Gridlock and Inefficient Policy Instruments[1]

David Austen-Smith
Kellogg School of Management, Northwestern University, USA

Wioletta Dziuda
Harris Public Policy, University of Chicago, USA

Bård Harstad
Department of Economics, University of Oslo, Norway

Antoine Loeper
Department of Economics, Carlos III University, Spain

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Abstract

Why do rational politicians choose inefficient policy instruments? Environmental regulation, for example, often takes the form of technology standards and quotas even when cost-effective Pigou taxes are available. To shed light on this puzzle, we present a stochastic game with multiple legislative veto players and show that inefficient policy instruments are politically easier than efficient instruments to repeal. Anticipating this, heterogeneous legislators agree more readily on an inefficient policy instrument. We describe when inefficient instruments are likely to be chosen, and predict that they are used more frequently in (moderately) polarized political environments and in volatile economic environments. We show conditions under which players strictly benefit from the availability of the inefficient instrument.

1 Introduction

Over the years, the economics profession has converged at a set of effective policy recommendations for a wide variety of policy areas. For example, there is widespread agreement that externalities can be more efficiently internalized with Pigou taxes than with command-and-control interventions, and that it is less distortionary to increase public revenue by eliminating economically unjustified tax deductions and exemptions than by raising tax rates. Likewise, economists have argued that some fiscal consolidation policies are less harmful than others. In practice, however, these recommendations are frequently ignored. Instead, policy makers often intervene with strictly less efficient policy instruments than others that are as readily available. This paper concerns why such apparently irrational (all else equal) political decisions might arise in a world of instrumentally rational agents.

To understand this puzzle, we present a dynamic political economy model in which the government reacts to external shocks by choosing a policy from a given menu of policies, one of which is unequivocally Pareto dominated by another available alternative. Indeed, the inefficient policy is not only Pareto dominated by the alternative at the time of adoption, but in all possible states of the world. Nevertheless, we show that the inefficient policy intervention can arise naturally from a simple legislative bargaining model without any extraneous frictions or informational asymmetries: it is the very inefficiency of the policy that makes it appealing to legislators. We further show that the availability of the inefficient policy instrument may, at least in equilibrium, improve the welfare of *all* policy makers.

Our legislative bargaining model rests on three characteristics. First, legislative policy decisions involve multiple pivotal players. In particular, policy change requires the consent of different veto players who may disagree on when to enact or repeal an intervention. Second, the status quo policy in a dynamic, multi-period setting is endogenous: the policy implemented in one period becomes the status quo in the next. Third, the environment is subject to shocks across time that affect the state-contingent policy preferences of the veto players in any period, thereby creating the need for periodic renegotiations.

In a closely related model with only one available policy intervention, Dziuda and Loeper (2016) observed that the three characteristics described above imply that the legislator who is pivotal for introducing the intervention is distinct from the legislator pivotal for repealing it. Consequently, the anticipation of her loss of political influence makes the legislator pivotal for implementing the intervention less inclined to introduce it in the first place, fearing that it will be hard to repeal should circumstances change. Loosely speaking, (Markov perfect) equilibrium behavior involves political gridlock. In this paper, however, we show that when players can choose not only whether, but also how, to intervene, the fear of future gridlock can induce an inefficient policy intervention. Intuitively, the veto players can agree on an inefficient policy instrument because the inefficient instrument will be easier to repeal. As a result, there may be states of nature in which the more economically efficient intervention is not politically feasible, whereas the less efficient intervention is approved by all veto players.

In particular, we characterize conditions under which the inefficient policy instrument is implemented with positive probability in equilibrium. Intuitively, for an inefficient policy response to be chosen by rational legislators, it must be the case that the inefficiency is sufficiently high for the more interventionist legislator to approve repealing the policy in some states, and sufficiently low for the less interventionist legislator so that the ease of repeal offsets the cost of inefficiency for using the inefficient policy in other states. For any strictly positive level of inefficiency, however, there are (nonpathological) distributions of the state of nature under which legislators use the inefficient instrument in all equilibria.

The theory links both the political system and the level of economic stability to the choice of policy instrument. To see this, fix the level of economic stability and consider the political system. If decisions require super-majorities or must be approved by multiple legislative chambers or interest groups, then there is a larger set of veto players, and the ideological distance between the two most extreme pivotal players is widened. Gridlock can be substantial in these circumstances, creating room for the use of easily repealable instruments. The relationship between ideological polarization and the use of inefficient instrument, however, turns out to be non-monotonic in our framework. The relative ease of repealing inefficient policies makes such policies attractive interventions for moderate levels of ideological polarization, but not when ideological polarization is small or when it is large. To see why, note that, when the pivotal players have sufficiently similar preferences, they are likely to agree on when to repeal an efficient intervention. Conversely, when their preferences are sufficiently polarized, they are likely to disagree on when to repeal either type of policy intervention, in which case the strategic benefit of an inefficient intervention is too small to outweigh its cost.

To understand the effect of economic volatility, fix the political system. For stable economic environments, the current state of nature changes little over time and any policy intervention can be expected to persist for quite some time. Consequently, the expected cost of intervening with an inefficient instrument is larger than the option value of being able to repeal such a policy more readily. On the other hand, if the current economic environment is volatile, the state of nature can vary considerably and the possibility of at least one veto player preferring to repeal an intervention relatively quickly can be high. The relative ease with which inefficient interventions are repealed, therefore, makes use of such instruments attractive in this situation.

We show that all veto players can be strictly better off in equilibrium if the inefficient intervention is available as a policy option. Hence, our paper not only offers a rationale for the use of inefficient policy instruments, but also implies that they can be beneficial given the political constraints induced by a collective choice mechanism with multiple veto players. The less interventionist player benefits from the availability of the inefficient intervention because it is easier to repeal, and the more interventionist player benefits because it makes policy intervention politically feasible.

The model's logic can be applied to a variety of settings, including infant industry protection, environmental regulation, fiscal consolidation and financial regulation.

Temporary protection of infant industries. Trade theorists (see, e.g., Bardhan 1971) have argued that in the presence of dynamic learning externalities, protecting an infant industry from foreign competition (or protecting an established industry from a temporary surge in foreign competition) can raise social welfare. The literature has further shown that subsidies are preferable to tariffs because they do not distort consumption and, because of the double-dividend effect, that tariffs are preferred to quotas and other non-tariff barriers to trade. However, an important condition for these measures to be socially desirable is that they must be repealed when the industry matures (or when the temporary increase in foreign competition vanishes), although policy makers do not know ex-ante when that will occur (Melitz 2005). The logic of our model suggests that the more free-trade oriented party might prefer to protect the domestic industry with inefficient non-tariff barriers to trade for fear that the more protectionist party will veto a repeal of more efficient protectionist policies. In fact, since WWII, governments have increasingly relied on non-tariff barriers to trade to adapt trade policies to changes in trade flows (Bagwell and Staiger 1990).

Environmental regulation. Despite the sometimes considerable differences in perspective, economists from left to right tend to recommend Pigou taxes to regulate an externality because they are cost-effective, require little information, and offer a "double dividend" whereby emission taxes generate public revenues that allow governments to reduce other distortionary taxes.[1] It is thus "a mystery", according to some economists,[2] why the Repub-

[1] Weitzman (1974) compared quotas and taxes in a setting with incomplete information and zero value of the tax revenues. But starting with Tullock (1967), there is a large literature in economics on the double dividend. While a "strong" version of it is controversial, the "weak" version—that the revenues reduce overall distortions compared to a setting without these revenues—is generally accepted. Only the weak version is required for the argument we make here. For surveys on the literature on the double dividend, see Bovenberg (1999), Sandmo (2000), Goulder (2002), or Jorgensen et al. (2013). In part because of the double dividend, all but four of fifty one prominent economists surveyed in 2011 agreed that a carbon tax would be the less expensive way to reduce carbon-dioxide emissions. (http://www.igmchicago.org/igm-economic-experts-panel/poll-results?SurveyID=SV_9Rezb430SESUA4Y)

[2] On this "mystery," see: http://www.nytimes.com/2015/07/01/business/energy-environment/us-leaves-the-markets-out-in-the-fight-against-carbon-emissions.html. While Pigou taxes are relatively rare also in-

lican Party in the US blocked such a market-based policy during the Obama administration, since doing so effectively led to the command-and-control regulation of power plants introduced by that administration in 2015. In line with our model, however, some key Republicans may have anticipated that the administration would impose some curbs on the energy sector regardless, forcing them to use inefficient (command-and-control) instruments that would be easier to repeal once Obama's term was completed.[3] And, at the time of writing, the Republican administration is indeed working to repeal the regulations. It is hard to envision that the same attempt would occur if the intervention to be repealed was an efficient carbon tax combined with a lump-sum subsidy or a tax offset.

Fiscal consolidation. Consider a country deciding how to consolidate its fiscal policy after a shock has put its public debt on an unsustainable path. Although its government can do so along a variety of policy dimensions, we illustrate the logic of our argument by considering only one possibility, namely whether to focus on increasing revenues or decreasing outlays. Suppose, as the existing empirical evidence suggests, that a spending cut is less contractionary and thus statically preferred by the policy makers to a tax increase. In that case, the less interventionist veto player is the veto player ideologically least inclined to implement a spending cut, that is, the liberal veto player. Our theory suggests that the liberal veto player may veto the spending cut and support instead a more costly tax increase in anticipation that, once the fiscal situation improves, it will be easier to convince the conservative player to decrease taxes than to increase spending to its pre-crisis level.[4] Consistent with this logic, there is empirical evidence indicating that fiscal adjustments

ternationally, a famous exception is British Columbia, which introduced a carbon tax in 2008. Although initially controversial, the tax has gained support from all important stakeholders thanks to the rebates in other taxes that the revenues permit (http://www.nytimes.com/2016/03/02/business/does-a-carbon-tax-work-ask-british-columbia.html?smid=pl-share&_r=0).

[3] Jim Manzi, a prominent conservative commentator on climate change, said openly that "a carbon tax would be, mostly likely, a one-way door: Once we introduce it we're stuck with it for a long time. What if our economic and climate models are too aggressive, and there is no practical economic justification for emissions reductions [...] There are very large potential regrets to a carbon tax." (In "Conservatives, Climate Change, and the Carbon Tax", *The New Atlantis*, 2008 (21), 15-25)

[4] See Alesina et al. (2017) for a recent literature review. The findings of that literature are still subject to intense debate, but we would like to point out that the logic of our model applies equally to the opposite case in which a tax increase is more efficient than a spending cut, the only difference being that the interventionist player is then the liberal veto player. Our findings can also be applied to the reverse problem of fiscal stimulus. In that case, the government must choose between a spending increase of a tax cut. The logic of our model does not depend on which of the two is more efficient in that case either, but suppose for concreteness that tax cuts are more expansionary, as is suggested by the existing empirical evidence (see, e.g., Mankiw 2010 and the references therein). In that case, our model offers the following rationale. After an economic contraction, the liberal veto player foresees that if a tax cut is implemented, the conservative veto player will be reluctant to increase taxes back to their pre-crisis level once the economy grows again. As a result, the liberal veto player prefers a spending increase, because it will be easier to convince the conservative veto player to cut spending once the crisis is over.

based on spending cuts are longer-lived than fiscal adjustments based on tax increases, i.e., the efficient adjustment is more persistent than the inefficient one (e.g., Alesina et al. 1998, Alesina and Ardagna 2013).[5]

Financial regulation. Financial regulation tends to respond to financial crises. For example, after the financial crisis of 2007-2008, the U.S. Congress passed the Dodd-Frank Wall Street Reform and Consumer Protection Act. Although hailed by many as a step in the right direction, it is a complex piece of legislation that others consider inefficient. Its provisions rely on heavy government regulation instead of price instruments recognized as more efficient at curbing systemic risk. Applying the same lens here as for the environmental regulation example, the inefficiencies can be viewed as a price paid by liberals to insure at least some regulation was implemented and, from the perspective of the more laissez faire members of Congress at the time, an acceptable intervention in response to the fallout from the crisis but one that can be more easily unpacked in economically calmer times. Consistent with this view, on June 9 2017, The Financial Choice Act, legislation that would "undo significant parts" of Dodd-Frank, passed the House 233-186.

Related literature. We are not the first ones to offer an explanation for why governments implement inefficient policies. However, the logic that underlies our argument is, to the best of our knowledge, novel: inefficient interventions are more likely to be repealed should circumstances change, making them more likely to be accepted in the first place by all veto players. On the abstract level, there are three main features that distinguish our paper from the literature. First, the mechanism does not depend on the specificities of the economic environment, or on how the policy interacts with the private sector or the electorate. Instead, inefficiency arises solely from the conflict of interests between legislators and the need to adapt the policy to a changing environment. Second, the inefficient policy in our model is inefficient in a static sense: there exists a policy that gives a strictly greater flow payoff to all relevant decision makers in all states of nature. Third, the inefficient policy is not only the result of status quo inertia whereby a previously optimal policy becomes obsolete. Instead, it is actively implemented by the policy makers.

The paper closest to ours is Dziuda and Loeper (2016), who analyze a similar model except that they restrict the policy set to policies that are statically Pareto undominated for some states. They show that inefficient inertia occurs because each pivotal player fears that policy changes approved by her will be hard to repeal when she wishes to do so. As a result, the status quo can persist even when Pareto dominated. Hence, any inefficiency

[5] An alternative explanation for why tax hikes might be chosen even when they are less efficient than spending cuts is that the latter hurt powerful constituencies such as retirees or unions. However, that explanation is harder to reconcile with empirical findings that governments whose austerity programs focus on spending cuts are no less likely to be reelected than those who focus on tax increase (Alesina et al. 1998).

takes form of status quo inertia and any policy change is a Pareto improvement. Riboni and Ruge-Murcia (2008), Zapal (2011), Duggan and Kalandrakis (2012), Bowen et. al. (2017), and Dziuda and Loeper (2018) consider related models of dynamic legislative bargaining which also lead to policy inertia. Relative to these papers, our contribution is to show that adding a Pareto inefficient alternative to the policy space can mitigate policy inertia. In particular, legislators may adopt a *Pareto-dominated* policy change.[6]

Since our model requires policy makers to respond to shocks, it is related to the literature on policy reforms. Alesina and Drazen (1991) show that legislators can engage in war of attrition over who should bear the costs of reform. Fernandez and Rodrik (1991) and Ali, Mihm and Siga (2017) show that uncertainty over the distributional impact of reforms may stifle them. Spolaore (2004) compares the likelihood of policy adjustment across three stylized institutions. In Strulovici (2010), an endogenous status-quo bias arises if a majority learns that the new policy is beneficial for them. Anticipating this situation, policy makers may not want to try out new policies in the first place. Unlike in our model, these papers consider the policy response to a single shock after which legislators' policy preferences are fixed over time and restrict attention to efficient policy adjustments. Hence, inefficiency only takes the form of delays or failure to intervene. In contrast, legislators act without delay in our model but the solution they adopt is inefficient.

There is also large political science literature that explores inefficient policy making due to structural characteristics of legislative decisionmaking (e.g., the filibuster or committee structure). Important examples here include Krehbiel (1998) and Brady and Volden (2006), who explore models of gridlock, Ortner (2017), who shows that gridlock is likely near the next election, and Weingast, Shepsle and Johnson (1981) and Cox and McCubbins (2000), who analyze legislative structure and inefficient public good provision more generally. To our knowledge, however, there is as yet no analysis focusing directly on the deliberate strategic choice of inefficient policy change when policy change occurs.

Inefficient policy choices by a unitary actor, whether a single legislator or a fully coordi-

[6] In a distributive environment, Bowen et. al. (2014) and Anesi and Seidmann (2015) show that endogenous status quo can lead to Pareto inefficient policies, because they allow the proposer or the supporting coalition to extract greater transfers in the future. In these papers, preferences do not evolve over time, as in most of the literature on dynamic policy making with an endogenous status quo. Policy dynamics occur because the proposer changes (e.g., Baron 1996, Bernheim, Rangel and Rayo 2006, Kalandrakis 2004, Anesi and Duggan 2017, Buisseret and Bernhardt 2017), or because the same proposer forms different coalitions over time (e.g., Diermeier and Fong 2011). In contrast, in our model, each proposer always seeks the support of the same policy maker and the qualitative nature of policy inefficiencies is, by and large, independent of the allocation of bargaining power. A different strand of literature assumes that the implemented policy influences future states (see Hassler et al., 2003, for example). Baldursson and Von Der Fehr (2007) is closer to our story, as they argue that a relatively "brown" party may prefer quotas rather than taxes, because the relatively inefficient quotas are essentially property rights that are difficult to tighten or remove later.

nated party or group, can be also explained by aspects of the political economic environment other than legislative design. In Coate and Morris (1995), Acemoglu and Robinson (2001), and to some extent Glaeser and Ponzetto (2014), inefficient policy choices arise from an incumbent legislator's efforts to retain office.[7] Whereas the Coate and Morris (1995) and Glaeser and Ponzetto (2014) accounts rest on asymmetric information, Acemoglu and Robinson (2001) generate policy inefficiency from a model in which distortionary transfer payments are designed to counter declining political support and influence. Tullock (1993), Grossman and Helpman (1994), Becker and Mulligan (2003) and Drazen and Limao (2008) argue that any resource transfer increases wasteful lobbying (rent-seeking) activity. By committing itself to inefficient transfers, the government can reduce the level of wasteful lobbying. More generally, there is an extensive literature on policy distortions induced through special interest groups' lobbying and campaign contribution activities (see Wright 1996 and Grossman and Helpman 2001 for overviews of the literature)

Aidt (2003) claims that inefficient command-and-control instruments are more bureaucracy intensive and, to the extent that bureaucrats influence policy design and derive value from implementing policy, such interventions are favored by bureaucrats. Alesina and Passarelli (2014) and Masciandaro and Passarelli (2013) offer explanations of socially suboptimal policies that hinge on the median voter failing to internalize the costs and benefits to others when policies have different distributional consequences. However, both available policy choices are Pareto optimal in these papers. In contrast to these approaches, inefficiency in our model does not depend on groups, informational asymmetries, or reelection concerns.

2 A Simple Example

In this section we present a stylized example to illustrate the key mechanism and to preview some of our results. The mechanism requires two pivotal players, or legislators, L and R, both of whom must approve any policy change. Specifically, at the start of any legislative period, nature randomly chooses one legislator to propose a change in, or maintain, the status quo policy. The other legislator has a veto right over any proposed change in the status quo. Legislators start with no intervention, denoted by n, and can intervene by introducing either an efficient instrument p or an inefficient instrument q. The flow-payoff from policy n is normalized at 0. Intervention gives everyone a benefit θ, and the costs associated with p and q are w_i and $w_i + e_i$, respectively, for $i \in \{L, R\}$. We assume that $e_i > 0$ for each i, so that e_i is the additional benefit of the efficient instrument.

[7] See also Canes-Wrone, Herron and Shotts (2001) and Buisseret and Bernhardt (2018) for how electoral incentives can lead to policy distortions.

We can immediately make some simple observations:

Proposition 0 *Suppose there is only one period. Then the inefficient policy q is never implemented in equilibrium.*

Suppose now that there are two periods and let $\delta > 0$ be the common discount factor. The status quo in the first period is n and the first-period policy becomes the status quo in the second period. Assume further that there are only two states, $\underline{\theta}$ and $\bar{\theta} > \underline{\theta}$, with $\bar{\theta}$ occurring with probability π, and suppose that the costs associated with p and q satisfy

$$w_L < \underline{\theta} < \min\{w_L + e_L, w_R\} \leq \max\{w_L + e_L, w_R + e_R\} < \bar{\theta}. \tag{1}$$

The last inequality in (1) means that in state $\bar{\theta}$, both players prefer any type of intervention to n. The first two inequalities in (1), however, imply that in state $\underline{\theta}$, the less interventionist player R prefers no intervention, while the more interventionist player L prefers to intervene but only if the intervention is with the efficient instrument. In terms of our environmental application, $\bar{\theta}$ can be interpreted as the usual state of the economy in which both parties agree that environmental interventions are desirable, while $\underline{\theta}$ can be interpreted as an economic downturn that makes the R party (but not the L party) want to repeal any regulation that can compromise economic growth. In the case of fiscal consolidation, $\underline{\theta}$ may be interpreted as a business-as-usual state in which parties differ ideologically on whether public spending should be cut, while $\bar{\theta}$ can be interpreted as a fiscal crisis state in which both parties are willing either to cut spending or to increase taxes to bring public debt under control.

Consider the last period in this game. In state $\bar{\theta}$, both players strictly prefer p to any other policy. So, independently of which player has proposal rights, p is implemented in that state. What is implemented in the low state $\underline{\theta}$, however, depends on the status quo. If n is the status quo, R does not approve (propose or accept, depending on the allocation of proposal rights) any change in policy. Similarly, if p is the status quo, L does not approve of any change in policy. Finally, if q is the status quo, then the two players agree that either n or p are better than the status quo, but they disagree on which is best. In this situation, p is implemented if L has the proposal power and n is implemented otherwise. This reasoning implies that, in the second period, p is not *repealable*, but q can be repealable when the realized state in that period is $\underline{\theta}$.

Consider now the first period, and suppose the state is $\bar{\theta}$ with status quo n. Both players' first period flow-payoffs are maximized by policy p. But since p is not repealable, R may be reluctant to approve such a change in policy. In particular, R does not approve p in the first period if the benefit of p relative to n in state $\bar{\theta}$ is outweighed by the expected cost of being stuck with p in state $\underline{\theta}$; that is, if:

$$\overline{\theta} - w_R < \delta \left(1 - \pi\right) \left(w_R - \underline{\theta}\right). \tag{2}$$

Thus, players fail to intervene efficiently in the first period if the disagreement state $\underline{\theta}$ is relatively likely (i.e., $1-\pi$ is high), if players are patient, and if R's preference for intervention in state $\overline{\theta}$ is relatively weak compared to R's preference for no intervention in state $\underline{\theta}$. On the other hand, because q is more easily repealed than p, R may be willing to intervene with q. Denoting by $b_L \in [0,1]$ the probability that L has the authority to make a take-it-or-leave-it policy proposal in the second period, R approves an intervention q in the first period if

$$\bar{\theta} - w_R - e_R \geq \delta \left(1 - \pi\right) b_L \left(w_R - \underline{\theta}\right). \tag{3}$$

And since L receives a higher flow-payoff from q than n in state $\bar{\theta}$, and the likelihood that a change from status quo q to p in the second period exceeds that from n to p, L *also* prefers and approves q over n in the first period. Finally, because players' share the same ordinal policy preferences over n and q in both states, we have the following proposition.

Proposition 1 *Suppose there are two periods and*

$$\delta \left(1 - \pi\right) b_L \left(w_R - \underline{\theta}\right) + e_R \leq \overline{\theta} - w_R < \delta \left(1 - \pi\right) \left(w_R - \underline{\theta}\right).$$

Then, in the first period of any subgame perfect equilibrium,
(i) intervention occurs in $\bar{\theta}$ when the policy menu is $\{n,p,q\}$ but not when the menu is $\{n,p\}$;
(ii) both players strictly prefer menu $\{n,p,q\}$ to menu $\{n,p\}$.

The fact that efficient policies are hard to repeal can make them politically impossible to agree upon in a dynamic setting. At the same time, as part *(i)* of the proposition states, it is precisely *because* of its inefficiency that both legislators may approve a first period intervention with q in state $\bar{\theta}$. And, as part *(ii)* of the proposition asserts, adding this statically Pareto-dominated choice to the policy menu may not only result in its use, but it also strictly improves equilibrium payoffs for both players.

To our knowledge, the preceding claims are new to the literature. They are, however, derived in a stylized example that raises a number of questions. For instance, the cost of instrument q is limited since q will always be replaced by either n or p in the last period. But what is the desirability of q in a dynamic model when there is no last period? Furthermore, the binary state space in the example implies that instrument p will never be repealed once implemented. But why should the players prefer q to p in a more general setting where both interventions may eventually be repealed? To explore these and several other issues further, the following section generalizes the model to an infinite number of periods and to more general distributions of states.

3 Model

Policies, payoffs, and players. Two infinitely lived players, L and R, must decide in each period $t \in \mathbb{N}$ which of three policies $\{n, p, q\}$ to implement. Their preferences over these policies in a given period t depend on the realization of the state of nature $\theta_t \in \mathbb{R}$, and are specified as in the preceding example. That is, normalizing both players' flow-payoff from policy n in any state θ to zero, $U_i(\theta, n) = 0$, $i \in \{L, R\}$, player i's flow-payoff from policies p and q relative to policy n in θ are, respectively,

$$\begin{aligned} U_i(\theta, p) &= \theta - w_i, \\ U_i(\theta, q) &= \theta - (w_i + e_i). \end{aligned} \tag{4}$$

Thus, e_i is the period flow-payoff gain for player i from implementing p instead of q in state θ. For simplicity, we assume this gain is independent of θ. More importantly, we assume that $e_i > 0$ for both players. That is, intervention p Pareto dominates intervention q in all states of nature. Although the state of nature does not affect which intervention is best, it affects whether an intervention is needed in the first place: given the zero flow-payoff from no intervention, n, player i gets a greater flow-payoff from policy p than from policy n when $\theta \geq w_i$, and a greater flow-payoff from q than from n when $\theta \geq w_i + e_i$. Importantly, we assume that $w_L \neq w_R$, that is, in some states of nature, players disagree whether the efficient intervention p is preferred to no intervention n. By convention, L denotes the more interventionist player, so $w_L < w_R$.[8]

Timing of the game. Every period $t \in \mathbb{N}$ starts with some status quo $s_t \in \{n, p, q\}$, with $s_0 = n$. At the beginning of period t, both players observe the state $\theta_t \in \mathbb{R}$. After θ_t is observed, L and R must collectively choose a policy from the set $\{n, p, q\}$. We assume that one player makes a take-it-or-leave-it offer to the other regarding which policy to implement. The recognition probability for player i is denoted by $b_i(\theta, s)$, which may depend on the current state θ and status quo s. We assume that b_i is bounded below by some $\underline{b} > 0$. The recognized *proposer* offers a policy $y_t \in \{n, p, q\}$. If the other player, the *veto-player*,

[8] The terms "instrument" and "intervention" are used exclusively in reference to alternatives p and q. The term "policy" may refer to any of the available alternatives, including n. This looseness should cause no confusion. Also, we say that a policy p or q is "repealed" when it is the status quo and players agree to replace it by n.

accepts this proposal, then y_t is implemented; otherwise, the status quo s_t stays in place. The policy implemented in t, whether the proposal y_t or the status quo s_t, generates the flow-payoff for that period, as measured by (4), and becomes the status quo in the next period, $t+1$. Each player maximizes her expected discounted payoff over the infinite horizon. The common discount factor is $\delta \in (0,1)$. For simplicity, we initially assume $\{\theta_t : t \geq 0\}$ are distributed identically and independently over time according to some continuous c.d.f. F with full support. The assumption is relaxed in section 5 to permit serial correlation.

Equilibrium concept. We denote the above game by Γ and restrict attention to stationary Markov-perfect equilibria, referred to as "equilibria" in what follows. A stationary Markov-perfect equilibrium is a subgame-perfect equilibrium in which players use stationary Markov strategies. In this game, a strategy is stationary Markov if it depends only on the current state, the current status quo, the identity of the proposer, and the current proposal at the action node of the veto player.[9] Let σ_i denote i's stationary strategy and write $\sigma = (\sigma_L, \sigma_R)$.

Since our interest regards the use of the inefficient instruments, throughout most of the paper we focus on equilibria with this property. To this end, the following definition is useful.

Definition 1 *Let σ be an equilibrium of Γ. Then, σ is an* ***instrument inefficient equilibrium*** *(IE) if q is implemented with positive probability on the equilibrium path; σ is an* ***instrument efficient equilibrium*** *(EE) otherwise.*

Remarks on the assumptions. Some of the assumptions are made for simplicity and relaxing them may not change the results. In particular, binary policy levels are not necessary for the results. To see this, suppose that players can choose any level of p or q and consider a two-period example again. The inefficient instrument q will not be used in the last period and the set of states under which p is repealed will be independent of its level. The inefficient q at any level, however, will still be easier to repeal, and the logic of our paper applies.

However, three assumptions are crucial. First, the mechanics of the model rests on multiple veto players. With a unicameral legislature taking decision under simple majority rule, the policy maker i with the median w_i would be the unique pivotal decision maker. However, multiple veto players are natural in politics. Bicameralism, supermajority requirements, presidential veto power, or powerful interests groups imply the existence of a set of

[9] Thus, a *stationary Markov strategy* for player $i \in \{L, R\}$ consists of two contingent actions. First, a function that maps the current state and status quo into a policy proposal conditional on i being the proposer. Second, a function that maps the current state, status quo, and proposal into a choice over accepting or rejecting the proposal conditional on i being the veto-player. Mixed strategies are admissible. More formally, writing ΔS for the set of probability distributions over a set S, i's proposal strategy takes $\mathbb{R} \times \{n, q, p\}$ into $\Delta \{n, q, p\}$; and i's veto strategy takes $\mathbb{R} \times \{n, q, p\}^2$ into $\Delta \{accept, reject\}$. The restriction to stationary Markovian strategies is standard in the literature on dynamic bargaining with endogenous status quo. We conjecture that qualitatively different equilibria would arise if history-dependence were allowed.

veto-players, or pivots, whose approval is necessary and sufficient to enact a policy change. In the case of a unicameral legislature taking decisions under a qualified majority $m \in \left(\frac{1}{2}, 1\right]$, player L is such that exactly a fraction m of the w_i's are larger than w_L and, for player R, exactly m of the w_i's are smaller than w_R. Thus, the degree of heterogeneity, or polarization, $w_R - w_L$, increases in the majority requirement m.

Second, we assume away explicit side-payments. If the players could make unlimited side-payments then only efficient policies would be implemented in equilibrium. In particular, policy p would be implemented when $\theta_t > (w_L + w_R)/2$, and policy n would be implemented otherwise. Explicit side payments, however, are rare and often unavailable in politics, partly because there may not exist a third party that can enforce agreements on such transfers.

Third, we assume that any implemented policy stays in place until it is actively changed. This is consistent with legislative practice. Most laws and policies enacted by the U.S. Congress, for example, are permanent: they remain in effect until a new legislative action is taken. This is the case for mandatory spending policies, which include all entitlements, currently about 60% of total federal spending (Austin and Levit 2010), constitutional amendments, most statutes in the U.S. code, the Senate's rules of proceedings, and international treaties. Likewise, changes to the tax code are permanent unless legislators decide to attach a sunset provision, that is, a clause that specifies a period after which the relevant legislative act automatically expires. Historically, attaching sunset clauses to legislation has been the exception rather than the norm.[10] Nevertheless, legislators are not prohibited from attaching such limitations to any legislation, in which case the availability of sunset clauses seems, at least prima facie, to remove the need for inefficient instruments. In the supplementary appendix, however, we present an example that this is not necessarily the case. A careful analysis of sunsets is beyond the scope of this paper.

4 Analysis

The following lemma states that an equilibrium exists, and that all equilibria have a relatively simple structure: in any equilibrium of Γ, players behave as if they were playing a static version of the game Γ (a single period) with flow-payoff parameters $(w_L^\sigma, w_R^\sigma, e_L^\sigma, e_R^\sigma)$ rather than (w_L, w_R, e_L, e_R).[11]

[10] See, e.g., Posner and Verneule (2002, pages 1672, 1694, and 1701) on the permanent nature of statutes, the Senate's internal rules, or international treaties. As for tax legislation, prior to the Bush administration, the use of sunsets for changes in the tax code applied mainly to relatively small provisions known as "tax extenders" and were of significantly smaller scale (Gale and Orszag 2003; Mooney 2004).

[11] This simple representation of the equilibria relies on the stationarity of σ and on $\{\theta_t\}$ being i.i.d. (an assumption we relax in Section 5). If σ was nonstationary, the same expression would hold but the function V_i^σ and the parameters e_i^σ and w_i^σ would have to be indexed by the period t from which the continuation

Lemma 1 *There exist equilibria in* Γ*. Moreover, for any equilibrium* σ*, there exists a unique tuple* $(w_L^\sigma, w_R^\sigma, e_L^\sigma, e_R^\sigma) \in \mathbb{R}^4$ *such that the behavior prescribed by* σ *is the same as the behavior prescribed by an equilibrium of the game in which players play a single period of* Γ *with payoffs* $V_i^\sigma(\theta, n) = 0$ *and*

$$\begin{aligned} V_i^\sigma(\theta, p) &= \theta - w_i^\sigma, \\ V_i^\sigma(\theta, q) &= \theta - (w_i^\sigma + e_i^\sigma). \end{aligned} \tag{5}$$

All proofs are in the Appendix. We call $\{w_L^\sigma, w_R^\sigma, e_L^\sigma, e_R^\sigma\}$ the continuation payoff parameters induced by σ and note that they reflect players' strategic preferences in the equilibrium, that is, their policy preferences given continuation play σ, as distinct from their exogenous "ideological" preference parameters $\{w_L, w_R, e_L, e_R\}$.[12]

The main goal of this paper is to understand the strategic underpinnings of IE and why they can be beneficial. Therefore, for the sake of exposition, we focus only on IE. We first derive the properties of IE (Section 4.1) and then focus on their existence (Section 4.2).

Before we proceed, however, let us say a few words about the EE. By definition, in any EE, the inefficient instrument is not used; hence, EE are essentially equivalent to the equilibria of the two-alternative game of Dziuda and Loeper (2016). They show that in such equilibria, the more pro-intervention player distorts her votes in favor of p, and the less pro-intervention player distorts her votes in favor of n. As a result, even EE are inefficient in that there is excessive status quo inertia: a policy that was once adequate for the environment becomes obsolete, but players do not act. This kind of inefficiency, however, does not explain the puzzle outlined in the introduction, namely, why interventions that are inefficient in any state of the world are implemented. So IE differ from EE not in whether the equilibrium path is inefficient, but in the nature of inefficiency.

4.1 Properties of IE

The following proposition summarizes the main qualitative properties of any IE.

Proposition 2 *Let* σ *be an IE. Then the corresponding continuation payoff parameters* $(w_L^\sigma, w_R^\sigma, e_L^\sigma, e_R^\sigma)$ *satisfy the following inequalities for some distinct* $i, j \in \{L, R\}$*:*
(i) $w_i^\sigma < w_j^\sigma$,
(ii) $e_j^\sigma \leq 0$ *and* $e_i^\sigma > 0$,
(iii) $w_i^\sigma < w_j^\sigma + e_j^\sigma$.

payoff is computed. Likewise, if $\{\theta_t\}$ was not i.i.d., e_i^σ and w_i^σ would depend on θ.

[12] Duggan and Kalandrakis (2012) provide a very general existence result for dynamic bargaining games with an endogenous status quo. But to apply their result directly here requires violating our assumption that $U(\theta, p) - U(\theta, q)$ is constant in θ. Although introducing some noise to the payoffs, to ensure the difference is not locally constant, and letting that noise tend to zero is possible, the result would be a correlated equilibrium that obscures the particular tradeoffs of interest here.

Part *(i)* of Proposition 2 states that in any IE, one player $i \in \{L, R\}$, is more interventionist than the other player, j. Part *(ii)* states that, in equilibrium, players disagree on the appropriate intervention. In all states, player i prefers using the efficient instrument p to the inefficient policy q, whereas player j weakly prefers intervening with q rather than p. Finally, part *(iii)* implies that, consistent with the intuition provided in the Introduction, the inefficient instrument q is easier to repeal than the efficient instrument p. To see this, assume p is the status quo. Then $e_i^\sigma > 0$ implies player i always vetoes proposal q and, by definition of w_i^σ, i also vetoes proposal n for all states above w_i^σ. Hence, in all such states, the status quo p remains unchanged. Conversely, for all states below w_i^σ, i prefers n to any instrument, so i prefers to repeal p. And since $w_i^\sigma < w_j^\sigma$, j also prefers to repeal p. Thus, p is repealed in all states below w_i^σ. Both players, however, prefer n to q in all states below $w_j^\sigma + e_j^\sigma$. So (iii) implies that q is repealed on a larger set of states than is p.

When $e_j^\sigma = 0$, the policy dynamics implied by Lemma 1 and Proposition 2 may be quite complex due to possible mixing by player j. For $e_j^\sigma < 0$, however, there are only two possible equilibrium paths, illustrated in Figures 1 and 2 below, depending on whether $\max_k\{w_k^\sigma + e_k^\sigma\} < w_j^\sigma$ (Figure 1) or $w_j^\sigma < \max_k\{w_k^\sigma + e_k^\sigma\}$ (Figure 2). For concreteness, in Figures 1 and 2 we illustrate these two possibilities for the equilibria in which $i = L$ and $j = R$.[13] States are measured along the horizontal axis and policies are indicated by the three shaded bars above this axis. An arrow identifies an equilibrium policy change, conditional on the status quo, for any realized state within the same interval of states as that in which the arrow is drawn. For intervals of states where there is no arrow drawn, there is no equilibrium policy change. The status quo at the start of any period in which there is a policy change is indicated by the origin of the relevant arrow; the arrow head indicates the policy instrument chosen to replace the status quo.

In Figure 1, for example, suppose that the status quo going into the current period is n and the realized state is $\theta \in (\max_k\{w_k^\sigma + e_k^\sigma\}, w_R^\sigma)$. Player L, prefers to intervene with p, but state θ is high enough so that she prefers to intervene with either instrument rather

[13]Since $w_L < w_R$, it is natural to conjecture that necessarily, $i = L$. However, one can show that if $e_R > e_L$, then for some F, there also exists an equilibrium in which $i = R$, so somewhat surprisingly $w_R^\sigma < w_L^\sigma$. In such equilibria, the relatively more interventionist L approves p *less* frequently than R in equilibrium. Intuitively, if e_R is sufficiently large, then whenever R prefers to intervene at all, R strictly prefers to intervene with p rather than q. But then, for status quo n and θ very large, R would approve a proposal q by L. To hedge against such a possibility, R's best response strategy involves a bias against n when p is the status quo; that is, if p is the status quo and the state is not-so-large as to warrant intervention (relative to w_R), R blocks repealing p. Hence, for large enough bias, $w_R^\sigma < w_L^\sigma$, which means that p stays in place for a larger set of states that L would like. Anticipating R's bias and the resulting inertia under status quo p, under status quo n, L proposes to intervene with q rather than with p, rationalizing R's best response bias. Nevertheless, one can show that there always exists an equilibrium such that $w_L^\sigma < w_R^\sigma$, though this equilibrium need not be IE. Details available upon request.

than stay with n. Player R prefers to intervene with q, as the expected benefit from q being easier to repeal outweighs the inefficiency cost, but the difficulty of repealing p makes her prefer no intervention n to p. Hence, q is the only alternative that dominates n for both players in that state. As a result, q is implemented independently of who is the proposer, as indicated by the upward-pointing arrow. On the other hand, if $\theta > w_R^\sigma$, the intervention is so desirable that both players prefer intervening with either policy instrument. Since they differ with respect to their ordering of p and q, the implemented policy depends on who has proposal power. This is indicated by labeling the upward-pointing arrows for such states by the proposer's identity. When the status quo is either p or q, the downward-pointing arrows indicate those states in which the relevant instrument is repealed. As stated in Proposition 2, q is repealed for a larger set of states.

Figure 2 differs from Figure 1 only in that the less interventionist player, R, is willing to accept p in some states for which L is unwilling to accept the inefficient alternative q. As a result, for $\theta \in (w_R^\sigma, \{w_L^\sigma + e_L^\sigma\})$, R is forced to propose p and, therefore, p is implemented independently of who has proposal power.

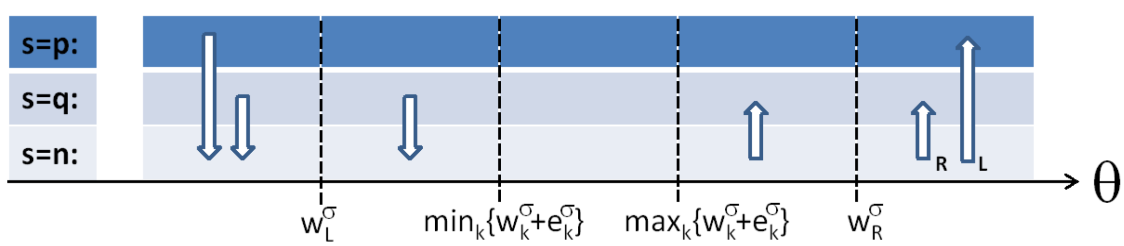

Figure 1. An example of IE with both players choosing q for some states.

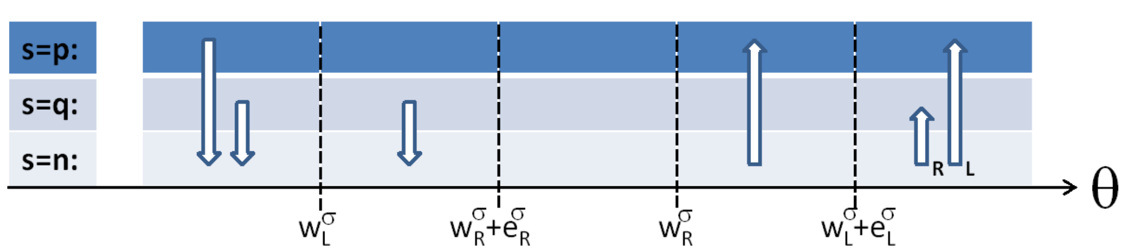

Figure 2. An example of IE with both players choosing p for some states.

4.2 Use of Inefficient Instruments in Equilibrium

In this section, we investigate under which condition *all* equilibria are IE. The following proposition shows that this can be the case for any payoff parameters (w_L, w_R, e_L, e_R). In particular, no matter how inefficient the policy instrument q is, all equilibria may be IE.

Proposition 3 *Let G be a c.d.f. with mean 0 and variance 1. For any (w_L, w_R, e_L, e_R), there exists $\bar{d} > 0$ and intervals of positive length, $\Delta \subset (0,1)$ and $M \subset \mathbb{R}$, such that, for any $\delta \in \Delta$, $m \in M$ and $d \in \left(0, \bar{d}\right)$, all equilibria of Γ are IE for the c.d.f. $F(\theta) \equiv G\left(\frac{\theta - m}{d}\right)$.*

The proposition is predicated on the observation that two things must be true of the distribution of states for all equilibria to be IE. First, extreme states must be sufficiently rare, i.e., d sufficiently small, else players would agree most of the time on whether or not to intervene. As a result, their preferences over which instrument to use would not be distorted by their expectation of future disagreements. Second, states in which players disagree about repealing p but agree on repealing q must be sufficiently likely. Using Figures 1 and 2, this means that for d sufficiently small, for all equilibria σ, the states around the mean m of the distribution must be in the interval $(w_i^\sigma, \min_k\{w_k^\sigma + e_k^\sigma\})$.[14]

In the supplementary appendix (see Proposition 8), we further show that q can be implemented arbitrarily more frequently than p for some distributions G. For example, if G is normal then, for any $\varepsilon > 0$, there exist an m and v such that, under status quo n and conditional on some intervention being implemented, the probability that q is implemented relative to p is greater than $(1 - \varepsilon)$.

The following proposition links the use of the inefficient instrument to the degree to which q is inefficient (parts *(i)* and *(ii)*) and to players' ideologies are polarized (part *(iii)*).

Proposition 4 *Fix $\delta \in (0,1)$, the recognition probability functions b_L and b_R, and the c.d.f. F. Then:*

(i) For any (w_L, w_R), if all equilibria are IE for some (e_L, e_R), then all equilibria are IE for all (e'_L, e'_R) such that $e'_L \geq e_L$ and $e'_R \leq e_R$.

(ii) For any (w_L, w_R) and any $e_L > 0$, there exists $\varepsilon > 0$ such that all equilibria are IE for any $e_R \leq \varepsilon$.

(iii) For any (e_L, e_R), and any average ideology $(w_L + w_R)/2$, there exists an EE as $(w_R - w_L) \to 0$. Furthermore, all equilibria are EE as $(w_R - w_L) \to \infty$.

The comparative statics in Proposition 4*(i)* on e_R is intuitive. If, for some e_R, R approves the inefficient intervention q in exchange for an increase in the likelihood that the intervention

[14] Similar result for the exclusive existence of EE holds. For instance, for any m and δ, one can show that as d becomes sufficiently large, that is, as extreme states become sufficiently likely, all equilibria are EE.

is repealed in the future, R also approves q for lower degrees of inefficiency. The claim in Proposition 4*(i)* regarding changes in e_L, however, is less obvious, since an increase in e_L has two effects. On the one hand, L approves repealing a status quo q for more states when e_L is large than when it is small. This, in turn, increases the strategic value of q for R. On the other hand, since a larger e_L implies that L's payoff from q is smaller, L approves any proposal to implement q in fewer states than when e_L is not so large. Regardless of the value of e_L, however, for θ large enough, L prefers q to n and R can be sure q is accepted and implemented on the equilibrium path. In particular, the inefficient policy instrument is used because it is costly for player L, not because it is costly for player R. Thus, as 4*(ii)* confirms, the inefficient instrument is always chosen (for sufficiently high states) if e_R is small enough (where "small enough" depends on w_R, w_L and e_L).

It is informative to reformulate Proposition 4*(i)* in terms of players' polarization. Recall that w_i and $w_i + e_i$ can be interpreted as the ideological position of player i on how often to intervene when using policy p and q, respectively. For a fixed (w_L, w_R), as e_L increases and e_R falls, the gap between the players' thresholds on policy p remains unaffected; for policy q, however, the difference between the two players decreases. Proposition 4*(i)* then says that as players become less ideologically polarized about intervention q, they agree to use the inefficient, but more consensual, policy instrument q more often.

Part *(iii)* of Proposition 4 concerns how the equilibria change with changes in legislative polarization on efficient intervention. Given an average ideology, existence of an EE is assured for sufficiently small polarization. Since the players' preferences are essentially aligned in this case, players are likely to agree on the repeal of p when necessary, so they do not need to resort to using inefficient q. On the other hand, when polarization is sufficiently large, the players are rarely aligned with respect to whether intervention is warranted. Thus, any policy is unlikely to be repealed and the players perceive the decision as virtually permanent. As in a static setting, both agree to an efficient intervention if they agree to intervene at all. In sum, therefore, part *(iii)* states that if an inefficient instrument is used in any equilibrium, then political polarization cannot be too small or too large.

4.3 The Value of Inefficient Instruments

In reality, the menu of available policy instruments is often endogenous. For example, if environmental policy is chosen at the local level, or by a regulatory body, then the instruments that are available may be restricted by the federal government and, in such cases, it is not at all clear whether an inefficient policy instrument would, or should, be made available to decision makers. As argued in Section 2, there is no social welfare gain to be had from the existence of q in a static environment; the question is whether this holds in dynamic settings.

The next result states that, under certain conditions, both players are strictly better off if the inefficient intervention q is available, even if the efficient intervention p is unavailable, than if they are constrained to choosing only between n and p. In other words, allowing for an inefficient policy instrument can lead to Pareto superior equilibria in the dynamic game.

Let $\Gamma(n,p,q)$ denote the original game, $\Gamma(n,p)$ the game in which the inefficient instrument is unavailable, and $\Gamma(n,q)$ the game in which the efficient instrument is unavailable. Say that one equilibrium is Pareto superior to another if both players get a strictly greater continuation payoff in the former.

Proposition 5 *For any (w_L, w_R, e_L, e_R) and for a nonnegligible set of $\delta \in (0,1)$, there exists an F such that any equilibrium of $\Gamma(n,p,q)$ and of $\Gamma(n,q)$ is Pareto superior to any equilibrium of $\Gamma(n,p)$.*

Proposition 5 states that for any payoff parameters, one can find an environment F in which having the inefficient instrument q available on the menu is welfare-improving. Proposition 5, however, does not shed light on how the benefits from q vary with the payoff parameters and F; that is, in which environments one should expect q to be welfare improving. Nevertheless, our previous results suggest some conjectures in this regard. Clearly, q can be beneficial to the players only if there is some ideological disagreement between them, as only then q can be used in equilibrium. At the same time, Proposition 4 implies that q is used only when players are not too polarized. Consequently, players may benefit from q for a given distribution F only when their ideological polarization is moderate. Similarly, players are also more likely to benefit from q if q is not too inefficient, but the inefficiency for the less interventionist player L must be sufficiently large to make q more easily repealed.

To investigate these conjectures, we consider an infinite horizon extension of the example in Section 2 with a distribution F having support $\{\underline{\theta}, \bar{\theta}\}$. In this simple environment, we have the following result.

Proposition 6 *Suppose there exists $\underline{\theta} < \bar{\theta}$ and $\pi \in (0,1)$ such that that, in every period t, with probability $1-\pi$, $\theta(t) = \underline{\theta}$ and with probability π, $\theta(t) = \bar{\theta}$.*
(i) No equilibrium of $\Gamma(n,p)$ is Pareto superior to any equilibrium of $\Gamma(n,p,q)$.
(ii) There exists an equilibrium of $\Gamma(n,p,q)$ that is Pareto superior to any equilibrium of $\Gamma(n,p)$ if and only if

$$w_L < \underline{\theta} \leq \min\{w_i + e_i\} \leq \max\{w_i + e_i\} < \bar{\theta}, \text{ and} \tag{6}$$

$$\delta(1-\pi)(w_R - \underline{\theta}) > (1 - \delta(1-\pi))\left(\bar{\theta} - w_R\right). \tag{7}$$

This statement holds unchanged if we replace $\Gamma(n,p,q)$ *by* $\Gamma(n,q)$*.*
(iii) Furthermore, all equilibria of $\Gamma(n,p,q)$ *are Pareto superior to all equilibria of* $\Gamma(n,p)$ *if and only if (6), (7) and the following hold:*

$$e_L > \frac{\delta\pi}{1-\delta}\left(\bar{\theta}-w_L\right)+\frac{1-\delta\pi}{1-\delta}\left(\underline{\theta}-w_L\right). \tag{8}$$

Proposition 6 part *(i)* states that adding the inefficient instrument to the menu of available policies cannot hurt both players. The intuition for this result is simple: in the two-state environment, either players agree when to implement and repeal p, in which case adding q does not affect the equilibrium paths, or they disagree, in which case the equilibria of $\Gamma(n,p)$ are gridlock equilibria and adding q can only help support a more responsive equilibrium, which is beneficial at least for the player who proposes q.

Part *(ii)* characterizes the conditions under which adding q can make both players strictly better off. To understand the intuition for, and implications of, these conditions, note first that condition (6) is essentially equivalent to the condition (1) in the two period example of Section 2, and has a similar interpretation as in the two period setup.[15] Specifically, the last three inequalities in (6) mean that, when choosing between n and q, players' static preferences are the same, so in the game $\Gamma(n,q)$, there exists an equilibrium in which players agree to implement n in state $\underline{\theta}$ and q in state $\bar{\theta}$. The first inequality in (6) means that p is L's most preferred policy in either state, so status quo p is never repealed. Hence, in the game $\Gamma(n,p)$, if n is the status quo and R's preference for n over p in the low state $\underline{\theta}$ is stronger than her preference for p over n in the high state $\bar{\theta}$, R will not approve intervening with p and in any equilibrium, players will stay at n forever. This is assured by condition (7), and in that case, both players are strictly worse off in this no-intervention equilibrium path than in the equilibrium path of $\Gamma(n,q)$, described above.

Now consider the game $\Gamma(n,p,q)$. Since, under (7), R does not approve p, adding p to the game leaves the incentives underlying the equilibria of $\Gamma(n,q)$ unchanged. Agreeing to implement n in state $\underline{\theta}$ and q in state $\bar{\theta}$, therefore, is also an equilibrium of $\Gamma(n,p,q)$ and both players strictly prefer it to the unique equilibrium path of $\Gamma(n,p)$. Hence, for the inefficient instrument to be beneficial in some equilibrium, players' degree of polarization must be sufficiently large so that they disagree sufficiently often on when to repeal p, but not so large that they agree sufficiently often on when to repeal q.

Part *(iii)* shows that the additional condition (8) is required to ensure both players are strictly better off in all equilibria of $\Gamma(n,p,q)$ relative to $\Gamma(n,p)$. The reason is that staying

[15]The only difference between condition (1) and (6) is that the former requires $\underline{\theta} < w_R$, but this extra condition is implied by (7).

with n indefinitely, as in the unique equilibrium path of $\Gamma(n, p)$, may remain an equilibrium in $\Gamma(n, p, q)$, leaving L and R indifferent between the two games. To see how this can occur, suppose that L threatens to approve only p if ever q is implemented. If such a threat is credible, R never approves q and no intervention is ever implemented. Condition (8) guarantees that no such threat is credible: when e_L is sufficiently large, L prefers to accept n rather than stay at q in state $\underline{\theta}$. Thus, to guarantee that both players benefit unambiguously from the availability of q, q must be sufficiently inefficient for the less interventionist players.

It is worth noting how the conditions of Proposition 6 depend on π and δ. From (7) and (8), the set of w_R, w_L and e_L that satisfy these conditions expands as π falls. Hence, both players are more likely to benefit from q if the state in which they disagree over intervening with the efficient policy becomes more likely. And as δ increases and players become more patient, the set of w_R satisfying (7) increases and a welfare improving IE equilibrium, therefore, becomes more likely. However, the right hand side of (8) first decreases and then increases with δ. Hence, to benefit unambiguously from the presence of q, players should be sufficiently, but not excessively, patient.[16]

5 Volatility and Persistent States

Until now, states, and thereby players' state-contingent preferences, have been assumed i.i.d. draws over time. In many applications, however, there may be periods of relative stability when players do not expect to change their positions, other things equal, and there may also be periods in which new information about the desirability of an intervention arrives frequently, resulting in frequent revisions of the relevant policy preferences.[17] Our main argument, therefore, that one reason for rational legislators to reject an efficient policy in favor of an inefficient policy is because inefficient policies are easier to repeal, is attenuated to the extent that it depends essentially on the i.i.d. assumption. Consequently, we extend the argument to a more general environment in which the i.i.d. assumption is relaxed to permit serial correlation.

[16] The nonmonotonic impact of δ on the value of inefficient instument is not an artifact of the two-state distributions considered in Proposition 6. Condition (6) guarantees that players are in full ideological agreement when choosing between the inefficient instrument q and no intervention n. This makes q particularly attractive. If the distribution of the state has full support, sufficiently patient players may introduce an inefficient gridlock also to the choice between n and q. As a result, unlike in the two-state case, q may not strictly improve both players' payoffs if players are sufficiently patient and, further, an IE may not exist.

[17] A painful example is the US Congressional response to the 2008 fiscal collapse. For some years before 2008, the US economy was growing strongly and atypical Congressional economic interventions were minimal. The fall of Lehman Brothers and the subsequent turmoil in much of the global economy led to serious Congressional disagreement regarding the appropriate level and duration of any extraordinary intervention, from whether to bail out banks or the car industry, to extensions of unemployment and welfare benefits.

To capture the possibility that states may persist across periods in an analytically tractable way, and to allow for players' expectations regarding the persistence of the current state to vary over time, consider, for every period t, a tuple $(\theta_t, v_t) \in \mathbb{R} \times [0,1]$. Assume the evolution of such tuples across periods satisfies the following transition property: for all t,

$$(\theta_{t+1}, v_{t+1}) = \begin{cases} (\theta_t, v_t) \text{ with probability } 1 - v_t \\ (\theta_{t+1}, v_{t+1}) \sim H \text{ with probability } v_t \end{cases},$$

where H is some joint c.d.f.. As before, θ_t is the underlying policy-relevant state. We interpret the additional variable v_t as a measure of volatility of the current policy-relevant state θ_t or, equivalently, of players' period t expectations over θ_{t+1}. In each period t, the state θ_t persists into period $t+1$ with probability $(1 - v_t)$ and, for simplicity, assume the volatility v_t also persists into period $t+1$; with probability v_t, θ_{t+1} and v_{t+1} are drawn according to the joint c.d.f. H. Thus, the volatility of future state-contingent policy preferences is redrawn if and only if the state-contingent policy preferences are redrawn.

Note that the evolution of the state collapses to the basic i.i.d. model if $v_t \equiv 1$ for all t. Similarly, $v_t \equiv 0$ for all t implies preferences never change, while $v_t \equiv v \in (0,1)$ for all t implies the degree of volatility is fixed. With this in mind, we have the following result.

Proposition 7 *In any equilibrium σ, there exists $\bar{v} \in (0,1]$ such that q is never implemented when the realization of volatility is $v < \bar{v}$, but q is implemented with positive probability under status quo n if $v > \bar{v}$. Moreover, for any (w_L, w_R, e_L, e_R), for all δ sufficiently close to 1, there exists distribution H with full support on $\mathbb{R} \times [0,1]$ such that, for all equilibria, $\bar{v} < 1$.*

Proposition 7 states that q is implemented on the equilibrium path only in sufficiently volatile economic environments. Intuitively, when players expect the state to remain fairly stable over time ($v_t \leq \bar{v}$), strategic concerns regarding the possibility of conflict over repealing today's intervention tomorrow, say, are muted and any intervention is efficient. When the economic environment is expected to be sufficiently volatile, however ($v_t > \bar{v}$), today's choice is likely to need revision in the next period, making salient exactly the sorts of strategic consideration underlying the use of inefficient interventions.

6 Conclusion

The continued and widespread use of inefficient policy interventions in more-or-less democratic political systems is a puzzle. For example, while economists uniformly recommend regulating emissions with Pigou taxes, technology and quantity controls are the most adopted instruments in reality. Why would rational politicians agree on the use of Pareto dominated policy instruments? In our model, an inefficient policy intervention may arise even when there is no salient legislative history or vested interest. Rather, an inefficient policy may be chosen precisely *because* it is inefficient and the environment is expected to change in the future. From this perspective, the puzzle alluded to above can be understood without pointing to informational asymmetries, interest group influence, or differential distributional implications of alternative policy instruments among the electorate at large.

With a heterogeneous legislature and multiple veto players, inefficient interventions are politically easier than efficient interventions to repeal in dynamic environments subject to policy-relevant stochastic shocks. And since inefficient interventions are easier to repeal, heterogeneous veto players, differentiated only by the threshold shocks beyond which they judge some policy intervention to be warranted, can be more willing to agree on responding to a sufficiently severe downside shock with an inefficient instrument. As a consequence, inefficient interventions are more likely to be used in (moderately) polarized political environments and for issues where the fundamentals are subject to change over time.

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8 Appendix

8.1 Notations and Lemmas

Notation 1 *For a given stationary Markov strategy profile σ, the policy outcome in some period $t \in \mathbb{N}$ with status quo $s(t) \in \{n, p, q\}$ can depend on $\theta(t)$, the identity $p(t)$ of the proposer recognized in period t, and the realization of players' private randomization devices $(\rho_L(t), \rho_R(t))$.*[18] *Let $\upsilon(t) \equiv (\theta(t), s(t), p(t), \rho_L(t), \rho_R(t))$ denote the random variable that encodes this information. We refer to $\upsilon(t)$ as the* state of the world *in period t. Let Υ denote the set of possible states of the world. Note that $\{\upsilon(t) : t \in \mathbb{N}\}$ is i.i.d.. Let μ denote its probability distribution. For any state of the world $\upsilon \in \Upsilon$, $\theta(\upsilon)$ denotes the corresponding realization of the* state of nature. *For all $s, x \in \{n, p, q\}$, $\Upsilon^\sigma(s, x)$ denotes the*

[18]Formally, $(\rho_i(t))_{i\in\{L,R\},t\in\mathbb{N}}$ is a collection of random variables that are i.i.d. across periods and players, and uniformly distributed on $[0,1]$. Each player i privately observes $\rho_i(t)$ at the beginning of period t. A mixed action for proposer (veto-player) i in period t can then be modelled as a piecewise constant function from $[0,1]$ to $\{n, p, q\}$ (to $\{accept, reject\}$).

set of realizations of the state of the world for which status quo s leads to outcome x.

The next lemma defines formally the continuation payoff parameters used in Lemma 1.

Lemma 2 (Continuation Payoff) *Let σ be a Markov strategy profile, and let $V_i^\sigma(\theta, x)$ denote the expected discounted payoff for player $i \in \{L, R\}$ from implementing policy $x \in \{n, p, q\}$ in period 1 conditional on $\theta_1 = \theta$, and on players playing σ from period 2 onwards. Then there exist unique $w_i^\sigma \in \mathbb{R}$ and $e_i^\sigma \in \mathbb{R}$ such that, for all $\theta \in \mathbb{R}$,*

$$\begin{aligned} V_i^\sigma(\theta, p) - V_i^\sigma(\theta, n) &= \theta - w_i^\sigma, \\ V_i^\sigma(\theta, p) - V_i^\sigma(\theta, q) &= e_i^\sigma. \end{aligned} \tag{9}$$

The parameters (w_i^σ, e_i^σ) correspond to the continuation payoff parameters introduced in Lemma 1. Moreover, $\frac{w_i^\sigma - w_i}{\delta}$ $(\frac{e_i^\sigma - e_i}{\delta})$ can be interpreted as the expected payoff gain for player i of having initial status quo n (p) instead of p (q) in the game Γ with continuation play σ.

Proof. By definition, $V_i^\sigma(\theta, p) - V_i^\sigma(\theta, n)$ is the sum of the flow payoff gain from implementing p instead of n in $t = 1$ when $\theta(1) = \theta$ and δ times the continuation payoff gain from period 2 onwards from having $s_2 = p$ instead of $s_2 = n$, given continuation play σ in $t \geq 2$. Using Notation 1,

$$V_i^\sigma(\theta, p) - V_i^\sigma(\theta, n) = \theta - w_i + \delta \sum_{x,y \in \{n,p,q\}} \int_{\Upsilon^\sigma(p,x) \cap \Upsilon^\sigma(n,y)} (V_i^\sigma(\theta(\upsilon), x) - V_i^\sigma(\theta(\upsilon), y))\, d\mu(\upsilon).$$

Therefore, to prove the first line of (9), it suffices to set

$$w_i^\sigma \equiv w_i - \delta \sum_{x,y \in \{n,p,q\}} \int_{\Upsilon^\sigma(n,x) \cap \Upsilon^\sigma(p,y)} (V_i^\sigma(\theta(\upsilon), x) - V_i^\sigma(\theta(\upsilon), y))\, d\mu(\upsilon). \tag{10}$$

An analogous reasoning on the continuation payoff gain from implementing p instead of q proves the second line of (9) with

$$e_i^\sigma \equiv e_i + \delta \sum_{x,y \in \{n,p,q\}} \int_{\Upsilon^\sigma(p,x) \cap \Upsilon^\sigma(q,y)} (V_i^\sigma(\theta(\upsilon), x) - V_i^\sigma(\theta(\upsilon), y))\, d\mu(\upsilon). \tag{11}$$

■

Definition 2 *Let $(w^c, e^c) \in \mathbb{R}^4$. We say that a stationary strategy profile σ is subgame perfect for the continuation payoff parameters $(w^c, e^c) \in \mathbb{R}^4$ if σ is a subgame perfect equilibrium of the game in which players play a single period of Γ with payoffs V such that for all $\theta \in \mathbb{R}$, $V_i(\theta, p) - V_i(\theta, n) = \theta - w_i^c$ and $V_i(\theta, p) - V_i(\theta, q) = e_i^c$.*

Lemma 3 (Equilibrium Continuation Payoffs) *A stationary strategy profile σ is an equilibrium if and only if σ is subgame perfect (in the sense of Definition 2) for the continuation payoff parameters w^σ and e^σ defined in Lemma 2. In that case,*

$$\frac{w_i^\sigma - w_i}{\delta} = \int_{\Upsilon^\sigma(p,p)\cap\Upsilon^\sigma(n,n)} \left(w_i^\sigma - \theta(\upsilon)\right) d\mu(\upsilon) - \int_{\Upsilon^\sigma(p,p)\cap\Upsilon^\sigma(n,q)} e_i^\sigma d\mu(\upsilon), \tag{12}$$

$$\begin{aligned}\frac{e_i^\sigma - e_i}{\delta} &= \int_{\Upsilon^\sigma(p,p)\cap\Upsilon^\sigma(q,q)} e_i^\sigma d\mu(\upsilon) + \int_{\Upsilon^\sigma(p,p)\cap\Upsilon^\sigma(q,n)} \left(\theta(\upsilon) - w_i^\sigma\right) d\mu(\upsilon) \\ &\quad + \int_{\Upsilon^\sigma(p,n)\cap\Upsilon^\sigma(q,q)} \left(w_i^\sigma + e_i^\sigma - \theta(\upsilon)\right) d\mu(\upsilon).\end{aligned} \tag{13}$$

Proof. The proof of the first claim follows immediately from our definition of subgame perfection and Lemma 2. Let us now derive (13). Consider the summands

$$e_i^\sigma(x,y) \equiv \int_{\Upsilon^\sigma(p,x)\cap\Upsilon^\sigma(q,y)} \left(V_i^\sigma(\theta(\upsilon),x) - V_i^\sigma(\theta(\upsilon),y)\right) d\mu(\upsilon) \tag{14}$$

on the R.H.S. of (11) for all policies x and y that are implemented in some state under status quo p and q, respectively. When $x = y$, $e_i^\sigma(x,y) = 0$. Consider next $(x,y) = (q,n)$, that is, status quo p is replaced by q, and status quo q is replaced by n. Since σ is an equilibrium, it is subgame perfect given continuation payoff parameters (w^σ, e^σ), so for all $\upsilon \in \Upsilon^\sigma(p,q)\cap\Upsilon^\sigma(q,n)$, both players must weakly prefer to implement q to p and n to q in period 1. Moreover, one of them must be indifferent between implementing n and q, because if both players strictly preferred to implement n to q in state υ, then under status quo p, either veto player would accept n, so proposing n instead of q in that period would be a profitable deviation for the proposer. Therefore, for all $\upsilon \in \Upsilon^\sigma(p,q)\cap\Upsilon^\sigma(q,n)$, $\theta(\upsilon) - w_i^\sigma - e_i^\sigma = 0$ for some player i. Since F is continuous, this implies that $\mu(\Upsilon^\sigma(p,q)\cap\Upsilon^\sigma(q,n)) = 0$, so $e_i^\sigma(q,n) = 0$. In the case $(x,y) = (p,n)$, an analogous reasoning implies that $\mu(\Upsilon^\sigma(q,p)\cap\Upsilon^\sigma(p,n)) = 0$ so $e_i^\sigma(p,n) = 0$. Consider now $(x,y) = (q,p)$, that is, status quo p is replaced by q and vice versa. Subgame perfection implies that for all $\theta \in \Upsilon^\sigma(p,q)\cap\Upsilon^\sigma(q,p)$, $V_i^\sigma(\theta,p) = V_i^\sigma(\theta,q)$, so from (9), $e_i^\sigma = 0$, and from (14), $e_i^\sigma(q,p) = 0$. The only remaining cases are (x,y) equal to (p,q), (p,n), and (n,q), so (11) becomes

$$\begin{aligned}\frac{e_i^\sigma - e_i}{\delta} &= \int\limits_{\Upsilon^\sigma(p,p)\cap\Upsilon^\sigma(q,q)} \left(V_i^\sigma(\theta(\upsilon),p) - V_i^\sigma(\theta(\upsilon),q)\right) d\mu(\upsilon) \\ &+ \int\limits_{\Upsilon^\sigma(p,p)\cap\Upsilon^\sigma(q,n)} \left(V_i^\sigma(\theta(\upsilon),p) - V_i^\sigma(\theta(\upsilon),n)\right) d\mu(\upsilon) + \int\limits_{\Upsilon^\sigma(p,n)\cap\Upsilon^\sigma(q,q)} \left(V_i^\sigma(\theta(\upsilon),n) - V_i^\sigma(\theta(\upsilon),q)\right) d\mu(\upsilon).\end{aligned}$$

Substituting (9) into the above expression, we obtain (13).

The proof of (12) follows a similar logic as the proof of (13). Consider the summands $w_i^\sigma(x,y) \equiv \int_{\Upsilon^\sigma(p,x)\cap\Upsilon^\sigma(n,y)} \left(V_i^\sigma(\theta(\upsilon),x) - V_i^\sigma(\theta(\upsilon),y)\right) d\mu(\upsilon)$ on the R.H.S. of (10) for all possible policies x and y that can be implemented under status quo p and n, respectively. Using the same steps as the ones used to prove $\mu(\Upsilon^\sigma(p,q)\cap\Upsilon^\sigma(q,n)) = 0$ and reversing the role of p and q, we obtain that $\mu(\Upsilon^\sigma(p,n)\cap\Upsilon^\sigma(n,q)) = 0$, so $w_i^\sigma(n,q) = 0$. An analogous reasoning implies that $\mu(\Upsilon^\sigma(p,q)\cap\Upsilon^\sigma(n,p)) = 0$, so $w_i^\sigma(q,p) = 0$. Moreover, for all $\upsilon \in \Upsilon^\sigma(p,n)\cap\Upsilon^\sigma(n,p)$, each player i must be indifferent between implementing n and p, so $\theta(\upsilon) - w_i^\sigma - e_i^\sigma = 0$. Since F is continuous, this implies that $\mu(\Upsilon^\sigma(p,n)\cap\Upsilon^\sigma(n,p)) = 0$, so $w_i^\sigma(n,p) = 0$. The only remaining cases left are (x,y) equal to (p,n), (p,q), and (q,n). Equation (12) follows then from substituting these three cases into the right-hand side of (10) and using (9). ■

Lemma 4 (Necessary and Sufficient Conditions for EE)

A) Let σ be an EE. Then $e_L^\sigma > 0$, $e_R^\sigma \geq 0$, $w_L^\sigma < w_L < w_R < w_R^\sigma$, $\Upsilon^\sigma(q,q) \subseteq \Upsilon^\sigma(p,p)$, and for all $i \in \{L,R\}$,

$$\frac{w_i^\sigma - w_i}{\delta} = \int_{w_L^\sigma}^{w_R^\sigma} (w_i^\sigma - \theta)\, dF(\theta). \tag{15}$$

B) For any EE σ, there exists an EE σ' such that players never propose q under any status quo, $w^{\sigma'} = w^\sigma$ and for all $i \in \{L,R\}$,

$$\frac{e_i^{\sigma'} - e_i}{\delta} = \int_{w_L^\sigma}^{\min\left\{w_L^\sigma + e_L^{\sigma'}, w_R^\sigma\right\}} b_R(\theta,q)(\theta - w_i^\sigma)\, dF(\theta). \tag{16}$$

C) Reciprocally, if there exists $(w^*, e^*) \in \mathbb{R}^4$ such that $e_R^* \geq 0$ and such that

$$\frac{w_i^* - w_i}{\delta} = \int_{w_L^*}^{w_R^*} (w_i^* - \theta)\, dF(\theta), \text{ and} \tag{17}$$

$$\frac{e_i^* - e_i}{\delta} = \int_{w_L^*}^{\min\left\{w_L^* + e_L^*, w_R^*\right\}} b_R(\theta,q)(\theta - w_i^*)\, dF(\theta), \tag{18}$$

then there exists an EE σ such that $(w^\sigma, e^\sigma) = (w^*, e^*)$.

Proof. Proof of part A: Let σ be an arbitrary EE.

Step A1: $w_L^\sigma < w_R^\sigma$ and for all $i \in \{L,R\}$,

$$\frac{w_i^\sigma - w_i}{\delta} = \int_{\Upsilon^\sigma(p,p)\cap\Upsilon^\sigma(n,n)} (w_i^\sigma - \theta(\upsilon))\, d\mu(\upsilon). \tag{19}$$

Since σ is an EE, $\mu\left(\Upsilon^{\sigma}(n,q)\right)=0$. Substituting the latter equality into (12), we obtain (19). Taking differences across players in (19) and solving for $w_R^{\sigma}-w_L^{\sigma}$, we obtain $w_R^{\sigma}-w_L^{\sigma}=\frac{w_R-w_L}{1-\delta\mu(\Upsilon^{\sigma}(p,p)\cap\Upsilon^{\sigma}(n,n))}<0$.

Step A2: $e_L^{\sigma}\geq 0$ *and* $e_R^{\sigma}\geq 0$

Suppose that $e_i^{\sigma}<0$ for some $i\in\{L,R\}$. Then in any period t in which $s(t)=n$, the proposer is i, and $\theta(t)>\max_{k\in\{L,R\}}\left\{w_k^{\sigma}+e_k^{\sigma}\right\}$, subgame perfection implies that the other player j must accept proposal q. Since q is the outcome that gives the greatest continuation payoff to i in that state, the only subgame perfect action for i is to propose q, which contradicts σ being an EE.

Step A3: modulo a zero measure set, $\Upsilon^{\sigma}(p,p)=\left\{\upsilon\in\Upsilon:\theta(\upsilon)>w_L^{\sigma}\right\}$ *and* $\Upsilon^{\sigma}(n,n)=\left\{\upsilon\in\Upsilon:\theta(\upsilon)<w_R^{\sigma}\right\}$.

For all $\upsilon\in\Upsilon$ such that $\theta(\upsilon)>w_L^{\sigma}$, L strictly prefers to implement p to n, so subgame perfection implies that $\upsilon\notin\Upsilon^{\sigma}(p,n)$. Since σ is an EE, $\mu\left(\Upsilon^{\sigma}(p,q)\right)=0$. By definition of Υ^{σ}, $\left(\Upsilon^{\sigma}(p,x)\right)_{x\in\{n,p,q\}}$ is a partition of Υ, so necessarily, $\upsilon\in\Upsilon^{\sigma}(p,p)$. Conversely, for all $\upsilon\in\Upsilon$ such that $\theta(\upsilon)<w_L^{\sigma}$, from Step A1, $\theta(\upsilon)<w_R^{\sigma}$, so both players strictly prefer implementing n to p. Therefore, subgame perfection implies that $\upsilon\notin\Upsilon^{\sigma}(p,p)$. Since F is continuous, the set of υ such that $\theta(\upsilon)=w_L^{\sigma}$ has probability 0, which proves the first equality in Step A3. For all $\upsilon\in\Upsilon$ such that $\theta(\upsilon)<w_R^{\sigma}$, R strictly prefers to implement n to p, so subgame perfection implies that $\upsilon\notin\Upsilon^{\sigma}(n,p)$. Since σ is an EE, $\mu\left(\Upsilon^{\sigma}(n,q)\right)=0$. Since $\left(\Upsilon^{\sigma}(n,x)\right)_{x\in\{n,p,q\}}$ is a partition of Υ, necessarily, $\upsilon\in\Upsilon^{\sigma}(n,n)$. Conversely, for all $\upsilon\in\Upsilon$ such that $\theta(\upsilon)>w_R^{\sigma}$ and, from Step A1, $\theta(\upsilon)>w_L^{\sigma}$, both players strictly prefer implementing p to n. Therefore, subgame perfection implies that $\upsilon\notin\Upsilon^{\sigma}(n,n)$. The second equality in Step A3 follows then from the continuity of F.

Step A4: $(w_L^{\sigma},w_R^{\sigma})$ *satisfies (15) and* $w_L^{\sigma}<w_L<w_R<w_R^{\sigma}$.

From Step A3, modulo a zero measure set, $\Upsilon^{\sigma}(p,p)\cap\Upsilon^{\sigma}(n,n)=\left\{\upsilon\in\Upsilon:\theta(\upsilon)\in(w_L^{\sigma},w_R^{\sigma})\right\}$. Substituting this equality into (19), we obtain (15). Step A1 and the assumption that F has full support imply $\int_{w_L^{\sigma}}^{w_R^{\sigma}}\left(w_L^{\sigma}-\theta\right)dF(\theta)<0<\int_{w_L^{\sigma}}^{w_R^{\sigma}}\left(w_R^{\sigma}-\theta\right)dF(\theta)$. Together with (15), the above inequalities imply $w_L^{\sigma}<w_L<w_R<w_R^{\sigma}$.

Step A5: $e_L^{\sigma}>0$

For all $\upsilon\in\Upsilon^{\sigma}(p,n)\cap\Upsilon^{\sigma}(q,q)$, both players strictly prefer implementing n to p and one player must weakly prefers implementing q to n, so $\min_i\left\{w_i^{\sigma}+e_i^{\sigma}\right\}\leq\theta(\upsilon)<w_L^{\sigma}$. Therefore, $\min_i\left\{w_i^{\sigma}+e_i^{\sigma}\right\}<w_L^{\sigma}$, a contradiction with Step A1 and A2. This proves that $\mu\left(\Upsilon^{\sigma}(p,n)\cap\Upsilon^{\sigma}(q,q)\right)=0$. Substituting the latter into (13), we get

$$\frac{e_i^{\sigma}-e_i}{\delta}=\int_{\Upsilon^{\sigma}(p,p)\cap\Upsilon^{\sigma}(q,q)}e_i^{\sigma}d\mu(\upsilon)+\int_{\Upsilon^{\sigma}(p,p)\cap\Upsilon^{\sigma}(q,n)}\left(\theta(\upsilon)-w_i^{\sigma}\right)d\mu(\upsilon). \tag{20}$$

From Step A3, for almost all $\upsilon \in \Upsilon^{\sigma}(p,p)$, $\theta(\upsilon) > w_L^{\sigma}$, so (20) implies $e_L^{\sigma} \geq e_L > 0$.

Step A6: $\Upsilon^{\sigma}(q,q) \subseteq \Upsilon^{\sigma}(p,p)$

For all $\upsilon \in \Upsilon^{\sigma}(q,q)$, subgame perfection implies that one player weakly prefers implementing q to n, so $\theta(\upsilon) \geq \max\{w_L^{\sigma} + e_L^{\sigma}, w_R^{\sigma} + e_R^{\sigma}\}$. From Step A1, A2, and A5, this implies that $\theta(\upsilon) > w_L^{\sigma}$ and so, as shown in Step A3, $\upsilon \in \Upsilon^{\sigma}(p,p)$.

Proof of part B: We construct the strategy profile σ' as the limit of a sequence $\left(\sigma^k\right)_{k\in\mathbb{N}}$. We set $\sigma^0 = \sigma$. For any $k \geq 0$, we define σ^{k+1} as a function of σ^k as follows. Consider first the action nodes of veto player $i \in \{L,R\}$. When comparing n and p, i plays σ. When comparing p and q, i votes for p. When comparing n and q, i votes for q when $\theta(\upsilon) > w_i^{\sigma} + e_i^{\sigma^k}$ and for n otherwise. Consider now the action nodes of proposer $i \in \{L,R\}$. When the status quo is n or p, i plays σ. When the status quo is q, L proposes n when $\theta \leq w_L^{\sigma}$, and p otherwise, and R proposes n when $\theta \leq \min\left\{w_L^{\sigma} + e_L^{\sigma^k}, w_R^{\sigma}\right\}$ and p otherwise.

Step B1: for all $k \in \mathbb{N}$, $w^{\sigma^k} = w^{\sigma}$ and $e_L^{\sigma^1} \leq e_L^{\sigma^0}$

By construction, for all $k \in \mathbb{N}$, σ^k prescribes the same actions as σ when neither the status quo nor the proposal is q, and these actions never lead to outcome q. So the value of the game Γ with initial status quo n (p) is the same with continuation play σ^k and σ. From Lemma 2, this implies $w^{\sigma^k} = w^{\sigma}$. When the status quo is q, by construction of σ^1, σ^1 always implements n or p, and σ^1 prescribes actions that are subgame perfect given continuation play σ. So the actions prescribed by σ and σ^1 under status quo q differ only when one player is indifferent between implementing two policies given continuation play σ. Since F is atomeless, Lemma 2 implies that this indifference can occur in a nonnegligibe set of states only between p and q (and when $e_R^{\sigma} = 0$). In those cases, σ^1 always prescribes R to choose p, which is what L prefers given continuation play σ since from part A, $e_L^{\sigma} > 0$. So under status quo q, L is weakly better off when σ^1 is played than when σ is played in the current period, given continuation play σ. Since the path of σ^1 and σ coincide once n or p is implemented, this implies that under status quo q, L is weakly better off when σ^1 is played in all periods than when σ is played in all periods. From Lemma 2, this means that $e_L^{\sigma^1} \leq e_L^{\sigma^0}$.

Step B2: for all $k \in \mathbb{N}$, $e_L^{\sigma^k} > 0$ and

$$\frac{e_i^{\sigma^{k+1}} - e_i}{\delta} = \int_{w_L^{\sigma}}^{\min\left\{w_L^{\sigma}+e_L^{\sigma^k}, w_R^{\sigma}\right\}} b_R(\theta,q)\left(\theta - w_i^{\sigma}\right) dF(\theta). \tag{21}$$

Since $\sigma^0 = \sigma$ is an EE, Part A implies $e_L^{\sigma^0} > 0$. To prove Step B2 by induction, suppose $e_L^{\sigma^k} > 0$ for some $k \in \mathbb{N}$, and let us show (21) and $e_L^{\sigma^{k+1}} > 0$. From Lemma 2, $\left(e_L^{\sigma^{k+1}} - e_i\right)/\delta$ is the expected payoff gain from having status quo p instead of q given continuation play

σ^{k+1}. To compute it, note that from Part A, $w_L^\sigma < w_R^\sigma$, and since $e_L^{\sigma^k} > 0$, we have $w_L^\sigma < \min\left\{w_L^\sigma + e_L^{\sigma^k}, w_R^\sigma\right\}$. So by construction of σ^{k+1}, on path, status quo q and p both lead to outcome n when $\theta < w_L^\sigma$, and they both lead to outcome p when $\theta > w_R^\sigma$, when $\theta \in (w_L^\sigma, w_R^\sigma)$ and L is the proposer, and when $\theta \in \left(\min\left\{w_L^\sigma + e_L^{\sigma^k}, w_R^\sigma\right\}, w_R^\sigma\right)$ and R is the proposer. Modulo a negligible set of states, the only remaining case is when R is the proposer and $\theta \in \left(w_L^\sigma, \min\left\{w_L^\sigma + e_L^{\sigma^k}, w_R^\sigma\right\}\right)$. In that case, status quo q and p lead to outcome n and p, respectively, so the relative payoff gain is $\theta - w_i^{\sigma^{k+1}} = \theta - w_i^\sigma$. Integrating over all these cases, we obtain (21). From Part A, $w_L^\sigma < w_R^\sigma$, so (21) implies $e_L^{\sigma^{k+1}} > e_L > 0$.

Step B3: for all $k \geq 1$, $e_L^{\sigma^{k+1}} \leq e_L^{\sigma^k}$ and $e_R^{\sigma^{k+1}} \geq e_R^{\sigma^k}$.

From Step B1 and B2, $e_L^{\sigma^1} \leq e_L^{\sigma^0}$. To prove Step B3 by induction, suppose $e_L^{\sigma^{k+1}} \leq e_L^{\sigma^k}$ for some $k \in \mathbb{N}$, and let us show that $e_L^{\sigma^{k+2}} \leq e_L^{\sigma^{k+1}}$ and $e_R^{\sigma^{k+2}} \geq e_R^{\sigma^{k+1}}$. From Step B2,

$$e_i^{\sigma^{k+2}} - e_i^{\sigma^{k+1}} = \delta \int_{\min\left\{w_L^\sigma + e_L^{\sigma^k}, w_R^\sigma\right\}}^{\min\left\{w_L^\sigma + e_L^{\sigma^{k+1}}, w_R^\sigma\right\}} b_R(\theta, q)(\theta - w_i^\sigma)\, dF(\theta).$$

Since $e_L^{\sigma^k} \leq e_L^{\sigma^{k-1}}$, R.H.S. of the above is negative for $i = L$ and positive for $i = R$.

Step B4: σ^{k+1} prescribes subgame perfect action given continuation play σ^k.

From Part A, $e_R^{\sigma^0} \geq 0$. Steps B2 and B3 imply then that for all $i \in \{L, R\}$ and $k \in \mathbb{N}$, $e_i^{\sigma^k} \geq 0$. So one can easily check that by construction, σ^{k+1} prescribes the veto player to approve (reject) the proposal when she strictly prefers the proposal (status quo) given continuation play σ^k, and that σ^{k+1} prescribes the proposer to propose the policy that she prefers among those accepted by the veto player, as needed.

Step B5: $\sigma' \equiv \lim_{k\to\infty} \sigma$ exists and satisfies the properties stated in Part B.

From Step B1 and B3, $w_i^{\sigma^k}$ and $e_i^{\sigma^k}$ have a limit as $k \to \infty$, and since σ^{k+1} is a continuous function of $w_i^{\sigma^k}$ and $e_i^{\sigma^k}$, it has a limit σ'. From Step B4, by continuity, σ' is an equilibrium, and by construction of $\left(\sigma^k\right)_{k\in\mathbb{N}}$, σ' never prescribes proposal q, so σ is an EE. Taking the limit in (21), we obtain (16).

Proof of part C: Suppose $(w^*, e^*) \in \mathbb{R}^4$ solves (17) and (18) with $e_R^* \geq 0$. Let us construct an EE σ such that $(w^\sigma, e^\sigma) = (w^*, e^*)$. Consider first the action nodes of veto player $i \in \{L, R\}$. When comparing n and p, i votes for p when $\theta > w_i^*$ and for n otherwise. When comparing p and q, i votes for p. When comparing n and q, i votes for q when $\theta(\upsilon) > w_i^* + e_i^*$ and for n otherwise. Consider now the the action nodes of proposer $i \in \{L, R\}$. When the status quo is n or p, i proposes p when $\theta > w_i^*$ and n otherwise, When the status quo is q, L proposes n when $\theta \leq w_L^*$, and p otherwise, and R proposes n when $\theta \leq \min\{w_L^* + e_L^*, w_R^*\}$ and p otherwise. By construction, q is never implemented on path. Therefore, to complete the proof, it suffices to show that σ is an equilibrium. From Lemma 3, it is equivalent to show that σ is subgame perfect given continuation payoffs (w^σ, e^σ), which is what Steps C1 to C3 below do.

Step C1: σ is subgame perfect for the continuation payoff parameters (w^*, e^*).
By assumption, $e_R^* \geq 0$, and since e^* satisfies (18), we have $e_L^* \geq e_L > 0$. So by construction, σ prescribe either veto players to accept (reject) a proposal when it gives her a greater (lesser) continuation payoff, given continuation payoff parameters (w^*, e^*). Likewise, σ prescribe either proposer to propose the policy that gives her the greatest continuation payoff among those accepted by the veto player, given continuation payoff parameters (w^*, e^*).

Step C2: $w^\sigma = w^*$.
Since w^* satisfies (17), the same reasoning as in Step A1 implies $w_L^* < w_R^*$. By construction of σ, q is never implemented on path, and status quo n and p lead to different outcomes only when $\theta \in (w_L^*, w_R^*)$, in which case either status quo stay in place, and the corresponding continuation payoff gain is $w_i^\sigma - \theta$. From Lemma 2, this means that $\frac{w_i^\sigma - w_i}{\delta} = \int_{w_L^*}^{w_R^*} (w_i^\sigma - \theta)\, dF(\theta)$. This equation can be viewed as a linear equation in w_i^σ, and it has a unique solution. From (17), w_i^* is also a solution to that equation, so $w_i^\sigma = w_i^*$.

Step C3: $e^\sigma = e^*$.
The same reasoning as in Step B2 (taking $w^\sigma = w^*$ and $e_L^{\sigma^k} = e_L^*$) shows that by construction of σ, modulo a negligible set of states, status quo p and q lead to different outcomes only when R is the proposer and $\theta \in (w_L^*, \min\{w_L^* + e_L^*, w_R^*\})$, in which case status quo p and q lead to outcome p and n, respectively, and the corresponding continuation payoff gain is $\theta - w_i^*$. Therefore, Lemma 2 implies that $\frac{e_i^\sigma - e_i}{\delta} = \int_{w_L^*}^{\min\{w_L^* + e_L^*, w_R^*\}} b_R(\theta, q)(\theta - w_i^*)\, dF(\theta)$. Together with (18), that equation implies $e_i^\sigma = e_i^*$. ■

Lemma 5 (Properties of IE) *Let σ be an IE and let $\Lambda, \varrho \in \{L, R\}$ be such that $w_\Lambda^\sigma \leq w_\varrho^\sigma$. Then $e_\varrho^\sigma \leq 0 < e_\Lambda < e_\Lambda^\sigma$, $w_\Lambda^\sigma < w_\varrho^\sigma$, $w_\Lambda^\sigma < \min_{i \in \{L,R\}} \{w_i^\sigma + e_i^\sigma\}$, and $\Upsilon^\sigma(q,q) \subset \Upsilon^\sigma(p,p)$.*

Proof. Let σ be an arbitrary IE.

Step 1: $\mu(\Upsilon^\sigma(p,p) \cap \Upsilon^\sigma(q,n)) > 0$.
In any state $\upsilon \in \Upsilon^\sigma(p,n)$, each player i must prefer implementing n to p so $\theta(\upsilon) \leq w_i^\sigma$ and, therefore, $w_i^\sigma + e_i^\sigma - \theta(\upsilon) \geq e_i^\sigma$. Substituting the latter inequality in the equilibrium formula for e_i^σ, (13), we obtain

$$\begin{aligned} \frac{e_i^\sigma - e_i}{\delta} &\geq \int_{(\Upsilon^\sigma(p,p) \cup \Upsilon^\sigma(p,n)) \cap \Upsilon^\sigma(q,q)} e_i^\sigma d\mu(\upsilon) + \int_{\Upsilon^\sigma(p,p) \cap \Upsilon^\sigma(q,n)} (\theta(\upsilon) - w_i^\sigma)\, d\mu(\upsilon) \\ &\Rightarrow\ e_i^\sigma \geq \frac{e_i + \delta \int_{\Upsilon^\sigma(p,p) \cap \Upsilon^\sigma(q,n)} (\theta(\upsilon) - w_i^\sigma)\, d\mu(\upsilon)}{1 - \delta\mu((\Upsilon^\sigma(p,p) \cup \Upsilon^\sigma(p,n)) \cap \Upsilon^\sigma(q,q))}. \end{aligned} \tag{22}$$

Suppose by contradiction that $\mu(\Upsilon^\sigma(p,p) \cap \Upsilon^\sigma(q,n)) = 0$. Then (22) implies $e_L^\sigma > 0$ and $e_R^\sigma > 0$, so both players always strictly prefer to implement p to q, and hence q is never implemented on path, which is impossible since σ is an IE.

Step 2: $w_\Lambda^\sigma < \min_{i\in\{L,R\}}(w_i^\sigma + e_i^\sigma)$.

For all $\upsilon \in \Upsilon^\sigma(p,p) \cap \Upsilon^\sigma(q,n)$, both players weakly prefer implementing n to q, i.e., $\theta(\upsilon) \leq w_i^\sigma + e_i^\sigma$, and at least one player weakly prefers implementing p to n, i.e., $\theta(\upsilon) \geq w_i^\sigma$, so $\min_{i\in\{L,R\}} w_i^\sigma \leq \theta(\upsilon) \leq \min_{i\in\{L,R\}}(w_i^\sigma + e_i^\sigma)$, so $w_\Lambda^\sigma \leq \theta(\upsilon) \leq \min_{i\in\{L,R\}}(w_i^\sigma + e_i^\sigma)$. From step 1, $\mu(\Upsilon^\sigma(p,p) \cap \Upsilon^\sigma(q,n)) > 0$, so the previous weak inequalities must be strict for some $\upsilon \in \Upsilon^\sigma(p,p) \cap \Upsilon^\sigma(q,n)$, which implies Step 2.

Step 3: $e_\varrho^\sigma \leq 0$ *and* $e_\Lambda^\sigma > e_\Lambda > 0$.

For all $\upsilon \in \Upsilon^\sigma(p,p)$, some player weakly prefers implementing p to n, so $\theta(\upsilon) \geq w_\Lambda^\sigma$. Since F is continuous, this inequality is strict for almost all $\upsilon \in \Upsilon^\sigma(p,p) \cap \Upsilon^\sigma(q,n)$. Substituting $\theta(\upsilon) > w_\Lambda^\sigma$ and Step 1 into (22), we get $e_\Lambda^\sigma > e_\Lambda > 0$. As argued in Step 1, since σ is an IE, $e_\Lambda^\sigma > 0$ implies $e_\varrho^\sigma \leq 0$.

Step 4: $\Upsilon^\sigma(q,q) \subset \Upsilon^\sigma(p,p)$.

Let $\upsilon \in \Upsilon^\sigma(q,q)$. Since σ is an equilibrium, in state $\theta(\upsilon)$, some player weakly prefer implementing q to n, so $\theta(\upsilon) \geq \min_{i\in\{L,R\}}\{w_i^\sigma + e_i^\sigma\}$. From Step 2, this implies that $\theta(\upsilon) > w_\Lambda^\sigma$, so one player must strictly prefer implementing p to n, and therefore $\upsilon \notin \Upsilon^\sigma(p,n)$. From Step 3, $e_\Lambda^\sigma > 0$ so $\upsilon \notin \Upsilon^\sigma(p,q)$. Since $(\Upsilon^\sigma(p,x))_{x\in\{n,p,q\}}$ is a partition of Υ, this implies $\upsilon \in \Upsilon^\sigma(p,p)$, as needed.

Step 5: $w_\Lambda^\sigma < w_\varrho^\sigma$.

From Step 3, $e_\varrho^\sigma \leq 0$, so (22) implies $\int_{\upsilon\in\Upsilon^\sigma(p,p)\cap\Upsilon^\sigma(q,n)} \left(\theta(\upsilon) - w_\varrho^\sigma\right) d\mu(\upsilon) < 0$. Therefore, there exists $\Upsilon^o \subset \Upsilon^\sigma(p,p) \cap \Upsilon^\sigma(q,n)$ such that $\mu(\Upsilon^o) > 0$ and for all $\upsilon^o \in \Upsilon^o$, $\theta(\upsilon^o) < w_\varrho^\sigma$, that is, ϱ strictly prefers implementing n to p in state υ^o. Since $\Upsilon^o \subset \Upsilon^\sigma(p,p)$ and since σ is an equilibrium, Λ must weakly prefer implementing p to n in state υ^o, i.e., $\theta(\upsilon^o) - w_\Lambda^\sigma \geq 0$, which with $\theta(\upsilon^o) < w_\varrho^\sigma$ implies $w_\Lambda^\sigma < w_\varrho^\sigma$. ■

The next lemma will be used to prove Proposition 4 Part (*iii*).

Lemma 6 *If σ is an equilibrium, then*

$$
\begin{aligned}
e_i^\sigma &= e_i + \delta \int_{\Upsilon^\sigma(q,q)} e_i^\sigma d\mu(\upsilon) + \int_{\Upsilon^\sigma(p,p)\cap\Upsilon^\sigma(q,n)} (\theta(\upsilon) - w_i^\sigma) d\mu(\upsilon), \qquad (23)\\
w_i^\sigma &= w_i + \delta \int_{\Upsilon^\sigma(p,p)\cap\Upsilon^\sigma(n,n)} (w_i^\sigma - \theta(\upsilon)) d\mu(\upsilon) - \delta \int_{\Upsilon^\sigma(n,q)} e_i^\sigma d\mu(\upsilon). \qquad (24)
\end{aligned}
$$

and for some D such that $(1-\delta)^2 \leq D \leq 1$,

$$\begin{aligned} w_R^\sigma - w_L^\sigma &= \tfrac{(1-\delta\mu(\Upsilon^\sigma(q,q)))[w_R - w_L] - \delta\mu(\Upsilon^\sigma(n,q))[e_R - e_L]}{D}, \\ e_R^\sigma - e_L^\sigma &= \tfrac{(1-\delta\mu(\Upsilon^\sigma(p,p)\cap\Upsilon^\sigma(n,n)))[e_R - e_L] - \delta\mu(\Upsilon^\sigma(p,p)\cap\Upsilon^\sigma(q,n))[w_R - w_L]}{D}, \\ w_R^\sigma + e_R^\sigma - w_L^\sigma - e_L^\sigma &= \tfrac{(1-\delta\mu(\Upsilon^\sigma(p,p)\cap\Upsilon^\sigma(q,\{n,q\})))[w_R - w_L] + (1-\delta\mu(\Upsilon^\sigma(p,p)\cap\{\Upsilon^\sigma(n,n)\cup\Upsilon^\sigma(n,q)\}))[e_R - e_L]}{D}. \end{aligned} \tag{25}$$

Proof. *Step 1: for any equilibrium σ, $\Upsilon^\sigma(q,q) \subset \Upsilon^\sigma(p,p)$, $\Upsilon^\sigma(n,q) \subset \Upsilon^\sigma(p,p)$, (24), and (23).*

That $\Upsilon^\sigma(q,q) \subset \Upsilon^\sigma(p,p)$ follows from Lemma 4A and Lemma 5. To show that $\Upsilon^\sigma(n,q) \subset \Upsilon^\sigma(p,p)$, let $\upsilon \in \Upsilon^\sigma(n,q)$. In state $\theta(\upsilon)$ both players must weakly prefer implementing q to n, so $\theta(\upsilon) \geq \max_{i\in\{L,R\}}\{w_i^\sigma + e_i^\sigma\}$. From Lemma 4A and Lemma 5,

$$\max_{i\in\{L,R\}}\{w_i^\sigma + e_i^\sigma\} \geq \min_{i\in\{L,R\}}\{w_i^\sigma + e_i^\sigma\} > \min_{i\in\{L,R\}}\{w_i^\sigma\},$$

so $\theta(\upsilon) > \min_{i\in\{L,R\}}\{w_i^\sigma\}$, which means that one player strictly prefers implementing p to n, and therefore $\upsilon \notin \Upsilon^\sigma(p,n)$. Since $\Upsilon^\sigma(p,q) = \emptyset$, and since $(\Upsilon^\sigma(p,x))_{x\in\{n,p,q\}}$ is a partition of Υ, necessarily, $\upsilon \in \Upsilon^\sigma(p,p)$, as needed.

Substituting $\Upsilon^\sigma(n,q) \subset \Upsilon^\sigma(p,p)$ into (12), we obtain (24). Finally, since $\Upsilon^\sigma(p,n) \cap \Upsilon^\sigma(p,p) = \emptyset$, $\Upsilon^\sigma(q,q) \subset \Upsilon^\sigma(p,p)$ implies $\Upsilon^\sigma(p,n) \cap \Upsilon^\sigma(q,q) = \emptyset$. Substituting the latter equality and $\Upsilon^\sigma(q,q) \subset \Upsilon^\sigma(p,p)$ into (13), we obtain (23).

Step 2: Proof of the first two lines of (25).

Subtracting (24) for $i = R$ from (24) for $i = L$, and doing the same for (23), we get

$$\begin{cases} w_R^\sigma - w_L^\sigma = w_R - w_L + \delta\mu(\Upsilon^\sigma(p,p)\cap\Upsilon^\sigma(n,n))(w_R^\sigma - w_L^\sigma) - \delta\mu(\Upsilon^\sigma(n,q))(e_R^\sigma - e_L^\sigma) \\ e_R^\sigma - e_L^\sigma = e_R - e_L + \delta\mu(\Upsilon^\sigma(q,q))(e_R^\sigma - e_L^\sigma) - \delta\mu(\Upsilon^\sigma(p,p)\cap\Upsilon^\sigma(q,n))(w_R^\sigma - w_L^\sigma) \end{cases}.$$

The above equations can be viewed as a linear system in $w_R^\sigma - w_L^\sigma$ and $e_L^\sigma - e_R^\sigma$. Straightforward algebra shows that its solution is given by the first two lines of (25) for

$$D \equiv (1-\delta\mu(\Upsilon^\sigma(p,p)\cap\Upsilon^\sigma(n,n)))(1-\delta\mu(\Upsilon^\sigma(q,q))) - \delta^2\mu(\Upsilon^\sigma(p,p)\cap\Upsilon^\sigma(q,n))\,\mu(\Upsilon^\sigma(n,q)).$$

Step 3: Proof of the third line of (25).

From Step 1, $\Upsilon^\sigma(q,q) \subset \Upsilon^\sigma(p,p)$ and, by definition of Υ^σ, $\Upsilon^\sigma(q,n)$ and $\Upsilon^\sigma(q,q)$ are disjoint, so

$$\mu(\Upsilon^\sigma(p,p)\cap\Upsilon^\sigma(q,n)) + \mu(\Upsilon^\sigma(q,q)) = \mu(\Upsilon^\sigma(p,p)\cap\{\Upsilon^\sigma(q,n)\cup\Upsilon^\sigma(q,q)\}). \tag{26}$$

From Step 1, $\Upsilon^{\sigma}(n,q) \subset \Upsilon^{\sigma}(p,p)$ and, by definition of Υ^{σ}, $\Upsilon^{\sigma}(n,n)$ and $\Upsilon^{\sigma}(n,q)$ are disjoint, so

$$\mu(\Upsilon^{\sigma}(p,p) \cap \Upsilon^{\sigma}(n,n)) + \mu(\Upsilon^{\sigma}(n,q)) = \mu(\Upsilon^{\sigma}(p,p) \cap \{\Upsilon^{\sigma}(n,n) \cup \Upsilon^{\sigma}(n,q)\}). \tag{27}$$

Adding up the first two lines of (25) and substituting (26) and (27) into the corresponding expression for $w_R^{\sigma} + e_R^{\sigma} - w_L^{\sigma} - e_L^{\sigma}$, we obtain the third line of (25) for the above D.

Step 4: $(1-\delta)^2 \leq D \leq 1$.

That $D \leq 1$ is obvious from the definition of D. To prove $D \geq (1-\delta)^2$, observe that $\Upsilon^{\sigma}(q,q)$ and $\Upsilon^{\sigma}(p,p) \cap \Upsilon^{\sigma}(q,n)$ are disjoint. So $\Upsilon^{\sigma}(q,q)$ is included into the complement of $\Upsilon^{\sigma}(p,p) \cap \Upsilon^{\sigma}(q,n)$. Likewise, $\Upsilon^{\sigma}(p,p) \cap \Upsilon^{\sigma}(n,n)$ and $\Upsilon^{\sigma}(n,q)$ are disjoint, so $\Upsilon^{\sigma}(p,p) \cap \Upsilon^{\sigma}(n,n)$ is included in the complement of $\Upsilon^{\sigma}(n,q)$. Therefore,

$$\begin{aligned}
D &= (1-\delta\mu(\Upsilon^{\sigma}(p,p) \cap \Upsilon^{\sigma}(n,n)))(1-\delta\mu(\Upsilon^{\sigma}(q,q))) \\
&\quad -\delta^2\mu(\Upsilon^{\sigma}(p,p) \cap \Upsilon^{\sigma}(q,n))\,\mu(\Upsilon^{\sigma}(n,q)) \\
&\geq (1-\delta(1-\mu(\Upsilon^{\sigma}(n,q))))(1-\delta(1-\mu(\Upsilon^{\sigma}(p,p) \cap \Upsilon^{\sigma}(q,n)))) \\
&\quad -\delta^2\mu(\Upsilon^{\sigma}(p,p) \cap \Upsilon^{\sigma}(q,n))\,\mu(\Upsilon^{\sigma}(n,q)) \\
&= (1-\delta)(1-\delta+\delta(\mu(\Upsilon^{\sigma}(n,q)) + \mu(\Upsilon^{\sigma}(p,p) \cap \Upsilon^{\sigma}(q,n)))) \geq (1-\delta)^2.
\end{aligned}$$

■

8.2 Proofs for Section 4

Proof of Lemma 1. The second claim of Lemma 1 follows from definition of (w^{σ}, e^{σ}) in Lemma 2, and from Lemma 3. The rest of the proof establishes equilibrium existence.

The strategy of the proof is to define an agent normal-form game $\hat{\Gamma}$ in which we create an agent for each Markov state of Γ[19] and give this agent payoff equal to the continuation payoff of the game starting from this Markov state, as in the standard proof of existence for finite Markov games. To make sure that this game has finitely many players, we assume that the same agent is playing in any two states that differ only in the realization of $\theta(t)$. We then show that the agent normal-form game admits a Nash equilibrium in simple strategies (defined below), and hence by Selten 1975, this equilibrium defines an equilibrium of Γ.

Step 1: Definition of the agent normal form game $\hat{\Gamma}$ *associated to* Γ.

Define the agent normal-form game $\hat{\Gamma}$ as follows. For all nodes in which $i \in \{L, R\}$ is the

[19] One can define the Markov states of Γ as follows. The Markov state associated to a given action node a is the set of action nodes a' of Γ such that the state of nature, the status quo, the identity of the proposer, and the player who is moving are the same in a as in a'.

veto player and chooses between $x, y \in \{n, p, q\}$ in Γ, replace i by a player called $\hat{\imath}(\{x, y\})$, and for all nodes in which $i \in \{L, R\}$ is the proposer facing the status quo $x \in \{n, p, q\}$, replace i by a player called $\hat{\imath}(x)$. That is, each $i \in \{L, R\}$ is replaced with 9 different players, so $\hat{\Gamma}$ has 18 players. A pure stationary strategy of $\hat{\imath}(x)$ ($\hat{\imath}(\{x, y\})$) is a mapping from the realization of θ in $\mathbb{R}$ to $\{n, p, q\}$ (to $\{x, y\}$). Note that a strategy profile of $\hat{\Gamma}$ defines a stationary strategy profile in Γ and vice versa. For any strategy profile $\hat{\sigma}$ of $\hat{\Gamma}$, we define the the payoff of $\hat{\imath}(x)$ (of $\hat{\imath}(\{x, y\})$) from alternative $z \in \{n, p, q\}$ as the expected discounted payoff of i from z in the game Γ in period 0, conditional on θ_0 being drawn from F. By construction of $\hat{\Gamma}$, a Nash equilibrium of $\hat{\Gamma}$ is a stationary Markov perfect equilibrium of Γ (Selten 1975). To complete the proof, we show below that $\hat{\Gamma}$ admits a Nash equilibrium.

Step 2: Definition of simple strategies.

Consider the following class of strategies for the players of $\hat{\Gamma}$, henceforth called simple strategies: for any $x \in \{p, q\}$, player $\hat{\imath}(\{n, x\})$ chooses n when $\theta \leq c_{\{n,x\}}$, and x otherwise, for some $c_{\{n,x\}} \in \mathbb{R}$; player $\hat{\imath}(\{p, q\})$ chooses p with probability (henceforth w.p.) $\pi_{\{p,q\}}$ and q with the remaining probability for some $\pi_{\{p,q\}} \in [0, 1]$; for any $x \in \{n, p, q\}$, player $\hat{\imath}(x)$ proposes n when $\theta \leq \underline{c}_x$, proposes p w.p. $\underline{\pi}_x$ and q w.p. $1 - \underline{\pi}_x$ when $\underline{c}_x < \theta \leq \bar{c}_x$, and proposes p w.p. $\bar{\pi}_x$ and q w.p. $(1 - \bar{\pi}_x)$ when $\theta > \bar{c}_x$, for some $\underline{c}_x \leq \bar{c}_x$ and some $\underline{\pi}_x, \bar{\pi}_x \in [0, 1]$. Note that the set of parameters $\left\{c_{\{n,x\}}\right\}_{x \in \{p,q\}}, \{\underline{c}_x, \bar{c}_x, \underline{\pi}_x, \bar{\pi}_x\}_{x \in \{n,p,q\}}$ satisfying the above restrictions is closed and convex.

Step 3: For any simple strategy profile $\hat{\sigma}$ and any player $\hat{\imath}$ of $\hat{\Gamma}$, there exists a simple strategy for $\hat{\imath}$ that is a best response to $\hat{\sigma}_{-\hat{\imath}}$.

Let $\hat{\sigma}$ be a simple strategy profile and let $\hat{\imath}$ be a player of $\hat{\Gamma}$. Finding a best response (not necessarily simple) for $\hat{\imath}$ to $\hat{\sigma}_{-\hat{\imath}}$ is a standard discrete dynamic programing problem, so a solution exists for the usual reason (see, e.g., Blackwell 1962). Moreover, any other best response must give $\hat{\imath}$ the same payoff in $\hat{\Gamma}$, and hence the same payoff for i in Γ. By Lemma 2, these payoffs can be summarized by continuation payoff parameters, and let $\left(w_{\hat{\imath}}^{\hat{\sigma}}, e_{\hat{\imath}}^{\hat{\sigma}}\right)$ denote these parameters when $\hat{\imath}$ best responds to $\hat{\sigma}_{-\hat{\imath}}$. Then choosing p when $\theta > w_{\hat{\imath}(\{n,p\})}^{\hat{\sigma}}$ and n otherwise is a simple best response to $\hat{\sigma}$ for $\hat{\imath}(\{n, p\})$. Choosing q when $\theta > w_{\hat{\imath}(\{n,q\})}^{\hat{\sigma}} + e_{\hat{\imath}(\{n,q\})}^{\hat{\sigma}}$ and n otherwise is a simple best response to $\hat{\sigma}$ for $\hat{\imath}(\{n, q\})$. And choosing p over q w.p. 1 if $e_{\hat{\imath}(\{p,q\})}^{\hat{\sigma}} > 0$, w.p. 0 if $e_{\hat{\imath}(\{p,q\})}^{\hat{\sigma}} < 0$, and with any probability in $[0, 1]$ if $e_{\hat{\imath}(\{p,q\})}^{\hat{\sigma}} = 0$ is a simple best response to $\hat{\sigma}$ for player $\hat{\imath}(\{p, q\})$. For all $x \in \{n, p, q\}$, the best response in simple strategies for player $\hat{\imath}(x)$ can be constructed by using the intuition provided in Section 4.1 and in Figures 1 and 2. Consider, e.g., $\hat{\imath}(n)$ in the case $e_{\hat{\imath}(n)}^{\hat{\sigma}} > 0$. Then a best response for $\hat{\imath}(n)$ is to propose p when she prefers implementing p to n (given continuation payoff parameter $w_{\hat{\imath}(n)}^{\hat{\sigma}}$ and $e_{\hat{\imath}(n)}^{\hat{\sigma}}$) and $\hat{\imath}(\{n, p\})$ accepts p, to propose q when she prefers implementing q to n and $\hat{\imath}(\{n, p\})$ rejects p, and to propose n otherwise. That is, if $c^{\hat{\sigma}}_{\{n,p\}}$denote the cutoff used by player $\hat{\jmath}(\{n,p\})$ in $\hat{\sigma}$, then a best response to $\hat{\sigma}$ for $\hat{\imath}(n)$ is to propose p when $\theta > \max\left\{w^{\hat{\sigma}}_{\hat{\imath}(n)}, c^{\hat{\sigma}}_{\{n,p\}}\right\}$, q when $\theta \in \left(w^{\hat{\sigma}}_{\hat{\imath}(n)} + e^{\hat{\sigma}}_{\hat{\imath}(n)}, c^{\hat{\sigma}}_{\{n,p\}}\right]$—with the convention that $(a,b) = \emptyset$ if $a \geq b$—and n otherwise, which is a simple strategy. The simple best response to $\hat{\sigma}$ of $\hat{\imath}(\{n\})$ when $e^{\hat{\sigma}}_{\hat{\imath}(n)} = 0$ and $e^{\hat{\sigma}}_{\hat{\imath}(n)} < 0$, and of $\hat{\imath}(p)$ and $\hat{\imath}(q)$ can be constructed in a similar fashion. We omit these descriptions for brevity.

Step 4: $\hat{\Gamma}$ admits a Nash equilibrium in simple strategies.

From (4), the payoff difference between two policies is bounded by $E\left[|\theta| + |w_i| + |e_i|\right]$, so the expected payoff difference between two policy paths is bounded by $\frac{E[|\theta|]+|w_i|+|e_i|}{1-\delta}$. Therefore, by definition of w^{σ}_i and e^{σ}_i (see Lemma 2), $[-|w^{\sigma}_i| - |e^{\sigma}_i|, |w^{\sigma}_i| + |e^{\sigma}_i|]$ must be included in the compact set $C \equiv \left[\frac{-E[|\theta|]-|w_i|-|e_i|}{1-\delta}, \frac{E[|\theta|]+|w_i|+|e_i|}{1-\delta}\right]$. So if $\hat{\sigma}$ is a simple strategy of $\hat{\Gamma}$ whose cutoff parameters $\left\{c_{\{n,x\}}\right\}_{x\in\{p,q\}}, \left\{\underline{c}_x, \bar{c}_x\right\}_{x\in\{n,p,q\}}$ are all in C, then from the construction in Step 3 there exist simple strategies that are best response to $\hat{\sigma}$ with cutoffs also in C. Hence, the set of parameters $\left(\left\{c_{\{n,x\}}\right\}_{x\in\{p,q\}}, \left\{\underline{c}_x, \bar{c}_x, \underline{\pi}_x, \bar{\pi}_x\right\}_{x\in\{n,p,q\}}\right)$ for which the corresponding simple strategies are best responses to some simple strategy $\hat{\sigma}$ is convex and upper-hemicontinuous in the parameters $\left\{c^{\hat{\sigma}}_{\{n,x\}}\right\}_{x\in\{p,q\}}, \left\{\underline{c}^{\hat{\sigma}}_x, \bar{c}^{\hat{\sigma}}_x, \underline{\pi}^{\hat{\sigma}}_x, \bar{\pi}^{\hat{\sigma}}_x\right\}_{x\in\{n,p,q\}}$ of $\hat{\sigma}$. Kakutani's fixed point theorem implies then that $\hat{\Gamma}$ admits an equilibrium in simple strategies. ■

Proof of Proposition 2. Simply set $i = \Lambda$ and $j = \varrho$ in Lemma 5. ■

Proof of Proposition 4 Parts (i) and (ii) and Part (iii) in the case $w_R - w_L \to 0$..

An equilibrium is either EE or IE, so all equilibria are IE if and only if no EE exist. Lemma 4 implies that no EE exist if and only if there exists no $(w^*, e^*) \in \mathbb{R}^2$ such that $e^*_R \geq 0$ and (17) and (18) hold. The proof proceeds by deriving conditions on the primitive parameters such that any solution (w^*, e^*) to (15) and (16) must be such that $e^*_R < 0$.

Step 0: Notations.

Condition (15) depends on (w,e) only through w, so let $W^*(w)$ denote the set of $w^* \in \mathbb{R}^2$ solving (17) for $i = L, R$. If we fix w^* and e_L, condition (18) for $i = L$ can viewed as a fixed point in e^*_L. It depends on (w,e) only through w^* and e_L, so let $E^*_L(w^*, e_L)$ denote the set of e^*_L solving (18) for $i = L$. Since the R.H.S. of (18) is continuous and bounded in e^*_L, $E^*_L(w^*, e_L)$ is nonempty and closed, so let $\underline{e^*_L}(w^*, e_L)$ denote its minimum. Finally, if we fix w^*, e^*_L, and e_R, (18) for $i = R$ pins down a unique e^*_R which we denote $e^*_R(w^*, e^*_L, e_R)$.

Step 1: for all $w^* \in W^*(w)$, $\underline{e^*_L}(w^*, e_L)$ is weakly increasing in e_L, and $e^*_R(w^*, e^*_L, e_R)$ is weakly increasing in e_R and weakly decreasing in e^*_L.

Since the R.H.S. of (18) for $i = L$ is continuous in (e_L, e^*_L) and weakly increasing in e_L, Theorem 1 in Villas Boas 1997 implies that the smallest fixed point $\underline{e^*_L}(w^*, e_L)$ is weakly increasing in e_L. The comparative statics on $e^*_R(.)$ follow readily from (18) for $i = R$ and the fact that for all $w^* \in W^*(w)$, $w^*_L < w^*_R$ (see Lemma 4A).

Step 2: no EE exists if and only if for all $w^* \in W(w)$, $e_R^*\left(w^*, \underline{e_L^*}(w^*, e_L), e_R\right) < 0$.
As explained at the beginning of this proof, no EE exists if and only if for all solutions to (17) and (18), $e_R^* < 0$. Using the notations of Step 0 and the comparative statics established in Step 1, no EE exists if and only if $e_R^*\left(w^*, \underline{e_L^*}(w^*, e_L), e_R\right) < 0$ for all $w^* \in W^*(w)$.

Step 3: Proof of Proposition 4 Parts (i) *and* (ii).
Part (i) follows from Step 2, together with the comparative statics established in Step 1. We now prove Part (ii). Let $w^* \in W^*(w)$. As shown in Step A1 in the proof of Lemma 4, $w_L^* < w_R^*$, so (18) for $i = L$ implies that $\underline{e_L^*}(w^*, e_L) \geq e_L > 0$. Finally, (18) for $i = R$ implies

$$\lim_{e_R \to 0} e_R^*\left(w^*, \underline{e_L^*}(w^*, e_L), e_R\right) = \delta \int_{w_L^*}^{\min\left\{w_L^* + \underline{e_L^*}(w^*, e_L), w_R^*\right\}} b_R(\theta, q)(\theta - w_R^*)\, dF(\theta).$$

Since F has full support and $b_R(\theta, q) > \underline{b} > 0$ for all $\theta \in \mathbb{R}$, the R.H.S. of the above equation is strictly negative. Step 2 implies then that for e_R sufficiently small, no EE exists.

Step 4: Proof of Proposition 4 Part (iii) *in the case* $w_R - w_L \to 0$.
Suppose Proposition 4 Part (iii) is false in the case $w_R - w_L \to 0$. Then from step 2, there exists $\left(w^k\right)_{k \in \mathbb{N}}$ such that $w_R^k - w_L^k \to 0$ and for all $k \in \mathbb{N}$, for all $w^{*,k} \in W^*\left(w^k\right)$, $e_R^*\left(w^{*,k}, \underline{e_L^*}\left(w^{*,k}, e_L\right), e_R\right) < 0$. Taking differences across players in (17), we obtain $w_R^{*,k} - w_L^{*,k} = \frac{w_R^k - w_L^k}{1 - \delta\left(F\left(w_R^{*,k}\right) - F\left(w_L^{*,k}\right)\right)} \leq \frac{w_R^k - w_L^k}{1-\delta}$, so $w_R^{*,k} > w_L^{*,k}$ and $w_R^{*,k} - w_L^{*,k} \to_{k \to \infty} 0$. Since $w_R^{*,k} > w_L^{*,k}$, (18) for $i = R$ implies $e_R^*\left(w^{*,k}, \underline{e_L^*}\left(w^{*,k}, e_L\right), e_R\right) \geq e_R + \delta \int_{w_L^{*,k}}^{w_R^{*,k}} b_R(\theta, q)\left(\theta - w_R^{*,k}\right) dF(\theta)$. Since $w_R^{*,k} - w_L^{*,k} \to_{k \to \infty} 0$, continuity of F implies that the R.H.S of the above inequality tends to $e_R > 0$ as $k \to \infty$, which contradicts that $e_R^*\left(w^{*,k}, \underline{e_L^*}\left(w^{*,k}, e_L\right), e_R\right) < 0$. ■

Proof of Proposition 4 Part (iii) **case** $w_R - w_L \to +\infty$**.**

Suppose by contradiction that Part (iii) is false in the case $w_R - w_L \to +\infty$. Then there exists $e \in (0, +\infty)^2$, $m \in \mathbb{R}$ and two sequences $\left(w^k\right)_{k \in \mathbb{N}}$ and $(\sigma(k))_{k \in \mathbb{N}}$ with the following properties: $w_R^k - w_L^k \to +\infty$, for all $k \in \mathbb{N}$, $w_L^k + w_R^k = m$, and $\sigma(k)$ is an IE for the flow payoff parameters $\left(e, w^k\right)$. Below, we show that for k sufficiently large, $e_L^{\sigma(k)} > 0$ and $e_R^{\sigma(k)} > 0$, which, from Lemma 4, contradicts the assumption that $\sigma(k)$ is an IE.

Step 1: for k *sufficiently large*, $w_L^{\sigma(k)} < w_L^{\sigma(k)} + e_L^{\sigma(k)} < w_R^{\sigma(k)} + e_R^{\sigma(k)} \leq w_R^{\sigma(k)}$.
Since $w_R^k - w_L^k \to +\infty$, Equation (25) and inequality $D \geq (1-\delta)^2$ in Lemma 6 imply that, for k large enough,

$$w_R^{\sigma(k)} - w_L^{\sigma(k)} \geq (1-\delta)\left(w_R^k - w_L^k\right) - |e_R - e_L|.$$

Hence, $w_R^{\sigma(k)} - w_L^{\sigma(k)} \to +\infty$ so for k large enough, $w_R^{\sigma(k)} > w_L^{\sigma(k)}$, and Lemma 5 implies that

$e_L^{\sigma(k)} > 0 \geq e_R^{\sigma(k)}$. Moreover, (25) and inequality $D \geq (1-\delta)^2$ in Lemma 6 imply

$$w_R^{\sigma(k)} + e_R^{\sigma(k)} - w_L^{\sigma(k)} - e_L^{\sigma(k)} \geq (1-\delta)\left(w_R^k - w_L^k\right) - \left|e_R - e_L\right|.$$

So $w_R^{\sigma(k)} + e_R^{\sigma(k)} - w_L^{\sigma(k)} - e_L^{\sigma(k)} \to +\infty$. Together with $e_L^{\sigma(k)} > 0 \geq e_R^{\sigma(k)}$, this implies Step 1.

Step 2: for k sufficiently large, $e_R^{\sigma(k)} \geq e_R + \delta\mu\left(\Upsilon^{\sigma(k)}(q,q)\right)e_R^{\sigma(k)} + \delta\int_{w_L^{\sigma(k)}}^{w_L^{\sigma(k)}+e_L^{\sigma(k)}}\left(w_L^{\sigma(k)} - w_R^{\sigma(k)}\right)dF(\theta)$.

Let $\upsilon \in \Upsilon^{\sigma(k)}(p,p) \cap \Upsilon^{\sigma(k)}(q,n)$. Since $\sigma(k)$ is an equilibrium, in state υ, both players must weakly prefer implementing n to q and one player must prefer implementing p to n. Thus, $\min_{i\in\{L,R\}} w_i^{\sigma(k)} \leq \theta(\upsilon) \leq \min_{i\in\{L,R\}}\left(w_i^{\sigma(k)} + e_i^{\sigma(k)}\right)$. Together with Step 1, this implies

$$\Upsilon^{\sigma(k)}(p,p) \cap \Upsilon^{\sigma(k)}(q,n) \subset \left\{\upsilon \in \Upsilon : w_L^{\sigma(k)} \leq \theta(\upsilon) \leq w_L^{\sigma(k)} + e_L^{\sigma(k)}\right\}. \tag{28}$$

From (28), for all $\upsilon \in \Upsilon^{\sigma(k)}(p,p) \cap \Upsilon^{\sigma(k)}(q,n)$, $\theta(\upsilon) - w_L^{\sigma(k)} \leq e_L^{\sigma(k)}$. Substituting this inequality into (23), we obtain

$$e_L^{\sigma(k)} \leq e_L + \delta\mu\left(\Upsilon^{\sigma(k)}(q,q)\right)e_L^{\sigma(k)} + \delta\mu\left(\Upsilon^{\sigma(k)}(p,p) \cap \Upsilon^{\sigma(k)}(q,n)\right)e_L^{\sigma(k)},$$

and therefore that $e_L^{\sigma(k)} \leq e_L/(1-\delta)$. Since $e_L^{\sigma(k)} > 0$, the latter inequality implies that $e_L^{\sigma(k)}$ is bounded. From Step 1 and (28), for all $\upsilon \in \Upsilon^{\sigma(k)}(p,p) \cap \Upsilon^{\sigma(k)}(q,n)$, $0 \geq \theta(\upsilon) - w_R^{\sigma(k)} \geq w_L^{\sigma(k)} - w_R^{\sigma(k)}$. The preceding inequality and (28) imply that

$$\int_{\Upsilon^\sigma(p,p)\cap\Upsilon^\sigma(q,n)}\left(\theta(\upsilon) - w_R^\sigma\right)d\mu(\upsilon) \geq \int_{w_L^{\sigma(k)}}^{w_L^{\sigma(k)}+e_L^{\sigma(k)}}\left(w_L^{\sigma(k)} - w_R^{\sigma(k)}\right)dF(\theta).$$

Substituting the above inequality into (23), we obtain Step 2.

Step 3: as $k \to +\infty$, $e_L^{\sigma(k)} \to -\infty$, $e_R^{\sigma(k)} \to +\infty$ *and* $w_R^{\sigma(k)} \sim \left|w_L^{\sigma(k)}\right|$.

As shown in Step 1, $w_R^k - w_L^k \to +\infty$. Therefore, to prove Step 3, it suffices to show that $w_L^{\sigma(k)} + w_R^{\sigma(k)}$ is bounded as $k \to \infty$. To do so, note that if we sum (23) and (24) across players and collect the terms in factor of $w_L^{\sigma(k)} + w_R^{\sigma(k)}$ and $e_L^{\sigma(k)} + e_R^{\sigma(k)}$, we obtain

$$\begin{pmatrix} 1 - \delta\mu\left(\Upsilon^{\sigma(k)}(p,p) \cap \Upsilon^{\sigma(k)}(n,n)\right) & \delta\mu\left(\Upsilon^{\sigma(k)}(n,q)\right) \\ \delta\mu\left(\Upsilon^{\sigma(k)}(p,p) \cap \Upsilon^{\sigma(k)}(q,n)\right) & 1 - \delta\mu\left(\Upsilon^{\sigma(k)}(q,q)\right) \end{pmatrix}\begin{pmatrix} w_L^{\sigma(k)} + w_R^{\sigma(k)} \\ e_L^{\sigma(k)} + e_R^{\sigma(k)} \end{pmatrix}$$
$$= \begin{pmatrix} m - \delta\int_{\Upsilon^{\sigma(k)}(p,p)\cap\Upsilon^{\sigma(k)}(n,n)}\theta(\upsilon)\,d\mu(\upsilon) \\ e_L + e_R + \int_{\Upsilon^{\sigma(k)}(p,p)\cap\Upsilon^{\sigma(k)}(q,n)}\theta(\upsilon)\,d\mu(\upsilon) \end{pmatrix}.$$

From Lemma 6, the determinant of that system, which is D, is bounded away from 0 as $k \to \infty$. Moreover, all the coefficients of the above system are bounded. Therefore, the solution $w_L^{\sigma(k)} + w_R^{\sigma(k)}$ must be bounded as $k \to \infty$, as needed.

Step 4: for k sufficiently large, $e_L^{\sigma(k)} > 0$ and $e_R^{\sigma(k)} > 0$.

Step 1 proves that for k large enough, $e_L^{\sigma(k)} > 0$. Let us now prove that $e_R^{\sigma(k)} > 0$. Step 3 together with the integrability of F implies that $\int_{w_L^{\sigma(k)}}^{w_L^{\sigma(k)}+e_L^{\sigma(k)}} w_L^{\sigma(k)} dF(\theta) \to 0$. Substituting this limit in Step 2, we obtain $e_R^{\sigma(k)} > 0$, as needed. ■

The proofs of Propositions 3 and 5 rely on the proof of Proposition 6, so we provide the latter before the former.

Proof of Proposition 6. Parts A and B below prove part (ii), part C proves part (i), and parts D and E prove part (iii).

Part A: If (6) and (7) hold, there exists an equilibrium of $\Gamma(n,p,q)$ and of $\Gamma(n,q)$ that is Pareto superior to any equilibrium of $\Gamma(n,p)$.

Assume (6) and (7) hold and define the strategy profile $\sigma^*(n,p,q)$ of $\Gamma(n,p,q)$ as follows: Under status quo n or q, in state $\underline{\theta}$ ($\bar{\theta}$), both players propose and accept n (q), veto q (n), and in both states, R vetoes p and L accepts p. Under status quo p, L proposes p and accepts only p, whereas R proposes p and accept any proposal. The path of $\sigma^*(n,p,q)$ is as follows: under status quo n or q, n is implemented in state $\underline{\theta}$ and q in state $\bar{\theta}$, and once p is implemented, p stays in place forever.

Below, we characterize the equilibria of $\Gamma(n,p)$ (Step A1), show that both players strictly prefer $\sigma^*(n,p,q)$ to any equilibrium of $\Gamma(n,p)$ (Step A2), and show that $\sigma^*(n,p,q)$ is an equilibrium of $\Gamma(n,p,q)$ (Steps A3 to A6). We then show that there exists an equilibrium $\sigma^*(n,q)$ of $\Gamma(n,q)$ that has the same path as $\sigma^*(n,p,q)$, and thus that also Pareto dominates any equilibrium of $\Gamma(n,p)$ (Step A7).

Step A1: In any equilibrium of $\Gamma(n,p)$, the initial status quo n stays in place forever.

Let σ be an equilibrium of $\Gamma(n,p)$. From (6), $w_L < \underline{\theta}$, so L gets a strictly greater flow payoff from p than from any other policy in either state. Therefore, on the path of σ, once implemented, p stays in place forever. Simple algebra shows that (7) means that in state $\bar{\theta}$ (and thus in state $\underline{\theta}$ as well), R strictly prefers implementing n forever to implementing p forever. Therefore, in either state, given continuation play σ, R strictly prefers implementing n forever to p forever. Therefore, the path of σ stays at the initial status quo n forever.

Step A2: Both players strictly prefer $\sigma^*(n,p,q)$ to any equilibrium of $\Gamma(n,p)$.

The path of $\sigma^*(n,p,q)$ implements n and q in state $\underline{\theta}$ and $\bar{\theta}$, respectively. From (6), both players get a strictly greater payoff from q than from n in state $\bar{\theta}$, so they strictly prefer the path of $\sigma^*(n,p,q)$ to staying at n forever. Step A2 follows then from Step A1.

Step A3: Given continuation play $\sigma^*(n,p,q)$, in either state, L strictly prefers implementing p to either n or q.

From (6), p gives a strictly greater flow payoff to L than any other policy in any state, and on the path of $\sigma^*(n,p,q)$, p stays in place forever once implemented.

Step A4: Given continuation play $\sigma^*(n,p,q)$, both players prefer implementing n to q (q to n) in state $\underline{\theta}$ ($\bar{\theta}$).

From (6), both players get a weakly greater flow payoff from n than from q (from q than from n) in state $\underline{\theta}$ ($\bar{\theta}$), and the same continuation value as under $\sigma^*(n,p,q)$, status quo n or q lead to the same outcome.

Step A5: Given continuation play $\sigma^*(n,p,q)$, in any state, R strictly prefers implementing n or q to p.

From Steps A1 and A2, R prefers implementing n and playing $\sigma^*(n,p,q)$ thereafter to implementing n forever. Moreover, (7) means that in either state, R strictly prefers n forever to p forever, which is what happens if p is implemented and $\sigma^*(n,p,q)$ is played thereafter. Therefore, given continuation play $\sigma^*(n,p,q)$, in either state, R strictly prefers implementing n to implementing p. From Step A4, in state $\bar{\theta}$, R strictly prefers implementing q to n, so from what precedes, she must strictly prefer implementing q to p in state $\bar{\theta}$. Since the flow payoff difference between q and p is independent of θ, R also strictly prefers implementing q to p in state $\underline{\theta}$.

Step A6: $\sigma^*(n,p,q)$ is an equilibrium of $\Gamma(n,p,q)$.

It is straightforward to check that given the preferences characterized in Steps A3-A5, behavior prescribed by $\sigma^*(n,p,q)$ is sequentially rational.

Step A7: there exists an equilibrium of $\Gamma(n,q)$ that is Pareto superior to any equilibrium of $\Gamma(n,p)$.

By construction, under status quo n or q, $\sigma^*(n,p,q)$ never prescribes player to make proposal p. So $\sigma^*(n,p,q)$ defines also an equilibrium of $\Gamma(n,q)$, and A7 follows from A2.

Part B: if there exists an equilibrium of $\Gamma(n,p,q)$ or $\Gamma(n,q)$ that is Pareto superior to any equilibrium $\sigma^*(n,p)$ of $\Gamma(n,p)$, then (6) and (7) must hold.

Let $\sigma(n,p,q)$ be an equilibrium of $\Gamma(n,p,q)$ that is Pareto superior to any equilibrium of $\Gamma(n,p)$ (argument for the case of $\Gamma(n,q)$ is analogous and is omitted for brevity).

Step B1: $w_L < \underline{\theta}$.

Suppose first that $w_L \geq \underline{\theta}$ and $w_R \geq \bar{\theta}$. In that case, player R gets a weakly greater flow payoff from n than from p in either state, so there exists an equilibrium of $\Gamma(n,p)$ in which R unilaterally decides to stay forever at the initial status quo n. This equilibrium implements the most preferred policy path of R, and thus cannot be dominated by any equilibrium of $\Gamma(n,p,q)$, a contradiction. Suppose now that $w_L \geq \underline{\theta}$ and $w_R < \bar{\theta}$. In that case, both players get a greater flow payoff from p (n) than from n (p) in state $\bar{\theta}$ ($\underline{\theta}$), so there exists an equilibrium of $\Gamma(n,p)$ in which players agree to implement n in state $\underline{\theta}$ and p in state $\bar{\theta}$. This equilibrium path is optimal for both players and can thus not be dominated by any equilibrium of $\Gamma(n,p,q)$, a contradiction.

Step B2: $w_R > \underline{\theta}$.

Suppose $w_R \leq \underline{\theta}$. Then both players get a weakly greater flow payoff from p than from n in either state of nature. So there exists an equilibrium of $\Gamma(n,p)$ in which both players' most preferred alternative p is implemented forever, so this equilibrium cannot be dominated by any equilibrium of $\Gamma(n,p,q)$, a contradiction.

Step B3: Given continuation play $\sigma(n,p,q)$, L strictly prefers implementing p to n and p to q, so under status quo p, p stays in place forever.

Step B3 follows from the observation that since $w_L < \underline{\theta}$, L gets a strictly greater flow payoff from p than from any other policy in either state of nature.

Step B4: Given continuation play $\sigma(n,p,q)$, R weakly prefers implementing q to p in either state, and R strictly prefers implementing n to q and n to p in state $\underline{\theta}$.

Observe first that q must be implemented on the path of $\sigma(n,p,q)$, otherwise $\Gamma(n,p)$ would admit an equilibrium with the same path as $\sigma(n,p,q)$, so those equilibria would be payoff equivalent. From Step B3, L must veto q under status quo p, so q must be implemented with positive probability in some state $\theta^* \in \{\underline{\theta}, \bar{\theta}\}$ under status quo n. Since from Step B3, L strictly prefers implementing p to q, R must weakly prefer implementing q to p in state θ^*. Since the flow payoff difference between p and q and the distribution of future states is independent of the current state, this must be the case in either state, which proves the first claim of Step B4. Since q gives a strictly lower flow payoff than p to R, the first claim of Step B4 implies that R strictly prefers status quo q to status quo p. So it must be that in some state θ', with positive probability, status quo q leads to some outcome x that R strictly prefers to p and q. So x must be n. From the first claim of Step B4, this implies R strictly prefers implementing n to p in state θ'. Since the flow payoff gain from n relative to p or q is decreasing in θ, this is true for $\theta' = \underline{\theta}$.

Step B5: Given continuation play $\sigma(n,p,q)$, R strictly prefers implementing n to p in any state, so under status quo n, p is never implemented.

Suppose Step B5 is false. Since from Step B4, R strictly prefers n to p in $\underline{\theta}$, it must be that R weakly prefers implementing p to n in state $\bar{\theta}$. Using Step B3, this means that in state $\bar{\theta}$, R weakly prefers implementing p forever to n forever, since R can unilaterally impose the latter policy path under status quo n. Since R gets a greater flow payoff from n than from p in state $\underline{\theta}$, and since L gets a strictly greater flow payoff from p than from n in either state, this implies that there exists an equilibrium of $\Gamma(n,p)$ in which the initial status quo n stays in place as long as the state stays at $\underline{\theta}$, and p is implemented at the first occurrence of $\bar{\theta}$ and stays in place forever after. This equilibrium implements L's most preferred policy path from the first occurrence of state $\bar{\theta}$ onwards. By assumption, L is strictly better off under

$\sigma(n,p,q)$. For that to be the case, it must be that at the initial status quo n and in state $\underline{\theta}$, $\sigma(n,p,q)$ implements q or p with positive probability, a contradiction with Step B4.

Step B6: Under status quo n or q, p is never implemented on the path of $\sigma(n,p,q)$.
Suppose Step B6 is false. From Step B5, this implies that p is implemented with positive probability under status quo q. From Step B4, this means that given continuation play $\sigma(n,p,q)$, R is indifferent between implementing q or p in any state. So from Steps B5, R strictly prefers implementing n to p and to q in any state, so R must weakly prefer staying at n forever to playing $\sigma(n,p,q)$. Since she can unilaterally impose n forever in $\Gamma(n,p)$, she must weakly prefer any equilibrium of $\Gamma(n,p)$ to $\sigma(n,p,q)$, a contradiction.

Step B7: Given continuation play $\sigma(n,p,q)$, both players strictly prefer implementing q to n in state $\bar{\theta}$.
Suppose Step B7 is false. Given the flow payoff specification, this means that some player weakly prefers implementing n to q in either state. From Step B6, this implies that she must weakly prefer staying at n forever to playing $\sigma(n,p,q)$. Since she can unilaterally impose n forever in $\Gamma(n,p)$, she must weakly prefer any equilibrium of $\Gamma(n,p)$ to $\sigma(n,p,q)$, a contradiction.

Step B8: With probability 1, *status quo n stays in place in $\underline{\theta}$ and is replaced by q in $\bar{\theta}$.*
That n stays in place in state $\underline{\theta}$ follows immediately from Step B4. That it is replaced by q with probability 1 follows from Steps B5 and B7.

Step B9: Status quo q stays in place with probability 1 *in state $\bar{\theta}$; in state $\underline{\theta}$, it is replaced with positive probability by n and stays in place with the remaining probability.*
That q stays in place with probability 1 in state $\bar{\theta}$ follows from Steps B5 and B7. Suppose now that status quo q is never replaced by n in state $\underline{\theta}$. From Step B6, this implies that q must stay in place with probability 1. But then both players strictly prefer implementing p forever, so $\sigma(n,p,q)$ is not an equilibrium, a contradiction.

Step B10: Condition (6) must be satisfied.
The first inequality of (6) comes from Step B1. For the path of play described by Steps B8 and B9 to be an equilibrium, both players must get a weakly greater (smaller) flow payoff from n than from q in state θ $(\bar{\theta})$. This means that the last three inequalities in (6) must hold weakly. Suppose by contradiction that the last inequality holds with equality. Then one player must be indifferent between staying at n forever and the equilibrium path. Since she can unilaterally impose n forever in the game $\Gamma(n,p)$, she must weakly prefer any equilibrium of $\Gamma(n,p)$ to $\sigma(n,p,q)$, a contradiction.

Step B11: Condition (7) must be satisfied.
Consider the strategy profile $\sigma(n,p)$ of $\Gamma(n,p)$ defined as follows: L always proposes p and accepts only p, and R proposes n (p) and accepts only n (p) in state $\underline{\theta}$ $(\bar{\theta})$. The path of play

of $\sigma(n,p)$ is such that n stays in place until the first occurrence of state $\bar{\theta}$, at which time n is replaced by p and p stays in place forever. Since the path of play of $\sigma(n,p)$ implements n until the first occurrence of state $\bar{\theta}$ and never implements p, L must strictly prefers $\sigma(n,p)$ to $\sigma(n,p,q)$, so by assumption, $\sigma(n,p)$ cannot be an equilibrium of $\Gamma(n,p)$. From Step B1, R gets a strictly greater flow payoff from p than from n in either state, so the actions prescribed by $\sigma(n,p)$ to L are subgame perfect given continuation play $\sigma(n,p)$. The actions prescribed by $\sigma(n,p)$ to R under status quo p are also subgame perfect since R is not pivotal under that status quo. From Step B2, R gets a greater flow payoff from n than from p in state $\underline{\theta}$, so in that state, it is subgame perfect for R not to propose p and to veto p under status quo n. The only remaining case is when the status quo is n and the state is $\bar{\theta}$. Since $\sigma(n,p)$ is not an equilibrium, it must be that R's actions in that case are not subgame perfect. This means that in state $\bar{\theta}$, R must strictly prefer staying at n forever than switching to p forever. Simple algebra shows that this condition is equivalent to (7).

Part C: Proposition 6 Part (i).

If $\bar{\theta} \leq w_R$, then implementing n forever is the unique most preferred path of R, and R can unilaterally impose that path in $\Gamma(n,p,q)$.

If $\underline{\theta} \leq w_L < w_R < \bar{\theta}$, then implementing n in state $\underline{\theta}$ and p in state $\bar{\theta}$ is the most preferred path of both players, so it is an equilibrium path of $\Gamma(n,p,q)$.

Consider now the case in which $w_L < \underline{\theta} < w_R < \bar{\theta}$ and (7) holds. Then as argued in Step A1, in any equilibrium of $\Gamma(n,p)$, the initial status quo n stays in place forever. This path can be unilaterally imposed by either player in $\Gamma(n,p,q)$, so either player must weakly prefer any equilibrium of $\Gamma(n,p,q)$ to the equilibrium of $\Gamma(n,p)$.

Consider then the case in which $w_L < \underline{\theta} < w_R < \bar{\theta}$ but (7) is violated. Since $w_L < \underline{\theta}$, p is the alternative that gives L the greatest flow payoff in both states, so in any equilibrium of $\Gamma(n,p)$ and $\Gamma(n,p,q)$, L always accepts proposal p and p stays in place forever once implemented. That (7) is violated means that in state $\bar{\theta}$, R prefers implementing p forever to implementing n forever. Therefore, the only equilibrium path of $\Gamma(n,p)$ stays at the initial status quo n as long as the state is $\underline{\theta}$, and moves permanently to p at the first occurrence of $\bar{\theta}$. Now let σ be an equilibrium of $\Gamma(n,p,q)$. Suppose first that in state $\bar{\theta}$, R accepts proposal p under status quo n. Then L will propose p, and R can unilaterally impose the same path as in $\Gamma(n,p)$ by proposing p in state $\bar{\theta}$ and imposing to stay at n in state $\underline{\theta}$. So R must weakly prefer σ to the equilibrium $\Gamma(n,p)$. Suppose now that in state $\bar{\theta}$, R refuses proposal p with positive probability under status quo n. Then R must weakly prefer staying at n in that state given continuation play σ to implementing p forever. So R must weakly prefer the path of σ to the equilibrium path of $\Gamma(n,p)$.

Finally, if $w_R < \underline{\theta}$, then implementing p forever is the unique most preferred path of both players, so this is the equilibrium path for $\Gamma(n,p,q)$ and $\Gamma(n,p)$.

Part D: If all equilibria of $\Gamma(n,p,q)$ are Pareto superior to any equilibrium of $\Gamma(n,p)$, then (6), (7), and (8) must hold.

Suppose all equilibria of $\Gamma(n,p,q)$ are Pareto superior to any equilibrium of $\Gamma(n,p)$. We already know from Part B that (6) and (7) must be satisfied. So to prove Part D, it suffices to prove (8). To do so, consider the following strategy profile $\sigma'(n,p,q)$. Under status quo p, both players propose p, L accepts only p, and R vetoes only q. Under status quo n, both players proposes n, R accepts only n, and L vetoes only q. Under status quo q, both players proposes p, L accepts only p, whereas R accepts any policy. By construction, the path of play of $\sigma'(n,p,q)$ implements n forever. Since either player can unilaterally impose that outcome in the game $\Gamma(n,p)$, $\sigma'(n,p,q)$ is not Pareto superior to any equilibrium of $\Gamma(n,p)$, so by assumption, it cannot be an equilibrium. In what follows, we assume that (8) is violated, we characterize players' incentives given continuation play $\sigma'(n,p,q)$ (Step D1) and show that $\sigma'(n,p,q)$ is an equilibrium (Step D2), a contradiction.

Step D1: If condition (8) is violated, then given continuation play $\sigma'(n,p,q)$, in either state, R prefers implementing n to p to q, whereas L prefers implementing p to q to n.

Note first that since status quo q and p both lead to policy p, and since p gives both players a greater flow payoff than q, both players prefer implementing p to q in any state. Moreover, (7) implies that in either state, R prefers implementing n forever to p forever, which is what happens if n and p are implemented, respectively, given continuation play $\sigma'(n,p,q)$. Therefore, R prefers implementing n to p to q. To complete the proof, it remains to show that L prefers implementing q to n in either state, given continuation play $\sigma'(n,p,q)$, or equivalently, that L prefers implementing q today and p forever after to implementing n forever. Simple algebra shows that this is equivalent to (8) being violated.

Step D2: If condition (8) is violated, $\sigma'(n,p,q)$ is an equilibrium.

It is straightforward to check that given the preferences characterized in Step D1, $\sigma'(n,p,q)$ prescribe either veto players to accept (reject) a proposal whenever it gives her a greater (lesser) continuation payoff, and it prescribe either proposer to propose the policy that gives her the greatest continuation payoff among those accepted by the veto player, as needed.

Part E: If (6), (7), and (8) hold, any equilibrium of $\Gamma(n,p,q)$ or $\Gamma(n,q)$ is strictly Pareto superior to any equilibrium of $\Gamma(n,p)$.

Assume (6), (7), and (8). From Part A, the strategy profile $\sigma^*(n,p,q)$ defined in Part A is an equilibrium that it is strictly Pareto better than any equilibrium of $\Gamma(n,p)$. So it suffices to show that any equilibrium of $\Gamma(n,p,q)$ and $\Gamma(n,q)$ must have the same path as $\sigma^*(n,p,q)$. Throughout, σ is an arbitrary equilibrium of $\Gamma(n,p,q)$.

Step E1: Given continuation play σ, in either state, L strictly prefers implementing p to

n or q, so given continuation play σ, status quo p always stays in place.
See the proof of Step A1.

Step E2: Given continuation play σ, in either state, R strictly prefers n to p.
Since σ is an equilibrium, R weakly prefers implementing n and playing σ thereafter to implementing n forever. Moreover, (7) means that in any state, R strictly prefers implementing n forever to p forever, which, from Step E1, is what happens if p is implemented and σ is played thereafter.

Step E3: p is never implemented on the path of σ.
From Step E1, p is never implemented under status quo n. So if Step E3 is false, it must be that (i) in some state θ', p is implemented with positive probability under status quo q, and that (ii) in some state θ'', q is implemented with positive probability under the initial status quo n. Since the flow payoff of q relative to p is independent of θ, (i) must be true for all $\theta' \in \{\underline{\theta}, \bar{\theta}\}$, so it must be true for $\theta' = \theta''$. Therefore, (i) and (ii) imply that R must strictly prefer implementing p to q and q to n in state θ'', a contradiction with Step E2.

Step E4: Given continuation play σ, in state $\underline{\theta}$, both players strictly prefer to implement n to q, so in state $\underline{\theta}$, status quo n or q lead to n.
Suppose by contradiction that given continuation play σ, some player i prefers implementing q to n in state $\underline{\theta}$. Since the flow payoff of n relative to q is decreasing in θ, i prefers implementing q to n in both states, so from Step E3, i weakly prefers implementing q forever to implementing n forever. But (7) means that in any state, R strictly prefers implementing n forever to p forever, and thus to q forever. So i must be L. Simple algebra shows that (8) implies that in state $\underline{\theta}$, L prefers implementing n forever to implementing q today and p forever after, so L must prefer n forever to q forever, a contradiction. The second claim in Step E4 follows from the first claim and Step E3.

Step E5: Given continuation play σ, in state $\bar{\theta}$, both players strictly prefer to implement q to n, so in state $\bar{\theta}$, status quo n or q lead to policy q.
From Step E4, in state $\underline{\theta}$, status quo n and q lead to the same outcome. Moreover, in state $\bar{\theta}$, from (6), both players get a strictly greater flow payoff from q than from n. Therefore, given continuation play σ, both players strictly prefer implementing q to n. The second claim in Step E5 follows from the first claim and Step E3.

Step E6: σ has the same path as $\sigma^* (n, p, q)$.
This Step follows directly from Steps E3, E4, and E5.

Step E7: any equilibrium of $\Gamma(n, q)$ has the same path as $\sigma^* (n, p, q)$.
This step follows directly from the proof of Step E4 and E5. ■

Proof of Propositions 3 and 5. In the proof below, we first show that for the two-state c.d.f. of Proposition 6, denoted now by $F_{\underline{\theta},\bar{\theta},\pi}$, for any (w, e), there exists $(\delta, \underline{\theta}, \bar{\theta}, \pi)$

such that (6), (7), and (8) are satisfied (Step 1). Under those conditions, we know from Proposition 6iii) that in any equilibrium, both players strictly benefit from the availability of q. We then add a small, full support perturbation to the degenerate c.d.f. $F_{\underline{\theta},\bar{\theta},\pi}$ so as to guarantee that $\theta(t)$ has full support, as implicitly assumed in Propositions 3 and 5 (Step 2). By continuity, when the perturbation is sufficiently small, in any equilibrium, players still unanimously benefit from the availability of q, which proves Proposition 5 (Step 3). To prove Proposition 3, note that a full support approximation F of the two-state process $F_{\underline{\theta},\bar{\theta},\pi}$ is compatible with the c.d.f. F assumed in Proposition 3 only when $\pi = 0$, that is, when $F_{\underline{\theta},\bar{\theta},\pi}$ puts probability 1 on $\underline{\theta}$. Step 1 shows that for a nonnegligible sets of $\left(\delta, \underline{\theta}, \bar{\theta}\right)$, (6), (7), and (8) are satisfied even for $\pi = 0$. Under those conditions, from Proposition 6 part (iii), all equilibria of $\Gamma(n, p, q)$ for the c.d.f. $F_{\underline{\theta},\bar{\theta},\pi}$ must be IE, not in the sense that q is implemented on the equilibrium path with positive probability as required by the definition, but in the weaker sense that conditional on $\theta(t) = \bar{\theta}$ (which occurs with probability $\pi = 0$), q is implemented. By continuity, that property must be true for a small enough perturbation of $F_{\underline{\theta},\bar{\theta},\pi}$. The equilibria of this perturbed game are IE in the usual sense, which proves Proposition 3 (Step 4).

Step 1: For any (w, e), *there exists* $\bar{\theta} \in \mathbb{R}$, $\bar{\pi} > 0$, *and closed intervals* $\Delta \subset (0,1)$ *and* $\underline{\Theta} \subset \mathbb{R}$ *of positive length s.t. for all* $\delta \in \Delta$, $\underline{\theta} \in \underline{\Theta}$, *and* $\pi \in [0, \bar{\pi}]$, *(6), (7), and (8) hold.*
For any $\underline{\theta} \in (w_L, \min\{w_L + e_L, w_R\})$ and $\bar{\theta} > \max\{w_L + e_L, w_R + e_R\}$, (6) holds. Moreover, simple algebra shows that (7), and (8) can be rewritten as

$$\pi < \frac{w_R - (1-\delta)\bar{\theta} - \delta\underline{\theta}}{\delta\left(\bar{\theta} - \underline{\theta}\right)}, \text{ and } \pi < \frac{(1-\delta)e_L + w_L - \underline{\theta}}{\delta\left(\bar{\theta} - \underline{\theta}\right)}. \tag{29}$$

The R.H.S. of the two inequalities above are strictly positive if and only if

$$\frac{\bar{\theta} - w_R}{\bar{\theta} - \underline{\theta}} < \delta < \frac{e_L + w_L - \underline{\theta}}{e_L}. \tag{30}$$

Note that for $\underline{\theta} = w_L$, the L.H.S. and R.H.S. of (30) are $\frac{\bar{\theta} - w_R}{\bar{\theta} - w_L} \in (0,1)$ and 1, respectively. Therefore, if we fix $\bar{\theta} > \max\{w_L + e_L, w_R + e_R\}$, by continuity, there exists closed intervals of positive length $\Delta \subset (0,1)$ and $\underline{\Theta} \subset (w_L, \min\{w_L + e_L, w_R\})$ such that for all $\delta \in \Delta$ and $\underline{\theta} \in \underline{\Theta}$, (30) holds. Since Δ and $\underline{\Theta}$ are closed, by continuity, the R.H.S. of the two inequalities in (29) are bounded below by some $\bar{\pi} > 0$ for all $\delta \in \Delta$ and $\underline{\theta} \in \underline{\Theta}$, as needed.

Step 2: Definitions.
Let $\{\theta(t) : t \geq 0\}$ denote the process for the two-state c.d.f. $F_{\underline{\theta},\bar{\theta},\pi}$ of Proposition 6, for some $\pi \in [0,1]$ and some $\underline{\theta} < \bar{\theta}$. Let $\{\varepsilon(t) : t \geq 0\}$ be a sequence of i.i.d. random variables distributed according to some c.d.f. G with full support, and for any $d \geq 0$, consider the

i.i.d. process $\{\theta(d,t) : t \geq 0\}$ where $\theta(d,t) \equiv \theta(t) + d\varepsilon(t)$. That is, $\theta(d,t)$ is the sum of $\theta(t)$ and a full support perturbation $d\varepsilon(t)$. For all $X \subseteq \{n,p,q\}$, let $\Gamma(d,X)$ denote the game Γ in which the set of available policies is X and the payoff state is $\{\theta(d,t) : t \geq 0\}$, and let $\sigma(d,X)$ be strategy profile of $\Gamma(d,X)$. Note that for the unperturbed game $\Gamma(0,X)$, one can view the realization of $\{\varepsilon(t) : t \geq 0\}$ as a payoff irrelevant, public signal. So for any $d > 0$, any behavioral strategy profile $\sigma(d,X)$ of $\Gamma(d,X)$ defines a (possibly correlated) behavioral strategy profile of $\Gamma(0,X)$. In what follows, we refer to this strategy profile as $\sigma^0(\sigma(d,X))$.

Step 3 (Proof of Proposition 5): Using the notations of Steps 1 and 2, there exists a closed interval $\Delta \subset (0,1)$ of positive length, $\underline{\theta}, \bar{\theta} \in \mathbb{R}$, $\pi \in (0,1)$ such that for d sufficiently small, for all $\delta \in \Delta$, both players strictly prefer any equilibrium of $\Gamma(d,\{n,p,q\})$ or $\Gamma(d,\{n,q\})$ to any equilibrium of $\Gamma(d,\{n,p\})$.

To see why Step 3 proves Proposition 5, simply set F equal to the c.d.f. of $\theta(d,t)$ for the values of $\underline{\theta}, \bar{\theta} \in \mathbb{R}$, $\pi \in (0,1)$ and d found in Step 3. To prove Step 3, suppose it is false. Then for all $\Delta \subset (0,1)$, $\underline{\theta}, \bar{\theta} \in \mathbb{R}$, $\pi \in (0,1)$, there exists a subsequence $d_k \to 0$ and a sequence $\delta_k \to \delta \in \Delta$, and a corresponding subsequence of equilibria $\sigma(d_k,\{n,p\})$, $\sigma(d_k,\{n,q\})$, and $\sigma(d_k,\{n,p,q\})$ such that for all $k \in \mathbb{N}$, some player weakly prefers $\sigma(d_k,\{n,q\})$ or $\sigma(d_k,\{n,p,q\})$ to $\sigma(d_k,\{n,p\})$. From Step 1, we can pick Δ, $\underline{\theta}$, $\bar{\theta}$, and π such that (6), (7), and (8) hold. For these parameters, for any $X \subseteq \{n,p,q\}$, consider the corresponding sequence of (correlated) strategy profiles $\sigma^0(\sigma(d_k,X))$ of the game $\Gamma(0,X)$, as defined in Step 2. Since $\Gamma(0,X)$ has finitely many Markov states, we can assume w.l.o.g. that $\sigma^0(\sigma(d_k,X))$ converges to some limit $\sigma(0,X)$. Since $\theta(d,t)$ converges in distribution to $\theta(t)$ as $d \to 0$, the limit strategy $\sigma(0,X)$ must be a correlated equilibrium of $\Gamma(0,X)$. Since players play sequentially in $\Gamma(0,X)$, it is easy to check that the arguments made in Steps A1 and E of the proof of Proposition 6 to characterize the unique equilibrium path under conditions (6) and (8) are valid also if one allows for correlated strategy profiles. So the path of $\sigma(0,X)$ must be the unique (uncorrelated) equilibrium path of $\Gamma(0,X)$. By continuity, some player must weakly prefer the equilibrium path of $\Gamma(0,\{n,p,q\})$ to the equilibrium path of $\Gamma(0,\{n,q\})$ or $\Gamma(0,\{n,p,q\})$. This contradicts Proposition 6 part (iii).

Step 4 (Proof of Proposition 3): Using the notations of Step 1, for $\pi = 0$, there exists $\bar{d} > 0$ such that for all $d \in (0,\bar{d})$, $\delta \in \Delta$ and $\underline{\theta} \in \underline{\Theta}$, all equilibria of $\Gamma(d,\{n,p,q\})$ are IE.

To see why Step 4 proves Proposition 3, simply set F equal to the c.d.f. of $\theta(d,t)$ for $\pi = 0$ and the values of $\underline{\theta}$ and d found in Step 4. This F satisfies conditions of Proposition 3 because its c.d.f. is $G\left(\frac{\theta - \underline{\theta}}{d}\right)$, and from Step 1, Δ and $\underline{\Theta}$ are of positive measure. Throughout this proof, we fix $\pi = 0$ and $X = \{n,p,q\}$ so we omit X from the notations. In what follows, we use results established in Step E of the proof Proposition 6. Careful reader will notice that

Proposition 6 assumes $\pi > 0$, but that inequality is not required for the proof of step E.[20] Suppose Step 4 is false. Then there exists a sequence $d_k \to 0$, $\delta_k \to \delta \in \Delta$ and $\underline{\theta}_k \to \underline{\theta} \in \underline{\Theta}$ such that for all $k \in \mathbb{N}$, there exists EE $\sigma(d_k)$ of $\Gamma(d_k)$ for these parameter values. Therefore, from Step 1, (6), (7), and (8) are satisfied for δ, $\underline{\theta}$, and for some $\bar{\theta}$. Consider now the sequence of strategy profiles $\sigma^0(\sigma(d_k))$ of the game $\Gamma(0)$, as defined in Step 2. Since (6), (7), and (8) are satisfied, using the same argument as in Step 3, we can assume that the path of $\sigma^0(\sigma(d_k))$ converges to the unique (uncorrelated) equilibrium path $\Gamma(0)$ for the parameters δ, $\underline{\theta}$, $\bar{\theta}$ and $\pi = 0$. Since $\sigma(d_k)$ is an EE, q is implemented with probability 0 under status quo n. Since $\theta(d_k, t)$ has full support, for either realization of $\theta(t)$, conditional on continuation play $\sigma(d_k)$, some player weakly prefers implementing n to q, or both weakly prefer implementing p to q. By continuity, in some equilibrium of $\Gamma(0)$, conditional on $\theta(t) = \bar{\theta}$, some player must weakly prefer implementing n to q, or both weakly prefer implementing p to q. This contradicts Step E (E2 and E5) of the proof Proposition 6. ■

8.3 Proofs for the Model of Section 5

Lemma 7 *Consider the model of Section 5. For every strategy profile σ, let V_i^σ denote the continuation payoff for player $i \in \{L, R\}$ of implementing policy $x \in \{n, p, q\}$ in some period t with state $(\theta_t, v_t) = (\theta, v) \in \mathbb{R} \times [0,1]$ until the next period in which the state is redrawn, given continuation play σ. There exist $(w_L, w_R, e_L, e_R) \in \mathbb{R}^4$ such that, for any equilibrium strategy profile σ and for all $i \in \{L, R\}$, $\theta \in \mathbb{R}$, and $v \in [0,1]$,*

$$\begin{aligned}(1-\delta(1-v))\left[V_i^\sigma(\theta, v, p) - V_i^\sigma(\theta, v, n)\right] &= \theta - (1-v) w_i - v w_i^\sigma, \\ (1-\delta(1-v))\left[V_i^\sigma(\theta, v, p) - V_i^\sigma(\theta, v, q)\right] &= (1-v) e_i + v e_i^\sigma.\end{aligned} \tag{31}$$

Proof. We use Notation 1 with the exception that a state of the world $\upsilon \in \Upsilon$ now also includes the realization of the volatility v. For any strategy profile σ, all $\upsilon \in \Upsilon$ and $s \in \{n, p, q\}$, let $X^\sigma(\upsilon, s)$ denote the policy outcome in a period in which the state of the world is υ, the status quo is s, and players play σ. By definition of V_i^σ,

$$\begin{aligned}&V_i^\sigma(\theta, v, p) - V_i^\sigma(\theta, v, n) \\ = \;& \theta - w_i + \delta \left\{ \begin{array}{c} (1-v)\left[V^\sigma(\theta, v, p) - V^\sigma(\theta, v, n)\right] + \\ v \int_{\upsilon\in\Upsilon} \left[V^\sigma(\theta, v, X^\sigma(\upsilon, p)) - V^\sigma(\theta, v, X^\sigma(\upsilon, n))\right] d\mu(\upsilon) \end{array} \right\} \\ = \;& \frac{\theta - (1-v) w_i - v\left\{w_i - \delta \int_{\upsilon\in\Upsilon} \left[V^\sigma(\theta, v, X^\sigma(\upsilon, p)) - V^\sigma(\theta, v, X^\sigma(\upsilon, n))\right] d\mu(\upsilon)\right\}}{1-\delta(1-v)}.\end{aligned}$$

[20] More precisely, $\pi > 0$ is needed in Step E of the proof of Proposition 6 only to guarantee that both players strictly prefer $\sigma^*(n,p,q)$ to any equilibrium of $\Gamma(n,p)$. But it is not needed to prove that all equilibrium of $\Gamma(n,p,q)$ have the same path as $\sigma^*(n,p,q)$, which is what we use here.

If we set w_i^σ equal to the term inside the curly brackets on the numerator of the above fraction, then w_i^σ depends neither θ nor on v, which proves the first line of (31). The proof for the second line of (31) follows an analogous argument and is omitted for brevity. ■

Proof of Proposition 7. Let σ be an equilibrium. For q to be implemented on some equilibrium path, some player i must weakly prefer implementing q to p in some state (v, θ) given continuation play σ. From Lemma 7, we have $V_i^\sigma(\theta, v, q) \geq V_i^\sigma(\theta, v, p)$ in some state $(v, \theta) \in [0, 1) \times \mathbb{R}$ if and only if $e_i^\sigma < 0$ and $v > \frac{e_i}{e_i - e_i^\sigma}$. So if $e_L^\sigma \geq 0$ and $e_R^\sigma \geq 0$, for all $(v, \theta) \in [0, 1) \times \mathbb{R}$, $V_i^\sigma(\theta, v, p) > V_i^\sigma(\theta, v, q)$, and the first part of Proposition 7 holds trivially for $\bar{v} = 1$. Suppose now that $e_i^\sigma < 0$ for some i and let $\bar{v} \equiv \frac{e_i}{e_i - e_i^\sigma} < 1$. Then from Lemma 7, for all $v < \bar{v}$ and all $\theta \in \mathbb{R}$, both players weakly prefer implementing p to q in state (v, θ) so q is not implemented on path in such states. When $v > \bar{v}$ and when θ is sufficiently large, i's most preferred policy given continuation play σ is q, and j prefers to accept q under status quo n, so q is implemented, as needed.

Consider a sequence of c.d.f. $(H_k)_{k \in \mathbb{N}}$ such that for all $k \in \mathbb{N}$, $H_k(v, \theta) = G_k(v) F(\theta)$, where G_k has full support and tends to the degenerate distribution which puts probability 1 on $v = 1$ as $k \to \infty$. Suppose the second claim of Proposition 7 is false. Then from what precedes, for all $k \in \mathbb{N}$, there exists an equilibrium $\sigma(k)$ such that $e_L^{\sigma(k)} \geq 0$ and $e_R^{\sigma(k)} \geq 0$. Together with (31), the latter inequalities imply that for all $\theta \in \mathbb{R}$ and $k \in \mathbb{N}$, both players strictly (weakly) prefer implementing p to q given continuation play $\sigma(k)$ when $v \in [0, 1)$ (when $v = 1$). So $\sigma(k)$ never prescribes players to propose q when $v \in [0, 1)$, and we can assume that this is true as well when $v = 1$. Since G_k puts probability 0 on $v = 1$, this deviation from $\sigma(k)$ does not affect the continuation payoff parameters $w^{\sigma(k)}$ and $e^{\sigma(k)}$, and the modified $\sigma(k)$ is still an equilibrium. W.l.o.g., we can restrict attention to a subsequence of $(\sigma(k))_{k \in \mathbb{N}}$ such that $w^{\sigma(k)}$ and $e^{\sigma(k)}$ converge to some w^∞ and e^∞.

Let us now show that the corresponding subsequence of $(\sigma(k))_{k \in \mathbb{N}}$ converges. For almost all (θ, v), $\theta - (1 - v) w_i - v w_i^\infty \neq 0$. Together with (31), this implies that for any such (θ, v), i strictly prefers to implement p to n for all k sufficiently large, or i strictly prefers to implement n to p for all k sufficiently large. An analogous statement holds between n and q, and between p and q. So for almost all (θ, v), the action prescribed by $\sigma(k)$ to veto player i and therefore to proposer i must be pure and constant in k for k sufficiently large.

Note finally that by continuity, $\lim_{k \to \infty}(\sigma(k))$ must be an equilibrium of Γ for the limit c.d.f. The limit of $(H_k(v, \theta))_{k \in \mathbb{N}}$ as $k \to \infty$ puts probability 1 on $v = 1$, so the limit game is equivalent to the game considered in Proposition 3, and by construction, q is never implemented on path, so it is an EE. Since δ can be chosen arbitrarily close to 1, and since F can be chosen arbitrarily, we obtain a contradiction with Proposition 3. ■

Supplementary Appendix to "Gridlock and Inefficient Policy Instruments"

Example 1 *Suppose that the support of F is concentrated on three states $\{\theta_0, \theta_1, \theta_2\}$ that occur with the respective probabilities $\{\pi_0, \pi_1, \pi_2\}$, and suppose that the payoff parameters are such that*

$$\theta_0 < w_L < w_L + e_L < \theta_1 < w_R < w_R + e_R < \theta_2. \tag{32}$$

That is, in state θ_0, both players most prefer no intervention (n) and, in state θ_2, both players prefer any intervention to n. In state θ_1, they disagree: L prefers any intervention while R prefers no intervention. Suppose that agents can choose between six alternatives $\{n, p, q, nS, pS, qS\}$, where xS is policy x with a sunset. That is, if in a period with status quo $s \in \{n, p, q\}$ a proposal xS is approved, then policy x is implemented for one period, after which the status quo reverts to s for the next period. Suppose further, for the sake of simplicity, that R has full proposal power in state θ_0, and L has full proposer power in states $\{\theta_1, \theta_2\}$. We show that one can find a set of $w_L \in (\theta_1, \theta_1)$, $w_R \in (\theta_1, \theta_2)$ so that for e_L and e_R sufficiently small, the following can be supported as an equilibrium.

- *When the status quo is n, then n is proposed in states θ_0 and θ_1 and q is proposed in state θ_2.*
- *When the status quo is q, then n is proposed in state θ_0 and pS is proposed in states θ_1 and θ_2.*
- *When the status quo is p, then nS is proposed in state θ_0 and p is proposed in states θ_1 and θ_2.*

The intuition for the use of q in state θ_2 when the status quo is n is as follows. Status quo p favors the more interventionist player L as it lets her defend the intervention when she needs it. As a result, in θ_0, she may be at most willing to accept a temporary change to n, that is, accept only nS. Since she prefers p to q, however, she may be less willing to defend q in state θ_0, so under status quo q, R may be able to implement a change to n without an attached sunset. Thus, implementing p results in p being the status quo forever, whereas implementing q can still lead to status quo n. Since n is a better status quo for R in states of disagreement, R is willing to propose q instead of p in state θ_2, as the equilibrium requires.

Proof of Example 1. Let $W_i(s)$ denote the value of the game for player $i \in \{L, R\}$ when the initial status quo is $s \in \{n, p, q\}$ and players play the proposed equilibrium, before knowing the realization of $\theta(0)$. Then

$$\begin{aligned}
W_i(n) - W_i(q) &= \pi_1 (w_i - \theta_1 + \delta (W(n) - W(q))) + \pi_2 (-e_i), \\
W_i(n) - W_i(p) &= \pi_0 \delta (W_i(n) - W_i(p)) + \pi_1 (w_i - \theta_1 + \delta (W_i(n) - W_i(p))) \\
&\quad + \pi_2 (\delta (W_i(q) - W_i(p)) - e_i), \\
W_i(p) - W_i(q) &= \pi_0 \delta (W_i(p) - W_i(n)) + (\pi_1 + \pi_2)(\delta (W(p) - W(q))).
\end{aligned}$$

Rearranging terms, we obtain

$$\begin{aligned}
W_i(n) - W_i(q) &= \frac{\pi_1}{1 - \pi_1 \delta}(w_i - \theta_1) - \frac{\pi_2}{1 - \pi_1 \delta} e_i, \\
W_i(n) - W_i(p) &= \frac{(1 - \delta(\pi_2 + \pi_1))}{(1 - \delta)(1 - \delta \pi_1)} (\pi_1 (w_i - \theta_1) - \pi_2 e_i), \\
W_i(p) - W_i(q) &= -\frac{\pi_0 \delta}{(1 - \delta)(1 - \delta \pi_1)} (\pi_1 (w - \theta_1) - \pi_2 e_i).
\end{aligned} \tag{33}$$

Together with (32), the above equations imply that $W_L(n) - W_L(q) < 0$, and $W_L(p) - W_L(q) > 0$, so L prefers status quo p to q to n . And for e_R sufficiently small, $W_R(n) - W_R(q) > 0$ and $W_R(p) - W_R(q) < 0$, so R prefers n to q to p as status quos. In what follows, we show that if players expect these continuation value, it is subgame perfect for them to play the proposed equilibrium.

Using (32) and (33), one can readily check that when $s = p$ and $\theta \in \{\theta_1, \theta_2\}$, it is optimal for L to unilaterally impose to stay at the status quo p. Likewise, when $s = n$ and $\theta \in \{\theta_0, \theta_1\}$, it is optimal for R to unilaterally impose to stay at the status quo n.

Consider now $s = n$ and $\theta = \theta_2$. For q to be implemented in such state, it must be that R rejects L's most preferred option p, which requires

$$w_R - \theta_2 + \delta (W_R(n) - W_R(p)) \geq 0. \tag{34}$$

Since R would always accept p with a sunset, it must be that L is weakly prefers implementing q to implementing pS, which requires

$$-e_L + \delta (W_L(q) - W_L(n)) \geq 0. \tag{35}$$

Together with (32), (35) implies that L prefers implementing q to implementing n, and to implementing qS. And finally, R also must prefer implementing q to staying at the status quo n, which requires

$$\theta_2 - (w_R + e_R) + \delta (W_R (q) - W_R (n)) \geq 0. \tag{36}$$

Consider now $s = q$ and $\theta = \theta_0$. Alternative n is R's best choice statically and as the status quo, and n is indeed accepted if L prefers implementing n to staying at q, which requires

$$w_L + e_L - \theta_0 + \delta (W_L (n) - W_L (q)) \geq 0. \tag{37}$$

Consider then $s = q$ and $\theta \in \{\theta_1, \theta_2\}$. Both players strictly prefer implementing pS to staying at q. However, L prefers a permanent change to p, so for L to propose pS instead, R must reject p, which means that she must prefer staying at q to implementing p. This requires that for $\theta \in \{\theta_1, \theta_2\}$,

$$V_R (\theta, q) - V_R (\theta, p) = -e_R + \delta (W_R (q) - W_R (p)) \geq 0,$$

which is true for e_R small enough.

Finally, consider $s = p$ and $\theta = \theta_0$. L clearly accepts nS (as n is statically better and p is her best status quo in θ_0), and R prefers implementing nS to qS. R's first best would be to implement n, so we have to make sure that L rejects that proposal, which requires

$$V_L (\theta_0, p) - V_L (\theta_0, n) = \theta_0 - (w_L + e_L) + \delta (W (p) - W (n)) \geq 0. \tag{38}$$

From (37) we see that L prefers n to q in θ_0, so she would also reject q.

Using (33) and isolating w_R, conditions (34) and (36) become

$$\begin{aligned} w_R &\geq \frac{(1-\delta)(1-\delta\pi_1)\theta_2 + \delta(1-\delta(\pi_2+\pi_1))\pi_1\theta_1}{1-\delta+\delta^2\pi_1(1-\pi_1-\pi_2)} + \frac{\delta(1-\delta(\pi_2+\pi_1))\pi_2}{1-\delta+\delta^2\pi_1(1-\pi_1-\pi_2)} e_R, \\ w_R &\leq (1-\delta\pi_1)\theta_2 + \delta\pi_1\theta_1 - e_R(1-\delta(\pi_2+\pi_1)). \end{aligned}$$

For $e_R = 0$, these conditions become

$$\theta_1 < \frac{(1-\delta)(1-\delta\pi_1)\theta_2 + \delta(1-\delta(\pi_2+\pi_1))\pi_1\theta_1}{1-\delta+\delta^2\pi_1(1-\pi_1-\pi_2)} \leq w_R \leq (1-\delta\pi_1)\theta_2 + \delta\pi_1\theta_1 < \theta_2,$$

where the two outer inequalities follow from (32). Simple algebra shows that the second term is strictly smaller than the fourth term, so one can find $w_R \in (\theta_1, \theta_2)$ and e_R small enough so that (32), (34), and (36) are satisfied.

Using (33) and $e_L = 0$, (37) becomes $w_L \leq \theta_1$, which from (32) holds strictly. So (37) holds for e_L sufficiently small.

Using again (33) and $e_L = 0$, (35) and (38) become

$$\theta_0 < (1 - \pi_1 \delta)\, \theta_0 + \delta \pi_1 \theta_1 \leq w_L \leq \frac{(1-\delta)(1-\delta\pi_1)\,\theta_0 + \delta\,(1 - \delta\,(\pi_2 + \pi_1))\,\pi_1 \theta_1}{1 - \delta + \delta^2 \pi_1\,(1 - \pi_1 - \pi_2)} < \theta_1.$$

where the two outer inequalities follow from (32). Simple algebra shows that the second term is strictly smaller than the fourth term, so one can find $w_L \in (\theta_0, \theta_1)$ and e_L small enough so that (32), (35), and (38) are satisfied, as needed. ■

The following proposition proves the claim made in the text right after Proposition 3.

Proposition 8 *Let G be the c.d.f. of the normal distribution. For any (w_L, w_R, e_L, e_R), for any $\varepsilon > 0$, there exists $\delta \in (0,1)$, $d > 0$ and $\underline{\theta} \in \mathbb{R}$ such that for the c.d.f. $F(\theta) \equiv G\left(\frac{\theta - \underline{\theta}}{d}\right)$, for any equilibrium of Γ, conditional on some intervention being implemented under status quo n, the probability (evaluated before the realization of $\theta(t)$) that p is implemented is smaller than ε.*

Proof. Throughout that proof, we use the same notations as in the proof of Proposition 3. From Step 1 of that proof, if we fix $\pi = 0$, we can pick $\delta \in (0,1)$, and $\underline{\theta} < \bar{\theta}$ such that conditions (6), (7), and (8) are satisfied. Using the same notations as in Step 3 of that proof, consider a sequence $d \to 0$, a corresponding sequence of equilibria $\sigma(d)$ of $\Gamma(d)$, and the corresponding sequence of strategy profiles $\sigma^0(\sigma(d_k))$ of the game $\Gamma(0)$, as defined in Step 2. As argued in Step 3, we can assume that $\sigma^0(\sigma(d_k))$ converges to an (uncorrelated) equilibrium $\sigma(0)$ of $\Gamma(0)$.

For all $d > 0$ along that sequence, consider the cutoff states $w_i^{\sigma(d)}$ and $w_i^{\sigma(d)} + e_i^{\sigma(d)}$ above which each player i prefers implementing p and q, respectively, to n, given continuation play $\sigma(d)$ in the game $\Gamma(d)$. Since the path of play of $\sigma(d)$ converges to the unique equilibrium path of $\Gamma(0)$, by continuity, these cutoff states must converge to $w_i^{\sigma(0)}$ and $w_i^{\sigma(0)} + e_i^{\sigma(0)}$. As argued in Step E of the proof of Proposition 6, the unique equilibrium path of $\Gamma(0)$ is such that status quo $s \in \{n, q\}$ leads to policy n and q in state $\underline{\theta}$ and $\bar{\theta}$, respectively. This implies that for both players,

$$\underline{\theta} < w_i^{\sigma(0)} + e_i^{\sigma(0)} = w_i + e_i < \bar{\theta}.$$

Moreover, it should be clear from Steps E1 and E2 of the proof of Proposition 6 that $w_R^{\sigma(0)} > \bar{\theta} > w_L^{\sigma(0)}$. Therefore,

$$\underline{\theta} < \max_i \left(w_i^{\sigma(0)} + e_i^{\sigma(0)} \right) = \max_i (w_i + e_i) < \bar{\theta} < \max_i \left(w_i^{\sigma(0)} \right). \tag{39}$$

So for d sufficiently small, $\max_i \left(w_i^{\sigma(d)} + e_i^{\sigma(d)} \right) < \max_i \left(w_i^{\sigma(d)} \right)$. As explained in Section

4.1, this implies that under status quo n, n stays in place if $\theta < \max_i \left(w_i^{\sigma(d)} + e_i^{\sigma(d)}\right)$, q is implemented when $\theta \in \left(\max_i \left(w_i^{\sigma(d)} + e_i^{\sigma(d)}\right), \max_i \left(w_i^{\sigma(d)}\right)\right)$ and p or q is implemented in states $\theta > \max_i \left(w_i^{\sigma(d)}\right)$, depending on who is the proposer. Therefore, under status quo n, conditional on some intervention being implemented, the probability that p is implemented is bounded above below by

$$\Pi(d) = \frac{1 - G\left(\frac{\max_i\left(w_i^{\sigma(d)}\right) - \underline{\theta}}{d}\right)}{1 - G\left(\frac{\max_i\left(w_i^{\sigma(d)} + e_i^{\sigma(d)}\right) - \underline{\theta}}{d}\right)}.$$

From (39), $\frac{\max_i\left(w_i^{\sigma(d)} + e_i^{\sigma(d)}\right) - \underline{\theta}}{d} \to_{d\to 0} \infty$ and $\frac{\max_i\left(w_i^{\sigma(d)}\right) - \underline{\theta}}{d} - \frac{\max_i\left(w_i^{\sigma(d)} + e_i^{\sigma(d)}\right) - \underline{\theta}}{d} \to_{d\to 0} \infty$. One can easily show that in the case of the normal distribution (as for many other standard distributions), for any two sequences $(a_n)_{n\in\mathbb{N}}$ and $(b_n)_{n\in\mathbb{N}}$ that tend to infinity, $\frac{1-G(a_n+b_n)}{1-G(a_n)} \to 0$ as $n \to \infty$, so $\Pi(d) \to 0$ as $d \to 0$, as needed. ■