where the inequalities bind if just \( w < \sqrt{3} - 1 \approx 0.73 \). Thus, focusing on equilibria that are not Pareto dominated can in some cases replace assumptions (A1) and (A2).

Remark on generality. Above, it was assumed that \( \partial U_i(\cdot)/\partial x_i < 0 \) and \( \partial U_j(\cdot)/\partial x_i > 0 \), \( j \neq i \). Although these assumptions simplify the expression of Theorem 1, a more general version of Theorem 1 is stated and proven in the Appendix and the additional assumptions are not needed for Theorem 2. Further, if the trivial equilibrium, \( x^b \), characterized by \( x_i^b = \arg \max_{x_i} U_i(x_i; x_{-i}^b) \forall i \), is such that \( U_j(x^b) > 0 \) for some \( i \), then it ceases to exist with assumptions (A1) or (A2) and, therefore, the word "nontrivial" in Theorem 2 is, in this case, redundant.

Remark on sufficiency. Condition (3) is necessary for \( x^* \) to be an equilibrium, but it may not be sufficient. Whether the second-order condition for an optimal deviation \( x_i \) holds globally depends on the pdf \( f \). If \( n = 2 \), a sufficient condition for the second-order condition to hold is that \( f_j \) is weakly increasing, as when \( \theta_{j,t} \) is uniformly distributed, for example.

The following sections [to be added] will investigate the consequences of (4) and of \( w \) in a climate-change model. Taking \( n \) as given, it will be shown that when \( w \) is small, parties contribute less to emission cuts, they (therefore) invest less in environmentally friendly technology, and (therefore) the optimal length of the commitment periods is long. However, these three results are reversed when \( n \) is endogenized by introducing a participation stage at the beginning of the game: the small \( w \) is then making it less costly to participate and a larger number of parties will find participation affordable. The larger \( n \) will raise total contributions and total investments in technology, and thus the optimal commitment period length is smaller. The predictions are consistent with several differences between the Kyoto and the Paris Agreement, it will be argued.

Proof of Theorem 1.

As advertised in Section 2, the following version of Theorem 1 is here proven without the additional assumptions \( \partial U_i(\cdot)/\partial x_i < 0 \) for \( x_i > 0 \), and \( \partial U_j(\cdot)/\partial x_i > 0 \), \( j \neq i \).

Theorem 1\(^G\). If \( x^* \) is a nontrivial stationary SPE in which \( U_i(x^*) > 0 \forall i \), then, for every \( i \in N \), we have:

(i) if \( \frac{\partial U_i(x^*)}{\partial x_i} \leq 0 \),

\[
- \frac{\partial U_i(x^*)}{\partial x_i} \leq \sum_{j \neq i} \max \left\{ 0, \frac{\partial U_j(x^*)}{\partial x_i} \right\} f_j(0) E(\theta_{i,t} | \theta_{j,t} = 0) \rho_i \Delta U_i(x^*) ;
\]

(ii) if \( \frac{\partial U_i(x^*)}{\partial x_i} > 0 \),

\[
\int f_j(\theta_j) d\theta_j > 1 , \text{ which is impossible for a pdf } f_j(\cdot).
\]

I thank Asher Wolinsky for making this observation.
\[
\frac{\partial U_i(x^*)}{\partial x_i} \leq \sum_{j \neq i} \max \left\{ 0, \frac{\partial U_j(x^*)}{\partial x_j} \right\} f_j(0) E(\theta_{i,t} | \theta_{j,t} = 0) \rho_i \Delta U_i(x^*).
\]

Note that with the additional assumptions \( \partial U_i(\cdot) / \partial x_i < 0 \) for \( x_i > 0 \), and \( \partial U_j(\cdot) / \partial x_i > 0, j \neq i \), the first-order condition of (3) is equivalent to (5).

(i) First, note that in any stationary SPE we must have \( U_i(x^*) \geq 0 \forall i \), since otherwise a party with \( U_i(x^*) < 0 \) would reject \( x^* \) in order to obtain the default payoff, normalized to zero. We will search for nontrivial equilibria in which \( U_i(x^*) > 0 \forall i \).

A stationary equilibrium \( x^* \), such that \( U_j(x^*) > 0 \forall j \), is accepted with probability 1 when \( \rho_{j,t} \geq 0 \). Therefore, \( i \) will never offer \( x_i > x_i^* \) when \( \frac{\partial U_i(x^*)}{\partial x_i} \leq 0 \), so to check when \( x^* \) is an equilibrium, it is sufficient to consider a deviation by \( i, x^i \), such that \( x_i^i < x_i^* \) while \( x_j^i = x_j^*, j \neq i \).

**Acceptable offers.** Let \( p(x^i; x^*) \) be the probability that at least one \( j \neq i \) rejects \( x^i \), and \( p_{-j}(x^i; x^*) \) the probability that at least one party other than \( j \) and \( i \) rejects \( x^i \).

Since party \( j \)'s discount factor is \( \delta_{j,t} = 1 - \rho_{j,t} \Delta = 1 - \theta_{j,t} \rho_{j,t} \Delta, j \neq i \) rejects \( x^i \) iff:

\[
(1 - p_{-j}(x^i)) U_j(x^i) + p_{-j}(x^i) (1 - \rho_{j,t} \Delta) U_j(x^*) < (1 - \rho_{j,t} \Delta) U_j(x^*) \Rightarrow \theta_{j,t} < \theta_j(x^i) \equiv \max \left\{ 0, \frac{U_j(x^*) - U_j(x^i)}{\rho_j \Delta U_j(x^*)} \right\}.
\]

When the joint pdf of shocks \( \theta_t = (\theta_{1,t}, ..., \theta_{n,t}) \) is represented by \( f(\theta_t) \), the probability that every \( j \neq i \) accepts \( x^i \) can be written as:

\[
1 - p(x^i) = G(\bar{\theta}_1(x^i), ..., \bar{\theta}_{i-1}(x^i), \bar{\theta}_{i+1}(x^i), ..., \bar{\theta}_n(x^i)) = \int_0^{\pi_i} \left[ \int_{\theta_{i,t}(x^i)}^{\theta_{i,t}(x^*)} \int_{\theta_{i-1,t}(x^i)}^{\theta_{i-1,t}(x^*)} \int_{\theta_{i+1,t}(x^i)}^{\theta_{i+1,t}(x^*)} \int_{\theta_{n,t}(x^i)}^{\theta_{n,t}(x^*)} f(\theta_t) d\theta_{i,t} \right] d\theta_i,
\]

which is a function of \( n - 1 \) thresholds. By taking the derivative w.r.t. \( x_i^i \) and using the chain rule,

\[
-\frac{\partial p(x^i)}{\partial x_i} = \sum_{j \neq i} \max \left\{ 0, \frac{\partial U_j(x^*)}{\partial x_j} \right\} G_j' \left( \bar{\theta}_1(x^i), ..., \bar{\theta}_{i-1}(x^i), \bar{\theta}_{i+1}(x^i), ..., \bar{\theta}_n(x^i) \right), \tag{6}
\]

and, at the equilibrium, \( x^i = x^* \),

\[
\frac{\partial p(x^i)}{\partial x_i} = \sum_{j \neq i} \max \left\{ 0, \frac{\partial U_j(x^*)}{\partial x_j} \right\} G_j'(0) = -\sum_{j \neq i} \max \left\{ 0, \frac{\partial U_j(x^*)}{\partial x_j} \right\} f_j(0), \tag{7}
\]

where, as written in the text already, \( f_j(0) \) is the marginal distribution of \( \theta_{j,t} \) at \( \theta_{j,t} = 0 \).

**Equilibrium offers.** When proposing \( x_i \), party \( i \)'s problem is to choose \( x_i \leq x_i^* \) so as to maximize

\[
(1 - p(x^i)) U_i(x^i) + p(x^i) \left( 1 - E\theta_{i,t}^R \rho_i \Delta \right) U_i(x^*), \tag{8}
\]

where \( E\theta_{i,t}^R \) is the expected \( \theta_{i,t} \) conditional on being rejected (this will be more precise below).

To derive the first-order condition w.r.t. \( x_i \), suppose \( i \) considers a small (marginal) reduction in \( x_i \) relative to \( x_i^* \), given by \( dx_i = x_i^i - x_i^* < 0 \). If accepted, this gives \( i \) utility \( U_i(x^i) \approx U_i(x^*) + dx_i \partial U_i(x^*) / \partial x_i > U_i(x^*) \), but it is rejected with probability

\[
\frac{\partial p(x^i)}{\partial x_i} dx_i = -\sum_{j \neq i} \max \left\{ 0, \frac{\partial U_j(x^*)}{\partial x_j} \right\} dx_i f_j(0),
\]

where each of the \( n - 1 \) terms represents the probability that \( \theta_{j,t} \) is so small that \( j \) rejects if \( x_i \) is modified by \( dx_i \), i.e., \( \Pr \left( \theta_{j,t} \leq \theta_j \right) \) for \( \theta_j = \frac{\partial U_j(x^*)}{\partial x_j} \rho_j \Delta U_j(x^*) \) \( dx_i \). Naturally, the probability that more than one party has such a small shock vanishes when \( |dx_i| \to 0 \) since \( f \) is assumed to have no mass point.
In combination, the reduction in \( x_i \) is not beneficial to \( i \) iff:

\[
- \sum_{j \neq i} \max \left\{ 0, \frac{\partial U_j(x^*)}{\partial x_i} / \rho_j \Delta U_j(x^*) \right\} dx_i f_j(0) U_i(x^*) \left( 1 - E \left( \theta_{i,t} \mid \theta_{j,t} \leq \hat{\theta}_j \right) \right) \leq U_i(x^*),
\]

where (7) has been substituted into the second line of (9), and where \( E \left( \theta_{i,t} \mid \theta_{j,t} \leq \hat{\theta}_j \right) \) follows from Bayes’ rule:

\[
E \left( \theta_{i,t} \mid \theta_{j,t} \leq \hat{\theta}_j \right) = \frac{\int_0^{\hat{\theta}_j} \int_{\theta_{i,t}} f(\theta_i) d\theta_i, \quad \text{and} \quad E \left( \theta_{i,t} \mid \theta_{j,t} = 0 \right) = \lim_{dx_i \downarrow 0} \frac{\int_0^{\theta_{i,t}} \int_{\theta_{i,t}} f(\theta_i) d\theta_i}{\int_0^{\theta_{i,t}} \int_{\theta_{i,t}} f(\theta_i) d\theta_i},
\]

and, as already defined, \( \Theta_{-j} \equiv \prod_{k \neq j} [0, \bar{\theta}_k] \) and \( \hat{\theta}_j \equiv \frac{\partial U_j(x^*)}{\rho_j \Delta U_j(x^*)} | dx_i | \).

When both sides of (9) are divided by \( |dx_i| \) and \( dx_i \uparrow 0 \), (9) can be rewritten as the first-order condition (5).

The proof of part (ii) is analogous and thus omitted. \( QED \)

**Proof of Theorem 2.**

(iii) A continuum of \( x^* \)'s can satisfy the equilibrium condition in Theorem 1, since it is not necessary for \( i \) to improve an offer relative to \( x^* \) when \( p(x^*) = 0 \). The idea of assumption (A1) or (A2) is to introduce trembles such that \( p(x^*) > 0 \) and thus we must check that \( i \) cannot benefit from marginally increasing or decreasing \( x_i^* \) from \( x_i^* \) to reduce \( p(x^*) \). With (A1) or (A2), \( i \) will strictly benefit from \( dx_i > 0 \) when (3) is strict, and thus it must hold with equality at \( x^* \).

(A1) The vector \( \epsilon_i \) is i.i.d. over time according to some cdf, \( H(\cdot) \). (For simplicity, I omit the superscript \( k \).) When \( j \) considers whether to accept \( U_j(x_i' + \epsilon_i) \), \( j \) faces the continuation value \( V_j(x^*) \) by rejecting, where \( V_j(x^*) \) takes into account that \( x^* \) may be rejected in the future (if the future \( \epsilon_t \)'s are sufficiently small).\(^9\) The shocks, combined with the possibility to reject, imply that \( V_j(x^*) > 0 \) even if \( U_j(x^*) = 0 \), so there is no need to assume \( U_j(x^*) > 0 \).

With this, party \( j \neq i \) rejects \( x_i' \) if and only if:

\[
(1 - p_j(x^*)) U_j(x_i' + \epsilon_i) + p_j(x^*) \left( 1 - \rho_j \Delta V_j(x^*) \right) V_j(x^*) < (1 - \rho_j \Delta V_j(x^*) ) V_j(x^*) \Rightarrow \\
1 - \rho_j \Delta V_j(x^*) < \theta_{i,t}^j \theta_{j,t} \Rightarrow \frac{V_j(x^*) - U_j(x_i' + \epsilon_i)}{\rho_j \Delta V_j(x^*)}.
\]

So, the probability that every \( j \neq i \) accepts is:

\[
1 - p(x^*) = \int G(\tilde{\theta}_1(x^*), \ldots, \tilde{\theta}_{i-1}(x^*), \tilde{\theta}_{i+1}(x^*), \ldots, \tilde{\theta}_n(x^*)) dH(\epsilon) = \\
\int_{\epsilon} \prod_{j=0}^{\tilde{\theta}_1(x^*), \ldots, \tilde{\theta}_{i-1}(x^*)} f(\theta_i) d\theta_i dH(\epsilon) \Rightarrow \\
\int_{\epsilon} \sum_{j \neq i} \left( \frac{-\partial U_j(x^* + \epsilon_i)}{\rho_j \Delta V_j(x^*)} G_j(\tilde{\theta}_1(x^*), \ldots, \tilde{\theta}_{i-1}(x^*), \tilde{\theta}_{i+1}(x^*), \ldots, \tilde{\theta}_n(x^*)) dH(\epsilon) \right) \Rightarrow \\
\int_{\epsilon} \sum_{j \neq i} \left( \frac{-\partial U_j(x^* + \epsilon_i)}{\rho_j \Delta V_j(x^*)} G_j(\tilde{\theta}_1(x^*), \ldots, \tilde{\theta}_{i-1}(x^*), \tilde{\theta}_{i+1}(x^*), \ldots, \tilde{\theta}_n(x^*)) dH(\epsilon) \right). \quad (10)
\]

\(^9\)It will be the combination of the \( \epsilon_t \)'s and the \( \theta_j \)'s that determines whether \( j \) rejects \( x^* \); let \( \Phi_1(x^*) \) be the set of \( \epsilon_t \)'s and \( \theta_j \)'s such that \( j \) accepts \( x^* \), while \( \Phi_R(x^*) \) is the complementary set. We then have \( p(x^*) = \Pr \{ (\epsilon, \theta) \in \Phi_R(x^*) \} \) and:

\[
V_j(x^*) = E_{\epsilon_j, (\epsilon, \theta) \in \Phi_{1}(x^*)} (1 - p(x^*)) U_j(x^* + \epsilon_i) + p(x^*) V_j(x^*) \in_{\epsilon_j, (\epsilon, \theta) \in \Phi_{1}(x^*)} (1 - \theta_{j,t} \rho_j \Delta V_j(x^*) ) \text{,}
\]

where the two expectations are taken over the set of parameters leading to acceptance vs. rejections, respectively.
The condition under which \( i \) does not benefit from a marginal change \( dx_i \) is given by the analogously modified (9),\(^{10} \) but now, since this inequality is continuous at \( dx_i = 0 \), it must hold whether \( dx_i \) is positive or negative; it thus has to hold with equality; and it thus holds with equality regardless of whether \( i \) could benefit from \( dx_i > 0 \) or \( dx_i < 0 \) (so, we do not need the assumptions \( \partial U_i (\cdot) / \partial x_j > 0 \) for \( j \neq i \) and \( < 0 \) for \( j = i \)).

When we let \( \epsilon \) vanish \( (E(\epsilon_i^2) \rightarrow 0 \) when \( k \rightarrow 0 \), we get \( p(X^*) \rightarrow 0 \) and \( V_j (X^*) \rightarrow U_j (X^*) \), and then the condition simplifies to

\[
- \frac{\partial U_i (X^*)}{\partial x_i} = \sum_{j \neq i} \frac{\partial U_j (X^*)}{\partial x_i} f_j (0) E(\theta_{i,t} | \theta_{j,t} = 0) \rho_i \Delta U_i (X^*) ,
\]

which is the first-order condition of

\[
\arg \max_{x_i} \prod_{j \in N} (U_j (x_i, x_{-i}^*))^{w_j} ,
\]

when \( w_j^i = \frac{\rho_i f_j (0)}{\rho_j} E(\theta_{i,t} | \theta_{j,t} = 0) \), \( \forall j \neq i \).

(A2) The proof when \( \theta_{i,t} < 0 \) is analogous: Now, \( p(X^*) = 1 - G(0) > 0 \) and \( p(X^*) \) is continuous at \( x_i^* = x_i^* \). Thus, the (modified) version of (9) must hold with equality, and it boils down to (3) holding with equality since, when \( \theta_{j,t} \uparrow 0 \forall j \), then \( p(X^*) \rightarrow 0 \). QED

References [Preliminary]


\(^{10}\)The modified version of (9) can be written as:

\[
\left( 1 - p(X^*) - \frac{\partial p(X^*)}{\partial x_i} dx_i \right) E_{\epsilon_i,t; (\epsilon, \theta) \in \Phi_A (x)} \left( U_i (x^* + \epsilon) + \frac{\partial U_i (x^* + \epsilon)}{\partial x_i} dx_i \right) + \int \sum_{j \neq i} \left[ \frac{\partial U_j (x^* + \epsilon)}{\partial x_i} \right] \frac{\theta_i}{\rho_j} \Delta V_j (x^*) d\theta_i \int_0^1 G_j \left( \frac{V_1 (x^*) - U_1 (x^* + \epsilon)}{\rho_1 \Delta V_1 (x^*)}, \frac{V_2 (x^*) - U_2 (x^* + \epsilon)}{\rho_2 \Delta V_2 (x^*)}, \ldots, \theta_i \right) d\theta_i .
\]

\( U_i (x^*) E_{\theta_{i,t; (\epsilon, \theta) \in \Phi_R (x^*)} (1 - \theta_{i,t} \rho_i \Delta) \leq U_i (x^*) .\)


