Compliance Technology and Self-enforcing Agreements[*]

Bård Harstad[†] Francesco Lancia[‡] Alessia Russo[§]

March 2, 2017

Abstract

We analyze a repeated game in which countries are polluting and investing in technologies. While folk theorems point out that the first best can be sustained as a subgame-perfect equilibrium when the players are sufficiently patient, we derive the second-best equilibrium when they are not. This equilibrium is distorted in that countries over-invest in technologies that are "green" (i.e., strategic substitutes for polluting) but under-invest in adaptation and "brown" technologies (i.e., strategic complements to polluting). Particularly countries that are small or benefit little from cooperation will be required to invest in this way. With uncertainty, such strategic investments reduce the need for a long, costly punishment phase and the probability that it will be triggered. The framework is consistent with the evolution from the 1997 Kyoto Protocol to the 2015 Paris Agreement.

Keywords: climate change, environmental agreements, green technology, repeated games, imperfect monitoring.

JEL: D86, F53, H87, Q54.

[*]We have benefitted from audiences at Columbia, Harvard, Yale, Oslo, and UC Berkeley, and from participants at the 11th World Congress of the Econometric Society, Montreal; Workshop on Rational Expectation Equilibrium in honor of Roy Radner, Vienna; the 10th Conference on the Economics of Energy and Climate Change, Toulouse; the Annual Conference of the European Association of Environmental and Resource Economists, Helsinki; CREE Workshop for Research on Environmentally Friendly Energy, Oslo; the 37th Meeting of the Norwegian Association for Economists, Bergen. We are particularly grateful for comments from Dilip Abreu, Sylvain Chassang, Prajit Dutta, Daniel Garcia, Marina Halac, Anirban Mitra, Francesco Nava, Francois Salanie, and Karl Schlag. This research received funding from the European Research Council under the EU's 7th Framework Programme, ERC GA no. 283236.

[†]University of Oslo, the Frisch Centre, and Toulouse School of Economics. Email: bard.harstad@econ.uio.no

[‡]University of Vienna. Email: francesco.lancia@univie.ac.at

[§]BI Norwegian Business School. Email: alessia.russo@bi.no

1 Introduction

To be successful, any environmental treaty must address two major challenges. First, in the absence of international enforcement bodies, an international treaty must be *self-enforcing*. That is, one must hope that countries will comply to the treaty simply because this will motivate other countries to cooperate in the future. This motivation, however, may not always be sufficiently strong. For example, for many years it was clear that Canada would not meet its commitments under the Kyoto Protocol. So, in 2011, it simply withdrew.

The second challenge is to develop new and environmentally friendly technology. The importance of new and green technology is recognized in climate treaties, but traditionally they do not quantify how much countries should be required to invest in these technologies.[1] Instead, negotiators focus on quantifying emissions or abatements and leave the investment decision to individual countries. Nevertheless, some countries do invest heavily in green technologies. The European Union aims for 20 percent of its energy to come from renewable sources by 2020, and to increase that number to 27 percent by 2030. China is a still larger investor in renewable energy and has invested heavily in wind energy and solar technology.[2] Other countries have instead invested in so-called "brown" technology: Canada, for example, has developed its capacity to extract unconventional oil such as tar sands while, simultaneously, "Canada risks being left behind as green energy takes off" (*The Globe and Mail*, September 21, 2009).

The interaction between the two challenges is poorly understood by economists and policymakers. To understand how treaties can address these challenges and how they interact, we need a model that allows technology investments and emission decisions to be made repeatedly. Since the treaty must be self-enforcing, strategies must constitute a subgame-perfect equilibrium (SPE).

There is no such theory in the literature, however, and many important questions have thus not been addressed. First, what characterizes the "best" SPE (i.e., the best self-enforcing treaty)? While folk theorems have emphasized that even the first best can be sustained if the players are sufficiently patient, what distortions occur if they are not? Which kinds of countries ought to invest the most, and in what kinds of technologies? And, finally, how should we understand the evolution from the 1997 Kyoto Protocol to the 2015 Paris Agreement?

[1] Chapter 16 of the Stern Review (2007) identified technology-based schemes as an indispensable strategy for tackling climate change. However, article 114 of the Cancun Agreement 2010, confirmed in Durban in 2011, states that "technology needs must be nationally determined, based on national circumstance and priorities." However, as discussed in Section 6, some of the pledges following the 2015 Paris Agreement relate to technology.

[2] On the European Union, see ec.europa.eu/energy/en/topics/energy-strategy/2030-energy-strategy. On China, see thediplomat.com/2014/11/in-new-plan-china-eyes-2020-energy-cap/.

To address these questions, we analyze a repeated extensive-form game where, in every period, countries can invest in technology before deciding on emission levels. In the basic model, all decisions are observable and investments are self-investments (i.e., there are no technological spillovers). Consequently, equilibrium investments would have been first best if the countries had committed to the emission levels. The first best can also be achieved if the discount factor is sufficiently high, in line with standard folk theorems. For smaller discount factors, however, the best SPE must be distorted. We show that the distortions take the form of over-investments in so-called "green" technologies (i.e., renewable energy or abatement technologies that can substitute for pollution). The reason is that such over-investments reduce a country's temptation to cheat by emitting more rather than less, and they are thus necessary to satisfy the compliance constraint at the emission stage. For so-called "brown" technologies, including drilling technologies and other infrastructure investments that are strategic complements to fossil fuel consumption, investments must instead be less than the first-best amount to satisfy the compliance constraint. Our most controversial result may be that countries should also be required to invest less than the first-best amount in adaptation technologies (i.e., technologies that reduce the environmental harm in a country).

Our comparative statics offer important policy implications. Naturally, it is harder to motivate compliance if the discount factor is small, the environmental harm is small, or each country is small. In these circumstances, the best SPE requires countries to invest more in green technologies and less in adaptation or brown technologies. If countries are heterogeneous, the countries that are small and more reluctant to cooperate, for example because they face less environmental harm, are the most tempted to free ride. Thus, for compliance to be credible, such countries must invest the most in green technologies or the least in adaptation and brown technologies. This advice contrasts the typical presumption that reluctant countries should be allowed to contribute less in order to satisfy their participation constraint. While incentives to *participate* require that a country's net gain of cooperating be positive, incentives to *comply* with emissions also require that this net gain outweigh the positive benefit of free riding for one period, before the defection is observed. The compliance constraint at the emission stage is therefore harder to satisfy than the participation constraint and to satisfy it, reluctant countries must invest more in green technologies and less in adaptation and brown technologies.

Simplicity and tractability are two assets of our workhorse model. Our main results are derived in a pedagogical way with binary emission levels and without imperfect monitoring, uncertainty, private information, renegotiation, technological spillovers, or policy instruments such as emission taxes or investment subsidies. These complicating factors are added in our later sections. When imperfect monitoring and uncertainty are added to the model, infinitely long punishments are not optimal since they may be triggered by mistake. Technology levels should then be chosen not only to motivate compliance, but also to allow countries to reduce the duration of the punishment period without violating the compliance constraints. If a country's cost of complying is stochastic, technologies should be chosen to increase the frequency of compliance. The necessity to invest strategically continues to hold if there are technological spillovers, but without strong intellectual property rights, spillovers can be so large, and the compliance constraint at the investment stage can be so hard to satisfy, that it is impossible to sustain an equilibrium with less emissions. When renegotiation is possible, the realistic penalty declines, free riding may become more attractive and, in response, the best renegotiation-proof SPE requires countries to invest even more in green technology and less in adaptation or brown technology.

Our most interesting extension might be to permit continuous emission levels. We then show that one may need to punish a country for cooperating *too much* (i.e., for investing a lot in green technology), since large investment levels could tempt other countries to free ride. With policy instruments, the first best requires an emission tax only, and no investment subsidy. If the discount factor is smaller, however, the emission tax that can be sustained in the repeated game is smaller, and an investment subsidy should be introduced.

Literature. It is widely accepted that international agreements must be self-enforcing.[3] The literature on repeated games is thus the relevant one, but this literature has mostly been concerned with folk theorems that state that any feasible and individually rational payoff is an equilibrium payoff when the discount factor approaches one (see, for example, Mailath and Samuelson, 2006).[4] However, such a large discount factor is unrealistic in many applications. Further, the benefit of cheating may depend on the technology. Thus, (i) we extend the standard repeated prisoner's dilemma by allowing agents to make technology investment decisions in each period, and (ii) we derive distortions that must occur when the discount factor is so small that the first best cannot be achieved. Note that each of these two extensions would be uninteresting in isolation: With high discount factors, the first best can always be sustained, even in a model with technology. Without technology and with small discount factors, only defect could be

[3] As Downs and Jones (2002: S95) observed, "a growing number of international relations theorists and international lawyers have begun to argue that states' reputational concerns are actually the principal mechanism for maintaining a high level of treaty compliance."

[4] See, among others, Fudenberg and Maskin (1986) for folk theorems which assume near perfect patience when monitoring is perfect. Abreu et al. (1990) developed a recursive methodology for computing the entire set of subgame-perfect equilibrium payoffs at fixed discount factor when there are finitely many actions, imperfect monitoring, and public randomization. Fudenberg et al. (1994) adopted a similar methodology to prove a folk theorem with imperfect public monitoring.

sustained in the repeated prisoner's dilemma game.[5]

By studying the second-best equilibrium in a repeated game with technology choices, our paper fills a gap between the literature on repeated games and the literature on green technology. The structure of our model draws on the structure in earlier papers. Also in Harstad (2012; 2016a) and Battaglini and Harstad (2016), countries sequentially pollute and invest in green technologies in every period. These papers, however, assume that countries can contract on and commit to emission levels and they study Markov-perfect equilibria: Harstad (2012) investigates how long-term commitments motivate technological investments; Harstad (2016a) analyzes hold-up problems before upcoming short-term agreements; while Battaglini and Harstad (2016) find participation to be motivated because only large coalitions prefer to sign the long-term agreements that circumvent the hold-up problem. This paper, in contrast, focuses on subgame-perfect equilibria and self-enforcing agreements. This focus leads to a completely new strategic effect of technology—namely that technology should be chosen to make future cooperation *credible*.[6]

Repeated games have often been used to analyze self-enforcing environmental agreements (Barrett, 1994; 2005), but the models used in these papers do not allow countries to invest in technologies along the way. Dutta and Radner (2004; 2006) study a dynamic game with emissions and technology choices. As we do, they refer to self-enforcing treaties as SPEs supported by trigger strategies. However, technology in these papers is either exogenous or chosen as a corner solution at the beginning of the game. In most of the literature, investments in green technology are typically studied in models with just a few stages.[7]

Our mechanism is somewhat related to the issues-linkage literature, where defecting on one issue is punished by ending cooperation on another (see, e.g., Bernheim

[5]Parts of the more applied literature have also allowed for fixed/smaller discount factors: On trade agreements, for example, see Bagwell and Staiger (1990), or the review by Maggi (2014).

[6]From the authors' point of view, the paper is a result of combining two independent and unrelated projects: Harstad (2016b) studies how green/brown technologies can be used as commitment devices for an isolated hyperbolic decision-maker (thus, in that paper, there is no self-enforcing treaty); Lancia and Russo (2016) study how agents exert effort strategically to signal their willingness to cooperate in a stochastic overlapping-generations model.

[7]For example, in two-stage games, Golombek and Hoel (2005) show that environmental agreements should be ambitious in order to induce R&D, while Hoel and de Zeeuw (2010) show that cooperation on R&D can increase participation when R&D reduces the cost of technology adoption. Investments are also permitted by Barrett (2006), studying the role of breakthrough technologies in environmental agreements. In these contributions, the presence of technological spillover plays a crucial role. Buob and Gunter (2011) allow for adaptation technology and point out that this is a strategic substitute to mitigation. Acemoglu et al. (2012) present a dynamic model with pollution and with investments in clean and dirty technology, but there is a single economy only, and the focus is on imperfections in the R&D market. In the dynamic game by Gerlagh and Liski (2011), technology investments are made by one agent while fossil fuel extraction is decided on by another: investments are then influenced by resource extraction instead of the need to make cooperation credible, as here. For surveys and overviews, see Barrett (2005) and Calvo and Rubio (2012).

and Whinston, 1990; Spagnolo, 1999; Ederington, 2001; Limao, 2005). In issues-linkage models, agents interact simultaneously on multiple issues, all characterized by the strategic structure of a prisoner's dilemma type of game. When issues are substitutes, linking them facilitates cooperation. The opposite holds when issues are complements. We depart from those models by allowing countries to make a self-interested investment decision when playing a prisoner's dilemma game of pollution. We find that setting technology at an inefficient level and linking actions can increase cooperation when decisions are substitutes as well as complements.

The role of technology in our paper is more similar to the role of capacity in industrial organization, and to the role of armament to sustain peace.[8] In these literatures, investments tend to be irreversible, and thus affect the sustainability of collusion/peace in two opposite ways. On the one hand, reductions in production capacity and in weapon stocks reduce the incentives to deviate, thereby reinforcing cooperation. On the other hand, less capacity or fewer arms weakens the severity of retaliation if one player deviates, and this weakening undermines cooperation. The total effect of technology on compliance is then generally non-monotonic and depends on the specific features of the model.[9] In contrast to these papers, we (i) allow countries to choose their technology level in every period, (ii) allow a general family of technologies so that we can focus on what *type* of technology countries should invest in, and (iii) explicitly focus on the second best (i.e., the best SPE that can be sustained when the discount factor is too small to sustain the first best). Our paper is also the first to study these kinds of mechanisms in a repeated game of pollution.[10]

The next section presents the stage game and discusses benchmark results. Section 3 derives the basic message of the paper: there is a unique Pareto-optimal SPE in which every country emits little, and to sustain it, technology investment will be strategically

[8]Spence (1977) and Dixit (1980) study how firms can deter entry by modifying capacity limits in a non-repeated setting. Fudenberg and Tirole (1984) discuss circumstances under which strategic investment may lead the incumbent to exploit strategic complementarity and to accommodate entrants rather than to exploit strategic substitutability and deter entry. Garfinkel (1990) is the first to study folk theorems for conflict models, establishing that peace can be supported for sufficiently patient players. The game of Jackson and Morelli (2009) is one of coordination where decisions of investments in weapons are made in each period.

[9]In a setting where firms first collude on capacity and then engage in an infinitely repeated game of price competition, Benoit and Krishna (1987) find that all equilibria exhibit excess capacity. When firms are asymmetric, however, investment in capacity unambiguously hinders collusion (see Lambson, 1994, and Compte et al., 2002). In their study of conflicts, Chassang and Padro i Miquel (2010) show that weapons unambiguously facilitate peace under complete information, but not under strategic risk.

[10]The idea that technology investments can relax compliance constraints is also present in the limited commitment literature on relational contracting. Ramey and Watson (1997) and Halac (2015) explore this idea in a model with repeated trading, where, before trade starts, the principal can make a noncontractible irreversible investment. In Che and Sakovics (2004), agents invest in each period until they agree on how to divide the trading surplus. Baker et al. (2002) and Halonen (2002) investigate the sustainability of cooperation in a repeated relationship where different ownership structures can modify enforcement constraints and affect the parties' ex-post incentive to renege.

distorted in specific directions. While we have attempted to keep the basic model simple, Section 4 shows that the main results continue to hold and that additional insights emerge when we extend the model to allow for imperfect monitoring, uncertainty, technological spillovers, and renegotiation. Section 5 allows for continuous emission levels and shows when it may be necessary to *penalize* a country for "cooperating too much," before the best combination of policy instruments is derived. Although our main goal is to advance on the theoretical front, the concluding section discusses how our framework can provide a rationale for why the international community left the 1997 Kyoto Protocol's approach and, with the 2015 Paris Agreement, adopted the European Union's approach of using sequential decisions and technology targets. The Appendix contains the proofs that are not in the text.

2 A Model of Compliance Technology

A repeated game consists of a stage game and a set of times when the stage game is played. While we in the next section focus on the dynamics and the subgame-perfect equilibria (hereafter SPEs), we here present the stage game and discuss important benchmark results.

There are n players or countries, indexed by i or $j \in N \equiv \{1, ..., n\}$. In this section, we allow the country size s_i to vary with i, although the average country size is normalized to one. At the emission stage, countries simultaneously make a binary decision $g_i \in \{\underline{g}, \overline{g}\}$ between emitting less (i.e., $g_i = \underline{g}$) or more (i.e, $g_i = \overline{g}$). Let $b_i(\cdot)$ be the *per capita* benefit as an increasing function of country i's per capita emissions g_i, while $c_i \sum_{j \in N} s_j g_j$ is the *per capita* environmental cost as a function of aggregate emissions.[11] We assume that countries' emission decisions constitute a prisoner's dilemma. That is, a country i benefits from emitting more for any fixed emission from other countries, $g_{-i} \equiv \sum_{j \neq i} s_j g_j$, but every country would be better off if everyone emitted less:

$$b_i(\underline{g}, r_i) - (s_i \underline{g} + g_{-i}) c_i < b_i(\overline{g}, r_i) - (s_i \overline{g} + g_{-i}) c_i \text{ and} \tag{1}$$

$$b_i(\underline{g}, r_i) - n \underline{g} c_i > b_i(\overline{g}, r_i) - n \overline{g} c_i. \tag{2}$$

Variable $r_i \in \Re_+$ is here capturing the fact that a country's benefit of emitting is not exogenous. We will refer to r_i as the country's *technology*, but r_i can actually be any variable which influences the benefit of emitting. In fact, we also allow r_i to influence a country's environmental cost by letting $c_i \equiv h_i c(r_i)$, where h_i measures country-specific

[11]The assumption that the country's cost of damage due to emissions is linear in the global stock of greenhouse gases is common in the literature and it is also approximately correct (see, for example, estimates by Golosov et al., 2014).

environmental harm and $c(r_i)$ may depend on r_i. To simplify, we assume $c(r_i)$ to be decreasing and convex in r_i. We let $b_i(\cdot)$ be increasing and concave in r_i.

Hereafter, unless otherwise specified, we use subscripts for derivatives. Moreover, we abuse notation by writing $b''_{i,gr}(r_i) \equiv \partial\left[\left[b_i(\overline{g}, r_i) - b_i\left(\underline{g}, r_i\right)\right] / \left(\overline{g} - \underline{g}\right)\right] / \partial r_i$, which captures how the benefit of emitting more rather than less varies with the level of technology. To illustrate the relevance of technologies, we will occasionally refer to the following special types:

Definition 1. *At some investment level* r_i*,*
(A) adaptation technology *is characterized by* $b''_{i,gr}(r_i) = 0$ *and* $c'(r_i) < 0$*;*
(B) brown technology *is characterized by* $b''_{i,gr}(r_i) > 0$ *and* $c'(r_i) = 0$*;*
(C) clean technology *is characterized by* $b''_{i,gr}(r_i) < 0$ *and* $c'(r_i) = 0$*.*

Adaptation technology refers to technologies which help a country to adapt to a warmer or more volatile climate. Such technologies include agricultural reforms or more robust infrastructure, and may even capture the effects of some geo-engineering practices that have strictly local effects. In other words, adaptation technology is useful because it helps the country to adapt to the emissions, since it reduces the environmental cost of emissions (i.e., $c'(r_i) < 0$). Brown technology can be interpreted as drilling technology, infrastructure that is helpful in extracting or consuming fossil fuel, or other technologies that are complementary to fossil fuel consumption: the complementarity explains $b''_{i,gr}(r_i) > 0$. In fact, investments in existing polluting industries are brown, according to our definition. Clean technology, in contrast, is a strategic substitute for fossil fuel and reduces the marginal value of emitting another unit. This is the case for abatement technology, or renewable energy sources, for example. Thus, for clean technologies, $b''_{i,gr}(r_i) < 0$. Both brown and clean technologies may still be beneficial in that $\partial b_i(\cdot)/\partial r_i > 0$. We could also allow for an entire vector of technologies, with small modifications of the results.

We endogenize the technology levels by considering a stage game where countries first decide simultaneously and non-cooperatively on investments, the r_i's, before they decide on whether to emit less or more. The sequential timing follows naturally whenever there is a minimum length of time $l > 0$ between the investment decision and the time at which the technology is ready to be used. The lag implies that if the actual marginal investment cost is, say, $\widehat{k}_i > 0$, the present discounted value of this investment cost is $k_i \equiv e^{\rho l}\widehat{k}_i$, when evaluated at the later time of the emission, and where ρ is the discount rate. With this reformulation, we do not need to discount explicitly between the two stages, and a country i's per capita utility can be written as:

$$u_i = b_i(g_i, r_i) - h_i c(r_i) \sum_{j \in N} s_j g_j - k_i r_i. \tag{3}$$

Note that it is without loss of generality to assume that the investment cost is linear in r_i, because r_i can enter a country's benefit function in arbitrary ways.[12] Since technological spillovers will not be introduced before Section 4, each country is here voluntarily investing the socially optimal amount, conditional on the emission levels. To see this result, note that if it happened that $g_i = g \ \forall i$, the first best would require investments to be given by:

$$r_i^* (g) \equiv \arg\max_{r_i} b_i (g, r_i) - ngh_i c (r_i) - k_i r_i.$$

Clearly, $r_i^* (g)$ coincides with the noncooperative choice of r_i when country i takes the emission levels as given and everyone emits g. In other words, if countries could solve their prisoner's dilemma by committing to low emission levels in advance, investments would be socially optimal and the first best would be implemented. These benchmark results provide some preliminary support for the typical presumption that it is not necessary to negotiate investments in addition to negotiating emissions.

Proposition 0.
(i) In the first best, investments are $r_i^* \equiv r_i^* (\underline{g})$ *and every* $g_i = \underline{g}$.
(ii) In the unique SPE of the stage game, investments are $r_i^* (g)$ *and every* $g_i = \overline{g}$.
(iii) If countries had committed to $g_i = \underline{g}$*, the outcome, including the equilibrium investments, would be first best.*

Remark on stocks and reversibility. It is straightforward to reformulate this model and allow for stocks. Suppose the pollution stock accumulates over time and depreciates only at rate $q^g \in [0, 1]$. As long as the marginal cost of pollution is constant, the stock is payoff-irrelevant in that it does not influence future decisions, and the long-lasting cost of emission can already be accounted for today. To see this result in the simplest way, suppose $\widetilde{h}_i$ were i's per-period cost of a marginally larger pollution stock. Then, the present-discounted cost of emitting another unit evaluated at the time of the emission would simply be the constant $h_i \equiv (1 - \delta q^g) \widetilde{h}_i$.

Analogously, suppose a fraction $q_i^r \in [0, 1]$ of country i's investments in technology survives to the next period. In this case, one benefit of investing today is that investments can be reduced in the next period. These cost-savings will not be payoff-relevant, however, in the sense that today's choice of r_i will not influence the level of technology in the future; it will only reduce the cost of obtaining that level of technology. Thus, if $\widetilde{k}_i$ were the actual cost of adding to the technology stock, we can already account for the future cost-savings today and write the net marginal investment cost as

[12] If the investment cost were another function $\kappa_i (r_i)$, we could simply define $\widetilde{b}_i (g_i, \kappa_i (r_i)) \equiv b_i (g_i, r_i)$ and $\widetilde{c}_i (\kappa_i (r_i)) \equiv c (r_i)$, treat $\kappa_i (r_i)$ as the decision variable, and then proceed as we do in the paper.

$k_i \equiv (1 - \delta q_i^r)\, \widetilde{k}_i$.[13]

If the q_i^r's were small, the analysis below would be unchanged since countries would need to invest in every period (even off the equilibrium path). The investments are then, in effect, reversible. These assumptions are reasonable in the very long-run context of climate change, where countries must expect to invest repeatedly, partly to maintain the infrastructure and the capacity to produce renewable energy, for example. If the q_i^r's were instead large, it would actually be easier to motivate countries to emit less (see the end of Section 3.2 for further discussion). By ignoring stocks and instead considering the one-period utilities given by (3), it is straightforward to interpret our dynamic game as a simple repeated game.

Remark on assumptions and extensions. In (3), we have assumed self-investments that affect only the investing country's technology. We have also abstracted away from uncertainty and policy instruments, and we permit only two possible emission levels. These assumptions allow us to derive key insights in a simple setting. In Sections 4-5, we relax all these assumptions and show that our main results continue to hold. The Appendix also discusses time-varying parameters, rather than the stationary ones in our basic model.

3 Self-enforcing Agreements

We now assume that the stage game described above is played repeatedly in every period $t \in \{1, 2, ..., \infty\}$. We let $\delta \in [0, 1)$ be the common discount factor and let $v_i^t \equiv (1 - \delta) \sum_{\tau=t}^{\infty} \delta^{\tau - t} u_i^\tau$ measure country i's continuation value at time t, normalized to per-period utility. Our goal in this section is to characterize a unique Pareto-optimal SPE in which every country emits less.

3.1 The Worst Equilibrium

Note that there is a unique SPE in the stage game described in Section 2. Given (1) and (2), emitting more at the emission stage is a dominant strategy for all countries, and at the investment stage, $r_i = r_i^* (\overline{g})$. Clearly, these strategies also survive as an SPE in the infinitely repeated game in which the stage game is played in every period. Using terminology standard in the literature of environmental agreements, we refer to this equilibrium as the business-as-usual (BAU) equilibrium and label it with superscript b. In fact, in every SPE in which $g_i = \overline{g}$, we *must* have $r_i = r_i^* (\overline{g})$. For any other

[13] Of course, the stocks would be payoff-relevant if the environmental harm or the investment costs were nonlinear functions. Allowing for payoff-relevant stocks is reasonable but they open up a host of other issues (some of them are discussed by Harstad (2012 and 2016a)) that are tangential to the strategic investments emphasized here.

equilibrium candidate r_i, country i could benefit from deviating to $r_i^*(\overline{g})$ without any risk of reducing v_i. Therefore, a country is always guaranteed the payoff from BAU. For every i, the minmax strategy is that everyone emits little, and BAU is the worst possible SPE.

Proposition 1. *The worst SPE is BAU:* $\left(r_i^b, g_i^b\right) = \left(r_i^*(\overline{g}), \overline{g}\right)$*. This equilibrium always exists.*

Thus, to derive better equilibria, we can without loss of generality focus on a simple trigger strategy where any deviation results in a permanent reversion to the worst SPE (i.e., BAU).

Corollary 1. *If a sequence* (r_i^t, g_i^t) *can be sustained as an SPE, it can be sustained as an SPE in which any deviation triggers an immediate reversion to BAU.*

3.2 The Best Equilibrium

Since the emission game is a prisoner's dilemma, we are particularly interested in Pareto-optimal SPEs in which every country emits little (i.e., where $(r_i^t, g_i^t) = (r_i^t, \underline{g})$ for some r_i^t). When such an equilibrium is unique, we refer to it as the "best" equilibrium.

Definition 2. *An equilibrium is referred to as* best *if and only if it is the unique Pareto-optimal SPE satisfying* $g_i^t = \underline{g}\ \forall i \in N, \forall t \geq 1$.

Since this definition requires that everyone emits little in the best SPE, we do not need to impose symmetry. However, note that a best equilibrium *also* coincides with the unique symmetric Pareto-optimal SPE (if "symmetry" requires the g_i's to be the same), since everyone is better off if everyone emits little than if everyone emits more. (Of course, there are also asymmetric Pareto-optimal SPEs where, for example, every $i \in N$ except for one emits little.) Since all parameters are invariant in time, it may not be surprising that the best equilibrium is in pure strategies and it is characterized by r_i^t's that are independent of t. Hence, we will skip t-superscript for brevity.

There are two decision stages in each period, and we must consider the temptation to deviate at each of them. At the investment stage, a country must compare the continuation value (v_i) it receives from complying with the SPE by investing r_i to the maximal continuation value it could possibly obtain by deviating. Since deviating at the investment stage implies that every country will emit more beginning from this period, the compliance constraint at the investment stage is the following:

$$\frac{v_i}{1-\delta} \geq \max_{r_i} b_i(\overline{g}, r_i) - h_i c(r_i) n\overline{g} - k_i r_i + \frac{\delta v_i^b}{1-\delta}. \qquad (\mathrm{CC}_i^r)$$

where $v_i^b \equiv b_i\left(\overline{g}, r_i^b\right) - h_i c\left(r_i^b\right) n\overline{g} - k_i r_i^b$ is i's continuation value with BAU. The right-hand side of (CC_i^r) is maximized when $r_i = r_i^*(\overline{g})$, implying that the right-hand side is simply v_i^b. Thus, (CC_i^r) simplifies to $v_i \geq v_i^b$, which actually is the participation constraint. In other words, as long as every country prefers the best equilibrium to BAU, the compliance constraint for the investment is trivially satisfied. Intuitively, in the absence of technological spillover, the temptation to free ride at the investment stage is weak since a country does not care about other countries' investment levels per se, but only about their emission levels. Thus, if a country deviates at the investment stage, the penalty is imposed before the free rider has benefitted.

At the emission stage, the investment cost for this period is sunk and the compliance constraint becomes:

$$b_i\left(\underline{g}, r_i\right) - h_i c\left(r_i\right) n\underline{g} + \frac{\delta v_i}{1-\delta} \geq b_i\left(\overline{g}, r_i\right) - h_i c\left(r_i\right)\left(s_i\overline{g} + (n-s_i)\,\underline{g}\right) + \frac{\delta v_i^b}{1-\delta}. \quad (\mathrm{CC}_i^g)$$

Given that $v_i \geq v_i^b$, (CC_i^g) implies that:

$$\delta \geq \widehat{\delta}_i\left(r_i\right) \equiv 1 - \frac{v_i - v_i^b}{b_i\left(\overline{g}, r_i\right) - b_i\left(\underline{g}, r_i\right) - s_i h_i c\left(r_i\right)\left(\overline{g} - \underline{g}\right) + v_i - v_i^b}. \quad (4)$$

In the limit, as δ tends to one, (CC_i^g) approaches the condition (CC_i^r). For any $\delta < 1$, however, (CC_i^g) is harder to satisfy than (CC_i^r) because of the free-riding incentive at the emission stage. It is not sufficient that the best equilibrium be better than BAU. In addition, the discount factor must be large or the temptation to free ride must be small.

As indicated in (4), the threshold for the discount factor generally depends on the equilibrium r_i. For first-best investments, where $r_i^* \equiv r_i^*\left(\underline{g}\right)$, the threshold is $\overline{\delta}_i \equiv \widehat{\delta}_i\left(r_i^*\right) < 1$. Thus, if $\delta \geq \overline{\delta}_i$ holds for every $i \in N$, every (CC_i^g) holds for first-best investment levels and the best equilibrium is simply the first best. If $\delta < \overline{\delta}_i$, however, (CC_i^g) does not hold for $r_i = r_i^*$. To ensure that the compliance constraint at the emission stage be satisfied, the temptation to free ride must be reduced by insisting on an r_i so that $\widehat{\delta}_i\left(r_i\right) \leq \delta$. Thus, we must have $r_i > r_i^*$ if $\widehat{\delta}_i'\left(r_i^*\right) < 0$, or $r_i < r_i^*$ if $\widehat{\delta}_i'\left(r_i^*\right) > 0$. It is easy to see that:

$$\widehat{\delta}_i'\left(r_i^*\right) < 0 \text{ if } b_{i,gr}''\left(r_i^*\right) < s_i h_i c'\left(r_i^*\right); \quad (\mathrm{G}_i)$$

$$\widehat{\delta}_i'\left(r_i^*\right) > 0 \text{ if } b_{i,gr}''\left(r_i^*\right) > s_i h_i c'\left(r_i^*\right). \quad (\mathrm{NG}_i)$$

Under condition (G_i) for "green" technology, making *more* investments relaxes the compliance constraint by reducing the threshold $\widehat{\delta}_i\left(r_i\right)$. Clearly, condition (G_i) is satisfied for what we referred to as clean technology in Definition 1, and above we explained that making more investments in clean abatement technology reduces the temptation to emit more. Under condition (NG_i) for "non-green" technologies, making *less* investments relaxes the compliance constraint. By Definition 1, adaptation and brown technologies are special cases of non-green technology. Large levels of these technology types make it tempting to emit more, and the compliance constraint is satisfied for a smaller set of discount factors.

As the discount factor $\delta < \overline{\delta}_i$ declines, (CC_i^g) becomes even harder to satisfy and requires investment levels that increasingly differ from the first-best level. Once the discount factor is smaller than a lower threshold referred to as $\underline{\delta}_i < \overline{\delta}_i$, the distortions are so large that $v_i < v_i^b$, and then $g_i = \underline{g}$ can no longer be sustained in an SPE.[14] The thresholds are discussed in the Appendix, which includes the proofs of the following results.

Proposition 2. *There exists a unique best equilibrium if and only if* $\delta \geq \max_i \underline{\delta}_i$. *It is stationary, and characterized as follows:*
(i) If $\delta \geq \overline{\delta}_i$, $r_i = r_i^*$ *and the best equilibrium is first best.*
(ii) If $\delta < \overline{\delta}_i$*:*[15]

$$r_i = \min \widehat{\delta}_i^{-1}(\delta) > r_i^* \text{ under } (G_i);$$
$$r_i = \max \widehat{\delta}_i^{-1}(\delta) < r_i^* \text{ under } (NG_i).$$

The result that the first best is achievable when the discount factor is sufficiently large is standard in the literature on repeated games.[16] Thus, the contribution of Proposition 2 is to characterize the distortions that must occur if the discount factor is small. The distortions are easy to understand given how (G_i) and (NG_i) are defined. If the first best cannot be achieved, countries are motivated to comply with an agreement and emit less only if they have, in advance, invested more in green technologies, or less in non-green technologies. Unless investments are distorted in this way, the temptation to emit more would be too large.

Corollary 2. *Compared to the first best, the best equilibrium requires countries to:*
(i) under-invest in adaptation technology;

[14] For $\delta < \underline{\delta}_i$, $g_i^t = \overline{g}$, at least for some periods t, in every SPE. If countries are very heterogenous, it might still be possible to sustain an SPE where some other countries emit less, but their compliance constraints become different (and harder to satisfy) as soon as country i will no longer emit less. It is beyond the scope of the present paper to characterize SPEs where some but not all countries emit less.

[15] In the following equations, the operators min and max are added since $\widehat{\delta}_i^{-1}(\delta)$ is a correspondence and, of the multiple values of $\widehat{\delta}_i^{-1}(\delta)$, it is optimal to select the one closest to r_i^*.

[16] Rubinstein and Wolinsky (1995) show that Fudenberg and Maskin's (1986) folk theorem can be generalized to repeated extensive-form games to account for subgame-perfection within periods.

(ii) under-invest in brown technology;
(iii) over-invest in clean technology.

These strategic investment levels, which are clearly inefficient conditional on the emission levels, must be part of the self-enforcing agreement in the same way as the small emission levels are: any deviation must be triggered by a reversion to BAU.

Distorting the choice of technology in this manner reduces the temptation to deviate from the equilibrium. Note that it is *not* necessary to require so little or so much investment that emitting less becomes a *dominant* strategy: it is sufficient to ensure that the benefit of emitting more be smaller (although still positive) than the present discounted value of continuing cooperation. Also, note that if technology were long-lasting and not reversible, it would be easier to satisfy the compliance constraint at the emission stage. The reason is simply that the deviation payoff would be less than the BAU payoff if the investments cannot adjust easily.

3.3 Comparative Statics

We are finally ready to discuss important comparative statics. The compliance constraints are not functions of the technology only. They also depend on other parameters of the model. Compliance is particularly difficult to motivate if the cost of reverting to BAU is small. The cost of BAU is small if relatively few countries are polluting (i.e., n is small), if the environmental harm is small (i.e., h_i is small), or if countries heavily discount the value of cooperating in the future (i.e., δ is small). In all these situations, a country i will not find it optimal to comply unless it is requested to invest less in adaptation and brown technologies, or more in clean technologies.

Some of our results are, at first, surprising. For example, the result that investments in clean technologies should decline with the discount factor is certainly at odds with the traditional intuition that investments should be larger when players are patient. In addition, it turns out that *all* investments will increase with the investment cost k_i. The intuition for this result is the following. For adaptation and brown technologies, we have $r_i < r_i^b$. A larger k_i thus reduces the value of BAU $\left(v_i^b\right)$ compared to the value of cooperating, and makes the compliance constraint at the emission stage easier to satisfy. Thus, when k_i increases, r_i can increase towards r_i^* without violating (CC_i^g). For clean technologies on the other hand, we have $r_i > r_i^b$, and a larger k_i again reduces the value of cooperating relative to the value of BAU. The compliance constraint becomes harder to satisfy. As a response, countries must invest even more in clean technologies to satisfy (CC_i^g) when k_i increases. This and other comparative statics are summarized in the following proposition.

Proposition 3. *Suppose* $\delta \in \left[\max_j \underline{\delta}_j, \overline{\delta}_i\right)$ *and consider the best equilibrium.*

(i) If k_i increases, r_i increases.
(ii) If δ or s_i decreases, $|r_i - r_i^*|$ increases.
(iii) If n or h_i decreases, r_i increases for clean technologies, while r_i decreases for brown technologies, and, assuming $(c')^2/c'' < c$, also for abatement technologies.[17]

Note that the comparative statics are country-specific. When environmental harm is heterogeneous, countries subject to the least harm (i.e., those with the smallest h_i) are most tempted to emit more. These countries, which are relatively reluctant to cooperate, will invest little in adaptation and brown technologies or more in green technologies to make compliance credible. Similarly, a small country is tempted to emit more because it internalizes less of the total harm if it free rides one period. Small countries will thus invest little in adaptation and brown technology or more in clean technology to counter the incentive to free ride.

Corollary 3. *In the best equilibrium, the smallest and the most reluctant countries invest the least in adaptation or brown technologies, or they invest the most in clean technology.*

The result that countries which are small or have high investment costs ought to invest more in clean technology is in stark contrast to the idea that countries should contribute according to ability and responsibility.

The result that countries which are reluctant to cooperate (in that the harm h_i is small) ought to invest more is similarly in contrast to the intuition that such countries must be given a better deal to make them cooperate.

It is true, of course, that countries that are reluctant either because they are small or have high investment costs, or because they are subject to less harm, have participation constraints (i.e., the constraint $v_i \geq v_i^b$) that are more difficult to satisfy than are the constraints for other countries. However, as we have shown above, the compliance constraint (CC_i^g) is more difficult to satisfy than the participation constraint. Although each country's benefit from cooperating, relative to BAU, must obviously be positive, this benefit must also be larger than the benefit from free riding one period before that deviation will be detected.

4 Uncertainty, Spillovers, and Renegotiation

The main results above are derived in a simple pedagogical model that can make it difficult to evaluate which assumptions are crucial and which can be relaxed. One

[17] If, instead, $(c')^2/c'' > c$, investing in adaptation technology is so productive that if n, g, or h_i increases, country i's environmental harm $h_i c(r_i) ng$ actually declines when the changes induce the country to invest more in adaptation technology. This is unrealistic, in our view.

crucial assumption is that some types of technology investments are observable. It is straightforward, however, to allow for non-observable investments in addition.[18] In the following sections, we extend the model in several other dimensions and show that the basic results continue to hold while additional insights emerge. We first allow private emissions, and private benefits from emissions, to be unobservable by others. In the former case, strategic technology investments can reduce the necessary duration of the punishment period; in the latter case, they reduce the probability of triggering the punishment, while keeping the incentives to comply at the emission stage. We also allow for technological spillovers and discuss renegotiation, before Section 5 allows for continuous emission levels and policy instruments such as emission taxes and investment subsidies. While our basic results are robust to all these extensions, each extension deepens our understanding of the strategic role of technologies. The reader is free to jump directly to the extension of interest, since all extensions build directly on the basic model presented in the previous sections. To isolate the insight of each variation, hereafter we assume that countries are homogeneous. Then, conditions (G_i) and (NG_i), for example, simplify to:

$$b''_{gr}(r) < hc'(r); \tag{G}$$

$$b''_{gr}(r) > hc'(r). \tag{NG}$$

4.1 Imperfect Monitoring and Duration of Punishment

In the basic model, grim-trigger strategies with infinitely long penalties come at no cost, since they will never occur in equilibrium. However, it is well documented how in reality direct monitoring of emissions is difficult, so that a permanent punishment would be too risky. Due to such monitoring imperfections, even if every country has the best of intentions, there is some chance that emission levels will appear to be higher than agreed upon. With such a risk, it is desirable to reduce the punishment length. When tracking the installation of technology is easier than tracking actual countries' emissions, which is generally the case (Sterner, 2003), we show how investments in technology can be strategically chosen to reduce the duration of the punishment. The equilibrium structure then becomes more compatible with the structure of actual treaties, as most sanctions for countries that violate international norms are temporary in nature (Hufbauer et al., 1990).

[18] Suppose a country can invest r in a technology type that is observable, as above, but also $\widetilde{r}$ in a different unobservable technology type at cost $\widetilde{k}\widetilde{r}$. If both technology types enter the benefit function $\widetilde{b}(g, r, \widetilde{r})$ (but, for simplicity, not the cost function), then our analysis above can remain unchanged if we just define $b(g,r) \equiv \max_{\widetilde{r}} \widetilde{b}(g, r, \widetilde{r}) - \widetilde{k}\widetilde{r}$ for $g \in \{\underline{g}, \overline{g}\}$, since the unobservable investment will always be set equal to the individually optimal level, conditional on the agreed-upon emission level.

To capture real-world uncertainty, we let total emission be given by $g \equiv \sum_{i=0}^{n} g_i$, where g_0, drawn from the cdf $F(\cdot)$ and i.i.d. over time, measures the net emission from *Nature*. In addition to the uncertain g_0, we also relax the assumption that the country-specific emission levels are observable. Instead, only the aggregate g is observed.[19] With these assumptions, punishments will be triggered even (in fact, only) after every country has emitted little, and it is thus optimal with a symmetric punishment.

We restrict attention to the set of public perfect equilibria (hereafter PPEs).[20] A "best" equilibrium is here a unique Pareto-optimal equilibrium among the PPEs in which everyone emits little. The best equilibrium $\left(r, \underline{g}\right)$ can be sustained by the following class of grim-trigger strategies: Comply by investing r and emitting $\underline{g}$ as long as (i) no country has deviated at the investment stage and (ii) the observed pollution level has been $g \leq \widehat{g}$, for some threshold $\widehat{g}$, in every earlier period. As soon as $g > \widehat{g}$, play BAU in $T \leq \infty$ periods before resuming cooperation. If one or more countries deviate at the investment stage, revert to BAU permanently.[21]

The presence of uncertainty leads to two types of errors. First, with probability $q \equiv 1 - F\left(\widehat{g} - n\underline{g}\right)$, we may have a *type I error* where cooperation ends even if every country polluted little. Second, with probability $1 - p = F\left(\widehat{g} - \left(\overline{g} + (n-1)\underline{g}\right)\right)$, we may have a *type II error* where cooperation continues even after a country has deviated by polluting more. We obviously have $p > q$ when $F(\cdot)$ is strictly increasing.

The compliance constraint at the emission stage requires that the one-shot benefit from free riding be smaller than the cost of increasing the chance of facing the punishment:[22]

$$b(\bar{g}, r_i) - b\left(\underline{g}, r_i\right) - hc(r_i)\left(\bar{g} - \underline{g}\right) \leq \frac{\delta\left(1-\delta^T\right)}{1-\delta}(p-q)\left(v_i - v_i^b\right), \text{ where} \qquad (\mathrm{CC}_F^g)$$

$$v_i = (1-\delta)\left[b\left(\underline{g}, r_i\right) - hc(r_i)\, n\underline{g} - kr_i\right] + \delta\left[(1-q)\, v_i + q\left(\left(1-\delta^T\right)v_i^b + \delta^T v_i\right)\right].$$

The last equation measures v_i, the continuation value if the penalty is not triggered. Also, note that $p - q < 1$ is the increased likelihood that the penalty is triggered if, at the emission stage, a country emits more rather than less. Clearly, the compliance

[19]Note that neither of the two modifications would play any role if introduced in isolation: If the g_i's were observable, the uncertain g_0 would play no role since the marginal cost of pollution is constant; if g_0 were deterministic or absent, it would be irrelevant whether the g_i's were observable as long as the aggregate g could be observed. Together, however, the two modifications turn out to be important and realistic.

[20]These are strategy profiles for the repeated game in which (i) each country's strategy depends only on public information, and (ii) no player wants to deviate at any public history. See Fudenberg and Tirole (1991) for a definition of this equilibrium concept.

[21]The equilibrium strategy is along the lines of Abreu et al. (1986) who characterize the optimal symmetric equilibrium under imperfect monitoring and binary actions. For a review, see Mailath and Samuelson (2006: 239).

[22]If condition (CC_F^g) holds, the compliance constraint at the investment stage, $v_i \geq v_i^b$, is, as before, satisfied. See the proof of Proposition 4 for the derivation of (CC_F^g) and the value function.

constraint at the emission stage is harder to satisfy than in the basic model. Both errors imply that the benefit of emitting less declines: penalties may be triggered in any case (when $q > 0$), or they may not be triggered even if a country emits more (when $p < 1$). The errors also reduce the continuation value v_i, which countries hope to receive in the next period. Finally, a shorter punishment period $T < \infty$ means that countries have less to fear from the penalty.

Condition (CC_F^g) can be written as $\delta \geq \widehat{\delta}(r_i, T)$, where $\widehat{\delta}(r_i, T)$ is the discount factor satisfying (CC_F^g) with equality. While $r_i = r^*$ and $T = 0$ would maximize the continuation value v_i, the compliance constraint at the emission stage would then be violated. At $r_i = r^*$, condition (CC_F^g) is weakened, and compliance is easier to achieve, for a larger investment $r_i > r^*$ (so $\widehat{\delta}'_r(r_i, T) < 0$) if and only if (G) holds. This strategic role of technology is the same as in the basic model.

The desire to reduce the punishment period, however, results in a new strategic role for technology. Starting at $T = \infty$, the equilibrium utility increases when T is reduced. However, a reduction in T makes (CC_F^g) harder to satisfy (so $\widehat{\delta}'_T(r_i, T) < 0$). To allow for a reduction in T, without violating the compliance constraint, it is necessary to invest even more in green technology or less in adaptation or brown technology. In other words, technology can be strategically chosen so as to allow for a reduction in the punishment length.

To be precise, we can solve a binding (CC_F^g) for δ^T and insert it in the expression for v_i, which then becomes:

$$v_i = b\left(\underline{g}, r_i\right) - hc\left(r_i\right) n\underline{g} - kr_i - \frac{q}{p-q}\left[b\left(\bar{g}, r_i\right) - b\left(\underline{g}, r_i\right) - hc\left(r_i\right)\left(\bar{g} - \underline{g}\right)\right]. \quad (5)$$

Clearly, the optimal emission cutoff level $\widehat{g}$ is simply given by:

$$\widehat{g}^* = \arg\min_{\widehat{g}} \frac{q}{p-q} = \arg\min_{\widehat{g}} \frac{1 - F\left(\widehat{g} - n\underline{g}\right)}{F\left(\widehat{g} - n\underline{g}\right) - F\left(\widehat{g} - \left(\overline{g} + (n-1)\,\underline{g}\right)\right)},$$

which implies that p and q are only functions of F, n, $\underline{g}$ and $\overline{g}$, making $\widehat{g}^*$ independent of any other parameter in the model.[23]

When $T < \infty$, the equilibrium investment level $\tilde{r}$ is equal to the arg max of (5). When $q > 0$, this implies $\tilde{r} > r^*$ under (G), and $\tilde{r} < r^*$ under (NG). When δ declines from 1, investment remains at $\tilde{r}$, which is independent of δ, while T must increase to satisfy a binding (CC_F^g), as illustrated in Figure 1. The constraint $\delta = \widehat{\delta}(r_i, T)$ implicitly defines T as a decreasing function of δ (i.e., $T(\delta)$). At the $\overline{\delta} \equiv \widehat{\delta}(\tilde{r}, \infty)$, the required T reaches infinity and, for even smaller discount factors, the compliance constraint cannot

[23] This equation is for the case in which it is optimal with $T < \infty$. If $T = \infty$, the derivation of the optimal $\widehat{g}$, p, and q is more complicated, as shown in the proof of Proposition 4 in the Appendix.

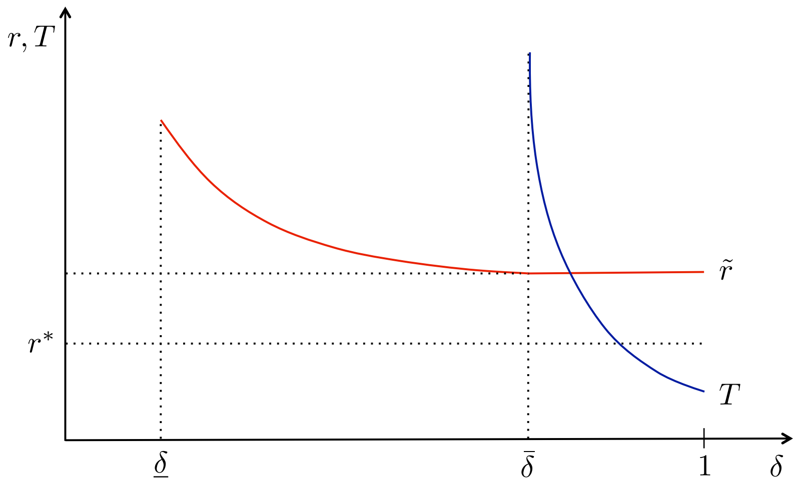

Figure 1: *Even for large discount factors, countries over-invest when (G) holds. The large investments permit a shorter punishment phase without violating the compliance constraint.*

be satisfied unless r_i is even larger than $\tilde{r}$ under (G), or even lower than $\tilde{r}$ under (NG). Therefore, at $T = \infty$, a binding constraint $\delta = \widehat{\delta}(r_i, \infty)$ is now implicitly defining the optimal investment as a function of δ (i.e., $r(\delta)$). Just as before, a lower boundary $\underline{\delta}$ may exist, since there is a limit to how large r_i can be in a treaty worth signing.[24] All this is proved in the Appendix.

Proposition 4. *There exists a unique best equilibrium if and only if $\delta \geq \underline{\delta}$ and it is characterized as follows:*
(i) If $\delta \geq \overline{\delta}$, $T = T(\delta)$ with $T'(\delta) < 0$, and investments are given by $r = \tilde{r}$ where:

$$\tilde{r} > r^* \text{ if (G);}$$
$$\tilde{r} < r^* \text{ if (NG).}$$

(ii) If $\delta \in [\underline{\delta}, \overline{\delta})$, $T = \infty$, and investments are given by $r = \tilde{r}$ where:

$$r(\delta) > \tilde{r} > r^* \text{ with } r'(\delta) < 0 \text{ if (G);}$$
$$r(\delta) < \tilde{r} < r^* \text{ with } r'(\delta) > 0 \text{ if (NG).}$$

The qualitative difference between Proposition 4 and the basic model without uncertainty is part (i). Since there is always a chance that the penalty will be triggered by mistake, the first best is impossible to sustain. The compliance constraint requires

[24] Of course, the thresholds $\underline{\delta}$ and $\overline{\delta}$ are different here compared to the thresholds in the basic model, and they are also different in the subsequent extensions. Since this difference is obvious, we do not add subscripts indicating that a threshold depends on the extension we are discussing.

a penalty, but the penalty duration should be reduced as much as the compliance constraint permits. By requiring the countries to invest strategically, the temptation to emit more declines and the penalty duration can be reduced without violating the compliance constraint.

Corollary 4. *With imperfect monitoring, one strategic role of technology is to reduce the duration of punishment that is necessary to motivate compliance.*

4.2 Technology and the Probability of Cooperation

It is well understood that climate policies are rife with uncertainties. The uncertainty may regard outcomes of technological progress and of the business cycle. When faced with such uncertainties, countries may not be motivated to comply with a climate treaty if the realized shock leads to higher compliance costs. For example, incentives to comply with low emission targets are likely to be influenced by changes in unemployment or by political pressure. With a stochastic compliance cost, the temptation to emit more will depend on the realization of the shock. We show that such a temptation can be strategically mitigated through technology decisions, since they reduce the frequency at which a punishment is triggered. With more investment in green technology, or less investment in adaptation and brown technology, the temptation to emit more decreases, as does the set of shock realizations that lead to non-compliance.

To illustrate this effect, suppose the benefit function is given by $\theta_i b(g_i, r_i)$, where the privately observed shock θ_i is distributed with mean $\overline{\theta}$ and with strictly positive density everywhere on the support $\Theta \equiv \left[\overline{\theta} - \sigma, \overline{\theta} + \sigma\right]$, where $\sigma > 0$ measures the uncertainty.[25] We continue to assume that the emission stage constitutes a prisoner's dilemma game for every θ_i. Let θ_i be i.i.d. in every period and let its realization be learned by i after the investment stage but before the emission stage.

Since a symmetric T-period reversion to BAU was an optimal punishment in Section 4.1, we will restrict attention to such a punishment also here in order to illustrate our point in the same simple framework. In equilibrium, a country complies with low emissions if and only if $\theta_i \leq \widehat{\theta}$ for some endogenous threshold $\widehat{\theta} \in \Theta$. Each country is thus complying with probability $\pi \equiv \Pr\left(\theta_i \leq \widehat{\theta}\right)$. For each $\theta_i \leq \widehat{\theta}$, the compliance constraint at emission stage becomes:

$$\theta_i \left(b\left(\overline{g}, r_i\right) - b\left(\underline{g}, r_i\right)\right) - hc\left(r_i\right)\left(\overline{g} - \underline{g}\right) \leq \frac{\delta\left(1 - \delta^T\right)}{1 - \delta}\pi^{n-1}\left(\mathbb{E}v_i - v_i^b\right), \qquad (\mathrm{CC}_\theta^g)$$

where π^{n-1} is the probability that all other countries comply. The first best can be

[25] If the shocks were publicly observed, it would be optimal to have "escape clauses" such as those that exist in trade agreements (Bagwell and Staiger, 1990).

sustained if (CC^g_θ) holds for $\theta_i = \overline{\theta} + \sigma$ when $\pi = 1$, $r_i = r^*$, and $T = \infty$. In this case, let (CC^g_θ) bind at discount factor $\overline{\delta}$. It is easy to see that $\overline{\delta} < 1$. When the discount factor falls below $\overline{\delta}$, the first best cannot be achieved, and the equilibrium outcome will necessarily be distorted. But while two distortions are possible, one has a first-order effect: if the compliance constraint is not satisfied for the highest realizations of θ_i, the punishment will be triggered with a strict positive probability (so, $\pi < 1$). Alternatively, one may require a larger r_i under (G), or a smaller r_i under (NG), and still ensure that (CC^g_θ) holds for every $\theta_i \in \Theta$. This distortion has a second-order effect on utilities, since the utility is continuously differentiable in r_i. For this reason, it is always optimal to distort r_i when δ falls (marginally) below $\overline{\delta}$, rather than letting $\widehat{\theta}$ and π decline.

Proposition 5. *Suppose θ_i is distributed with strictly positive density on Θ.*
(i) A threshold $\overline{\delta}$ exists such that the best equilibrium is first best if $\delta \geq \overline{\delta}$.
(ii) When δ falls below $\overline{\delta}$, the unique best equilibrium requires $r_i > r^*$ under (G), and $r_i < r^*$ under (NG).
(iii) The larger the uncertainty σ, the larger is $\overline{\delta}$, and the larger is the necessary distortion $|r_i - r^*|$.

The last part of the proposition requires countries to invest even more in green technology, or even less in adaptation and brown technology, if the compliance cost is highly uncertain. The proposition follows straightforwardly from (CC^g_θ).[26]

Corollary 5. *With stochastic compliance costs, one strategic role for technology is to raise the probability for continuing cooperation.*

4.3 Technological Spillovers

Cooperation on environmental policies may be plagued with free-riding problems arising from two sources of externalities. The first source is the global negative pollution externality emphasized above. The second results from the imperfect appropriability of knowledge required to develop a new technology, as R&D effort in one country is also beneficial for other countries through technological spillovers, because of imperfect intellectual property rights, for example. We now aim to demonstrate how the strategic role of technology is affected by this second source of externality.

[26]For details, see our working paper version (Harstad et al., 2015), where we also discuss the best equilibrium for discount factors that are strictly (and not only marginally) below $\overline{\delta}$. When δ continues to fall below $\overline{\delta}$, satisfying (CC^g_θ) requires strategic investments that eventually have first-order effects on the utility levels. It may then be optimal to give up on the compliance constraint for the highest realizations of θ_i. It continues to be true, of course, that one strategic role of choosing r_i different from its first-best level is to satisfy the compliance constraint for a larger set of shocks.

To better understand the interaction between technological spillovers and self-enforcing agreements, let $e \in (0,1)$ be the *fraction* of a country's investment that benefits the others instead of the investor. Country i's utility is:

$$u_i = b(g_i, z_i) - hc(z_i) \sum_{j \in N} g_j - kr_i, \text{ where } z_i \equiv (1-e) r_i + \frac{e}{n-1} \sum_{j \neq i} r_j.$$

Here, the term $(1-e)$ is just a normalization and it can be removed without affecting the results.[27] Furthermore, the term $(1-e)$ is natural when a reduction in e comes from stronger intellectual property rights, implying that the neighboring countries must pay the innovator when using the technology. With this formulation, the first-best r^* stays unchanged as we vary e. In BAU, in contrast, countries invest less the larger e is. Thus, it is no longer true that countries will invest the optimal amount conditional on g_i.

Since first-best investments are larger than non-cooperative investments (conditional on g_i), countries may be tempted to deviate even at the investment stage. A country that deviates at the investment stage will not only enjoy its BAU continuation value, but it may also benefit if other countries invest more than they would in BAU.

To simplify, we here refer to the "best" SPE as the SPE that maximizes the sum of payoffs under the threat that any deviation from the SPE will lead to BAU forever (and continuation value v^b).[28] The compliance constraint at the investment stage can then be written as:

$$\frac{v_i}{1-\delta} \geq \frac{e}{1-e} k \left(r_i - r^b\right) + \frac{v^b}{1-\delta}. \tag{CC$_e^r$}$$

Condition (CC_e^r) is trivially satisfied if $e = 0$ or if $r_i \leq r^b$. When $e > 0$ and $r_i > r^b$, (CC_e^r) can be written as:

$$\delta \geq \widehat{\delta}^r (r_i) \equiv 1 - \frac{1-e}{e} \frac{v_i - v^b}{r_i - r^b} \frac{1}{k} < 1.$$

The compliance constraint at the emission stage is similar to the compliance constraint in the basic model and therefore is not reported here, but we denote the threshold (4) by $\widehat{\delta}^g (r_i)$ to distinguish it from $\widehat{\delta}^r (r_i)$. The first best $\left(r^*, \underline{g}\right)$ can be supported as an SPE if and only if both $\delta \geq \overline{\delta}^r \equiv \widehat{\delta}^r (r^*)$ and $\delta \geq \overline{\delta}^g \equiv \widehat{\delta}^g (r^*)$. The Appendix shows that with small spillovers, that is $e < \overline{e}$ for some threshold $\overline{e} > 0$, the compliance constraint

[27] If instead the relevant utility function were $u_i = \widetilde{b}(g_i, \widetilde{z}_i) - h\widetilde{c}(\widetilde{z}_i) \sum_{j \in N} g_j - kr_i$, where $\widetilde{z}_i = r_i + \frac{\widetilde{e}}{n-1} \sum_{j \neq i} r_j$, we could define e from $e/(1-e) \equiv \widetilde{e}$ and then define $z_i \equiv \widetilde{z}_i (1-e) = (1-e) r_i + \frac{e}{n-1} \sum_{j \neq i} r_j$, $c(z_i) \equiv \widetilde{c}(z_i/(1-e)) = \widetilde{c}(\widetilde{z}_i)$, and $b(g_i, z_i) \equiv \widetilde{b}(g_i, z_i/(1-e)) = \widetilde{b}(g_i, \widetilde{z}_i)$, to write $u_i = b(g_i, z_i) - hc(z_i) \sum_{j \in N} g_j - kr_i$.

[28] Although this punishment was actually optimal in Sections 3 and 4.1, it is here a simplification since with technological spillovers, the minmax strategies are different.

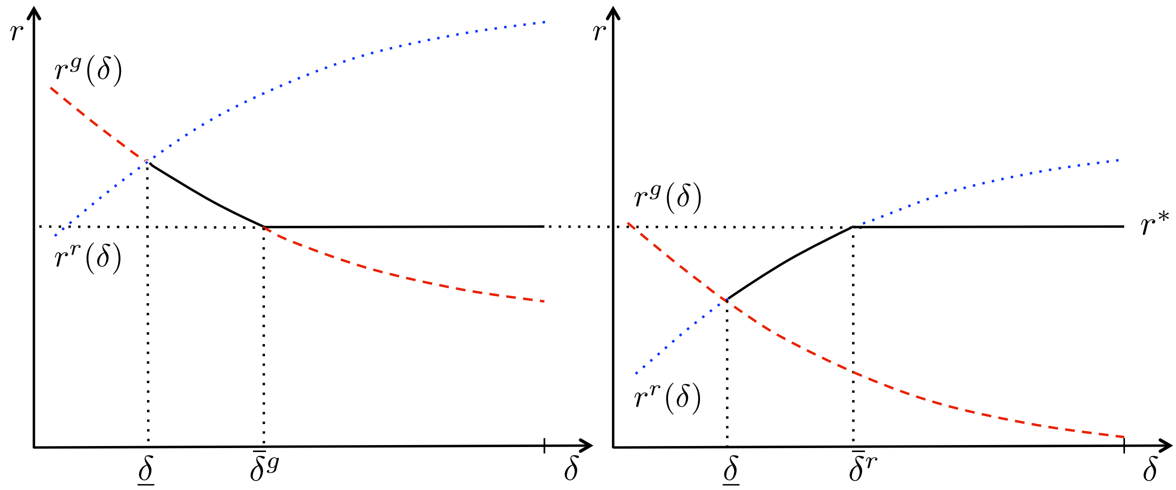

Figure 2: *With small spillovers (left panel), the emission-stage compliance constraint (dashed line) will bind first and over-investments may be necessary. With large spillovers (right panel), the investment-stage compliance constraint (dotted line) becomes tougher to satisfy, and investments may be suboptimally small.*

at the emission stage binds first (so, $\overline{\delta}^g > \overline{\delta}^r$), and to satisfy it, one must require that the optimal investments be larger than r^* under (G) and smaller than r^* under (NG). However, if spillovers are large, that is $e \geq \overline{e}$, the compliance constraint investment stage binds first (so, $\overline{\delta}^g < \overline{\delta}^r$), and to satisfy it, investments must be smaller than r^* when $\delta < \overline{\delta}^r$ for any type of technology.

We can write the compliance constraint at the investment stage as $r_i \leq r^r(\delta) \equiv \widehat{\delta}^{r-1}(\delta)$, and the compliance constraint at the emission stage as $r_i \geq r^g(\delta) \equiv \widehat{\delta}^{g-1}(\delta)$ under (G), or as $r_i \leq r^g(\delta)$ under (NG). Figure 2 shows how different levels of technological spillovers affect strategic investments in the case of green technology.

If $\delta \geq \max\left\{\overline{\delta}^r, \overline{\delta}^g\right\}$, $r^* \in [r^g(\delta), r^r(\delta)]$. With green technologies, the minimum level $r^g(\delta)$ increases as δ decreases, while the maximal level $r^r(\delta)$ declines with δ. So, for $\delta < \max\left\{\overline{\delta}^r, \overline{\delta}^g\right\}$, r_i must increase if $e < \overline{e}$, just as in the basic model, but must decrease if $e > \overline{e}$. For sufficiently small discount factors, $\delta \leq \underline{\delta}$, the interval $[r^g(\delta), r^r(\delta)]$ is empty and it is not possible to sustain low emissions as an SPE. For non-green technology satisfying (NG), both compliance constraints define upper boundaries for r_i, namely $r_i \leq r^r(\delta)$ and $r_i \leq r^g(\delta)$. Regardless of what constraint binds, the optimal investment r must fall whenever δ falls. For δ less than some threshold, $\underline{\delta}$, no r_i can satisfy both compliance constraints.

Proposition 6. *There exists a unique best equilibrium if and only if $\delta \geq \underline{\delta}$ and it is characterized as follows:*
(i) If $\delta \geq \max\left\{\overline{\delta}^r, \overline{\delta}^g\right\}$, $r_i = r^*$ and the best equilibrium is first best.

(ii) If $\delta \leq \max\left\{\overline{\delta}^r, \overline{\delta}^g\right\}$*:*

$$r = r^g(\delta) > r^* \text{ if } e \leq \overline{e} \text{ under (G),}$$
$$r = r^r(\delta) < r^* \text{ if } e > \overline{e} \text{ under (G), and}$$
$$r = \min\{r^g(\delta), r^r(\delta)\} < r^* \text{ under (NG).}$$

Compared to Proposition 2, the qualitative difference is that green investments fall when δ falls if $e > \overline{e}$. Such a large spillover is always harmful since it imposes a constraint on the investment level that can be sustained as SPEs. Specifically, requiring a high level of investment in clean technology to motivate compliance at the emission stage may not be possible if the spillover is large. Thus, with a policy that reduces the spillover, for example by strengthening intellectual property rights, countries can require more investments in green technology without fearing that the compliance constraint at the investment stage will be violated.

Corollary 6. *Stronger intellectual property rights may be necessary to sustain a self-enforcing treaty.*

4.4 Renegotiation Proofness and Compliance Technology

So far, the goal of our analysis has been to describe the best equilibrium. The game has included neither any negotiation, nor an explanation for *how* or *why* countries are able to negotiate or coordinate on the best equilibrium. If we introduced such negotiations, it may also be natural to allow countries to renegotiate later on. While there is no need to renegotiate when all countries comply with an agreement, countries do have an incentive to renegotiate as soon as a defection is observed, and before triggering a costly and long-lasting punishment phase.[29]

Our results continue to hold (or they are strengthened) if we introduce renegotiation. An especially simple case arises if a defecting country has no bargaining power in the renegotiation game. Suppose that if a country $i \in N$ emits more, everyone will play BAU forever unless countries renegotiate. If the coalition $N \backslash i$ of $n-1$ countries has all the bargaining power in this renegotiation game, the coalition will ensure that country i does not receive more than the BAU continuation value. If side payments are possible, this can be achieved by requesting the deviator to pay other countries before cooperation is restored. If side payments are impossible, the coalition $N \backslash i$ may request country i to

[29] Several solution concepts for repeated games have been proposed to incorporate renegotiation. In this discussion, we rest on the one by Farrell and Maskin (1989), according to which a renegotiation–proof equilibrium is defined as a subgame perfect equilibrium where, given any two continuation equilibria, neither Pareto dominates the other.

invest a particular amount at the next investment stage before cooperation continues.[30] When free riding one period leads to a continuation value identical to the value of BAU for the deviator, the compliance constraint remains identical to that of the case without renegotiation. In this case, allowing for renegotiation does not restrict the set of SPEs.

In our working paper version (Harstad et al., 2015), we consider the possibility that the deviator has some bargaining power in the renegotiation game.[31] This possibility implies that if country $i \in N$ deviates, it will receive more than its BAU continuation value beginning from the next period. The larger the bargaining power of i, the larger the continuation value i will receive if i defects by emitting more. This effect means that the compliance constraint is harder to satisfy in the case with renegotiation than in the case without renegotiation and, to satisfy it, $|r_i - r^*|$ must increase. The larger the bargaining power of a defecting country i in the renegotiation game, the larger $r_i > r^*$ under (G) or the smaller $r_i < r^*$ under (NG), if $\left(r_i, \underline{g}\right)$ is to be sustained as an SPE. In other words, if renegotiation is possible, the strategic role of compliance technology is strengthened.

5 Continuous Emission Levels and Policies

This section relaxes the assumption of binary emission levels in order to obtain two novel results: First, we show that the best equilibrium may require the countries to punish cooperation. If the compliance constraint at the investment stage binds, a country cannot be allowed to invest as much as it would like in green technology, since if it did, other countries would be tempted to deviate at the investment stage. Second, we introduce policy instruments and show that when the discount factor falls, the carbon tax implementing the best equilibrium falls while the investment subsidy increases.

To simplify and follow the same line of reasoning as in the rest of the paper, we here consider only SPEs enforced by the threat of reverting to BAU, despite the fact that BAU is no longer the harshest penalty when $g_i < g^b$ is possible. Furthermore, we consider only symmetric SPEs, although there can also be asymmetric SPEs that are Pareto optimal. This assumption is partially justified since we, as in Section 4, consider homogenous countries only. If there is such a unique Pareto-optimal SPE, we refer to it as the "best" equilibrium.

[30]That is, the coalition $N \backslash i$ may propose a take-it-or-leave-it offer to i involving a large r_i under (G), or a small r_i under (NG), where the choice of r_i is so costly for i that i is receiving just the outside option continuation value (which is the value of BAU). For $N \backslash i$, this is one (out of several) optimal renegotiation offers when r_i is self-investment; with technological spillovers (discussed in the previous subsection), requesting such a large level of r_i is strictly beneficial to $N \backslash i$ under (G).

[31]The role of bargaining power in renegotiation-proof equilibria is explored in depth in Miller and Watson (2013).

5.1 The Need to Punish Cooperative Investments

Allowing for a continuous g_i complicates the analysis. To proceed, we restrict attention to the case in which g_i and r_i are perfect substitutes in a linear-quadratic utility function:

$$u_i = -\frac{B}{2}\left(\overline{y} - (g_i + r_i)\right)^2 - \frac{K}{2} r_i^2 - c \sum_{j \in N} g_i,$$

where B and K are positive constants.[32] Here, $\overline{y}$ is a country's bliss level for consumption, and consumption is the sum of g_i (energy from fossil fuels) and r_i (energy from renewable energy sources). Since $\partial^2 u_i / \partial g_i \partial r_i < 0$, we explicitly consider only clean technology. We can easily reformulate the utility function such that the investment cost becomes linear, although there is no need to do so here.[33]

Note that the first-best outcome is:

$$r^* = \frac{cn}{K} \text{ and } g^* = \overline{y} - \frac{cn}{B} - r^*,$$

while the BAU equilibrium (the unique SPE in the stage game) is:

$$r^b = \frac{c}{K} \text{ and } g^b = \overline{y} - \frac{c}{B} - r^b.$$

If countries took their emission levels as given (for example, if i had committed to emission level g_i in advance), equilibrium investments would be socially optimal conditional on g_i and could be written as:

$$r^*(g_i) = \frac{B\left(\overline{y} - (g_i + r_i)\right)}{K} = \frac{B\left(\overline{y} - g_i\right)}{B + K}.$$

So, just as with binary actions, with commitment to some level of g_i, there would be no need to cooperate also on investments.

Nevertheless, the compliance constraint at the investment stage is no longer innocuous. Larger investments reduce the need to emit even after cooperation breaks down, so a country that deviates at the investment stage enjoys a payoff that is strictly higher than its BAU payoff. If technology is important (in that K/B is small), free riding may be more tempting at the investment stage than at the emission stage.

The compliance constraint at the investment stage, $\delta \geq \widehat{\delta}^r(g, r)$, and the one at the emission stage, $\delta \geq \widehat{\delta}^g(g, r)$, are both derived in the Appendix.

Proposition 7. *There exists a unique best equilibrium.*

[32] This utility function is also considered in Battaglini and Harstad (2016), who do not study SPEs, but instead the Markov-perfect equilibria when countries can commit to the emission levels. The first best and the BAU equilibrium are as in that paper, of course.

[33] To see this, simply define $\widetilde{r}_i = r_i^2/2$ and rewrite to $u_i = -\frac{B}{2}\left(\overline{y} - \left(g_i + \sqrt{2\widetilde{r}_i}\right)\right)^2 - K\widetilde{r}_i - c\sum_{j \in N} g_i$.

(i) The best equilibrium is first best if $\delta \geq \max\left\{\overline{\delta}^g, \overline{\delta}^r\right\}$, *where*

$$\overline{\delta}^g \equiv \widehat{\delta}^g\left(g^*, r^*\right) = \frac{K}{B+2K} \text{ and } \overline{\delta}^r \equiv \widehat{\delta}^r\left(g^*, r^*\right) = \frac{B-K}{2B}.$$

(ii) Investments should be larger if $K/B > 1/2$*: then,* $\overline{\delta}^r < \overline{\delta}^g$ *and, if* $\delta \in \left[\widehat{\delta}^r\left(g,r\right), \overline{\delta}^g\right)$*, we have:*

$$\begin{aligned} r &= r^*\left(g\right) + \frac{\phi\left(\delta\right)}{B+K} > r^*\left(g\right) \textit{ and} \\ g &= g^* + \frac{\phi\left(\delta\right)}{B} > g^*, \textit{ where} \\ \phi\left(\delta\right) &\equiv c\left(n-1\right)\left[1-\delta-\sqrt{\delta^2+\delta B/K}\right] > 0. \end{aligned}$$

(iii) Investments should be smaller if $K/B < 1/2$*: then,* $\overline{\delta}^r > \overline{\delta}^g$ *and, if* $\delta \in \left[\widehat{\delta}^g\left(g,r\right), \overline{\delta}^r\right)$*, we have:*

$$\begin{aligned} r &= r^*\left(g\right) - \frac{\psi\left(\delta\right)}{K} < r^*\left(g\right), \textit{ and} \\ g &= g^* + \frac{\psi\left(\delta\right)}{K} > g^*, \textit{ where} \\ \psi\left(\delta\right) &\equiv c\left(n-1\right)\left[1-\delta-\sqrt{\delta^2+K/B}\right] > 0. \end{aligned}$$

In part (ii), when $K/B > 1/2$, the relevant compliance constraint is the one at the emission stage, just as before. As the discount factor falls, cooperation becomes harder to sustain and g must be allowed to increase. On the one hand, the larger g means that it becomes optimal to invest less in substitute technologies. On the other hand, countries can dampen the increase in g by requesting other countries to invest more in green technology up front. These two effects cancel out each other when g and r are perfect substitutes, and r stays unchanged as δ declines, although g increases, as illustrated by the horizontal arrow in Figure 3. However, compared to the ex-post optimal level, it is clear that $r - r^*\left(g\right)$ is positive and increases as δ falls. Technically, this result follows since the function $\phi\left(\delta\right)$, defined in Proposition 7, declines to zero as δ increases to $\overline{\delta}^g$.

Part (iii) is new to the literature, as far as we know. When $K/B < 1/2$, the emission level is very sensitive to the choice of technology, so the temptation to free ride is larger at the investment stage than at the emission stage. As δ falls, motivating compliance requires that investment must be smaller than $r^*\left(g\right)$ while g grows, as illustrated by the downward-sloping arrow in Figure 3. Technically, this result follows since the function $\psi\left(\delta\right)$, defined in Proposition 7, declines to zero as δ increases to $\overline{\delta}^r$.

So, when $K/B < 1/2$, countries must be required to invest less than they would have

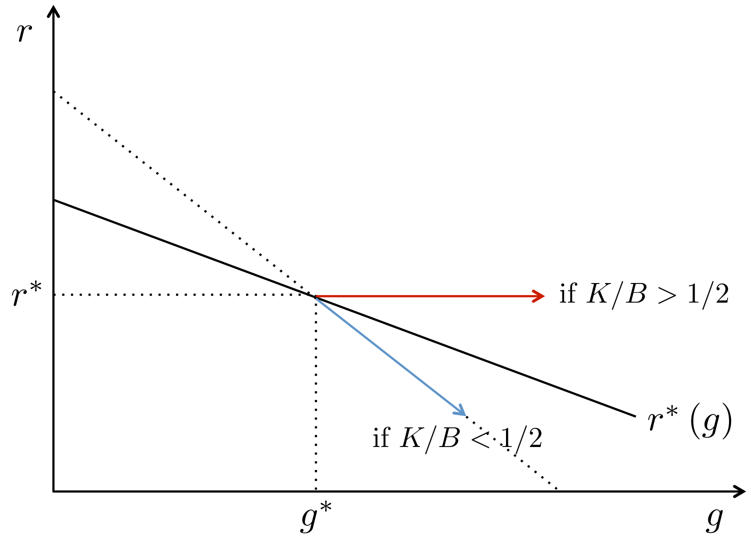

Figure 3: *If δ falls, the distortions follow the horizontal arrow if $K/B > 1/2$, and the 45-degree arrow if $K/B < 1/2$. In the latter case, optimal green investments must be penalized with BAU.*

found optimal conditional on g, since the smaller r is necessary to satisfy $\delta \geq \widehat{\delta}^r(g, r)$. For fixed g's, allowing countries to invest more would have been a Pareto improvement. The larger investment levels cannot be allowed in equilibrium, however, and must be punished by a reversion to BAU. The explanation for this result is that if a country i were allowed to invest more (for example, as much as $r^*(g)$), other countries would believe that i would find it optimal to emit little even if cooperation broke down. This belief would make it too tempting for other countries to deviate at the investment stage, and their incentive constraint would be violated.

5.2 Carbon Taxes and Investment Subsidies

Policies, such as emission taxes and investment subsidies, can be used to implement the best equilibrium. We assume that country i's investment subsidy, ς_i, is set by i just before the investment stage in each period, and it is observable by all countries. The actual investment is made by private investors who receive the subsidy ς_i in addition to the price paid by consumers. The emission tax, τ_i, is set just before the emission stage, and it represents the cost of polluting paid by consumers. If taxes are collected and subsidies are paid by national governments, they do not represent actual costs or revenues—from the government's perspective—and their only effect is to influence the decisions g_i and r_i. The agreement between countries then amounts to setting domestic taxes/subsidies such that the desired SPE is implemented. While the marginal benefit of consuming fossil fuel equals the emission tax, the marginal investment cost equals the marginal benefit of consuming energy plus the investment subsidy when investors can sell green technology to fossil fuel consumers:

$$B\left(\overline{y}-\left(g_i+r_i\right)\right)=\tau_i \text{ and } Kr_i=B\left(\overline{y}-\left(g_i+r_i\right)\right)+\varsigma_i. \tag{6}$$

The use of emission taxes does not change the compliance constraint at the emission stage: the policies are set simultaneously, so that if one country deviates by skipping the emission tax, it is too late for other countries to revise their emission taxes before the next period.

The use of an investment subsidy, in contrast, does relax the compliance constraint at the investment stage. As soon as one country deviates by setting a different investment subsidy, investors in all countries anticipate that cooperation will break down and that the demand for their technology at the emission stage will be reduced. This anticipation lowers investments everywhere, not only in the deviating country. Deviating at the investment stage immediately gives the deviator the BAU payoff, plus the benefit of other countries' larger investments induced by their subsidies. These subsidies are zero for $\delta \geq \overline{\delta}^g$ and they are small for discount factors close to $\overline{\delta}^g$. Consequently, there exists some $\underline{\delta} < \overline{\delta}^g$ such that the compliance constraint at the investment stage is not binding when $\delta \in \left(\underline{\delta}, \overline{\delta}^g\right]$.[34] Combining Proposition 7 and Eq. (6), we get:

Proposition 8. *Suppose the best equilibrium described in Proposition* 7 *is implemented by an emission tax* τ *and an investment subsidy* ς.
(i) The first compliance constraint to bind is always the one at the emission stage when δ *declines from* 1.
(ii) If $\delta \geq \overline{\delta}^g$, *the best equilibrium is first best and implemented by* $\tau = cn$ *and* $\varsigma = 0$.
(iii) If $\delta \in \left[\underline{\delta}, \overline{\delta}^g\right)$, *the best equilibrium is implemented by* $\tau = cn - \phi(\delta)$ *and* $\varsigma = \phi(\delta)$, *where* $\phi(\delta)$ *is defined in Proposition 7.*

In the first best, which can be sustained when the discount factor is sufficiently large, the emission tax is set at the Pigouvian level and there is no need to regulate investments in addition, since investors capture the entire profit associated with their technology investments.

When the discount factor falls and cooperation becomes difficult, the equilibrium emission tax must fall. But, for a given emission tax, the emission level can also be reduced by a larger stock of green technology. Thus, when emissions must be larger than the first-best level to motivate compliance, the best equilibrium requires countries to subsidize investments in green technology. The smaller the discount factor is, the

[34] The compliance constraint at the investment stage is $\frac{v_i}{1-\delta} \geq \varsigma(n-1)\frac{c}{K} + \frac{v^b}{1-\delta}$, which always holds when $\varsigma = \phi(\delta) \to 0$. When δ falls, v_i declines and $\varsigma = \phi(\delta)$ increases. The threshold $\underline{\delta}$ is defined implicitly by requiring that such constraint hold with equality at $\underline{\delta}$.

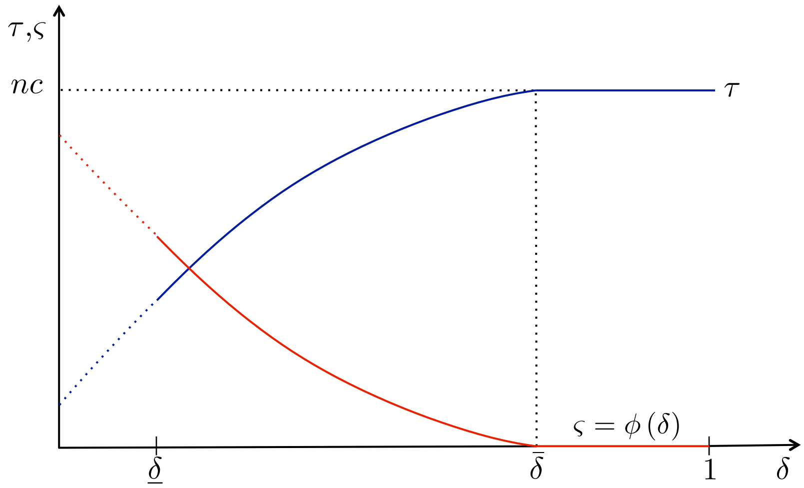

Figure 4: *When the discount factor falls and free riding becomes more tempting, the emission tax must be allowed to fall, but the investment subsidy must increase.*

smaller the equilibrium emission tax is, but the larger, therefore, is the investment subsidy. The sum of the emission tax and the investment subsidy is always given by nc (i.e., the first-best Pigouvian tax level). Both policies are drawn as functions of the discount factor in Figure 4.[35]

6 Lessons and Predictions

This paper analyzes a repeated prisoner's dilemma game with fixed discount factors and endogenous technology. Both assumptions are natural in a climate change context where countries over time invest in "green" and "brown" technology and emission agreements must be self-enforcing. When free riding is tempting and cooperation difficult to sustain, the best self-enforcing treaty requires countries to over-invest in "green" technology, which reduces the temptation to pollute, or under-invest in adaptation or "brown" technology that would have made free riding more attractive. The analysis provides positive predictions as well as normative policy lessons.

One basic positive prediction is that to motivate compliance with a climate treaty, one must not only ensure that the decisions are repetitive and observable and that the number of participants is large. In addition, countries must invest sufficiently in green technology.

The main climate agreement to date, the relatively unsuccessful Kyoto Protocol of 1997, does not fit these requirements. The Protocol specified emission caps for two subsequent commitment periods and for relatively few (35) countries. As discussed in

[35] Also Golosov et al. (2014) derive the optimal taxes in the presence of substitutability between clean and dirty energy, but they do not consider subsidies on technology investments.

the Introduction, China and the European Union have nevertheless invested substantially in environmentally friendly technology and, in line with our theory, the European Union has been complying well to the Protocol. Other countries, as Canada, were free to instead invest heavily in brown technology, and it ended up withdrawing rather than complying.

Remarkably, the relatively successful climate policies are more in line with our predictions. Among the successes, a particularly important example is the "2020 climate & energy package" adopted by the European Union in December, 2008. This agreement shares several features of the optimal self-enforcing treaty studied in this paper. First, together with emission targets, the European agreement also required countries to develop renewable energy sources of at least one fifth of the total energy mix by 2020. Second, the effectiveness of the European agreement has most likely relied on the sequential nature of investment and emission decisions. While member states were required to submit their national plans to meet the technology investment targets by 2010, they were required to limit their emissions to meet the annual limit only starting from 2013.[36] Having installed technologies before the actual emission abatement, member states could then build enforcement by conditioning cooperation on prior technology installation, as our theory suggests.

Interestingly, the 2015 Paris Agreement "vindicates the European approach," by requiring countries to submit "their nationally determined contributions [...] at least 9 to 12 months in advance of the relevant meeting of the Conference of the Parties serving as the meeting of the Parties to the Paris Agreement."[37] Many of the pledged contributions specify technology and investments: India, China, Indonesia, and Brazil do as the European Union and specify targets for renewable energies, while Canada and the U.S. have made promises to regulate brown investments.[38] The Paris agreement is also more likely to succeed than the Kyoto Protocol because it has a larger number of participants, it resembles a repeated game with its periodic pledge-and-review mechanism, and it enhances on transparency by requiring similar reporting methods for all parties.[39] Our theory can thus rationalize the evolution of climate policies from the failing Kyoto Protocol to the praised Paris Agreement with these characteristics.

The fact that our basic results are consistent with this evolution makes it worthwhile

[36] On investment targets, see ec.europa.eu/energy/en/topics/renewable-energy/national-action-plans. On emissions targets, see ec.europa.eu/clima/policies/effort/framework_en.

[37] See "EU approach to the Paris climate conference", retrieved from www.europarl.europa.eu/RegData/etudes/ATAG/2015/569005/EPRS_ATA(2015)569005_EN.pdf and unfccc.int/resource/docs/2015/cop21/eng/l09r01.pdf.

[38] On national climate plans, see www.wemeanbusinesscoalition.org/sites/default/files/The-Paris-Agreement.pdf.

[39] On the main features of the Paris Agreement as compared to the Kyoto Protocol, see www.europarl.europa.eu/RegData/etudes/BRIE/2016/573910/EPRS_BRI(2016)573910_EN.pdf.

to also consider our normative findings. For example, we have derived a large number of comparative statics. When countries are heterogeneous, it is particularly countries that are small or reluctant to cooperate (because their environmental harm is relatively small, for instance) that are most tempted to pollute and who ought to invest the most in green technology, or the least in adaptation and brown technology. With uncertainty, such investments are important to reduce the necessary punishment period or the likelihood that the punishment is triggered. To motivate the large green investments, it may also be necessary to strengthen intellectual property rights, and to offer larger investment subsidies when the countries are tempted to repeal high carbon taxes.

Our simple workhorse model is sufficiently tractable to be extended in many directions. So far, we have abstracting away from payoff-relevant stocks of pollution or technologies. We have focused exclusively on the best subgame-perfect equilibrium, although the actual transition toward such a treaty appears to be characterized by high transaction costs and wars of attritions. By focusing on the best subgame-perfect equilibrium, we have also ignored the possibility of opting out of the negotiations at the beginning of the game. When countries are heterogeneous, it may actually be optimal—even for the countries that cooperate—to exclude certain reluctant countries, since these countries may, with some chance, cheat and thus trigger a costly and long-lasting punishment phase. One of the goals with this project has been to provide a tractable workhorse model that can be further developed along these lines.

7 Appendix

The proofs of the basic results, i.e., Propositions 2 and 3, allow for time-varying parameters.

Proofs of Propositions 0-1.

These proofs are in the text.

Proof of Proposition 2.

The Pareto-optimal SPE satisfying $g_i = \underline{g}$ for each $i \in N$ requires r_i^t to be the closest to $r_i^{*,t}$ (because u_i^t is concave and single-peaked in r_i^t) with the compliance constraints at both the investment and the emission stages being satisfied. Formally, such a Pareto-optimal SPE solves:

$$\max_{r_i^t} \; u_i^t = b_i^t \left(\underline{g}, r_i^t\right) - h_i^t c \left(r_i^t\right) n^t \underline{g} - k_i^t r_i^t \quad \text{s.t.}$$

$$v_i^t - v_i^{b,t} \geq 0, \tag{CC$_i^{r,t}$}$$

$$\Delta_i^t \equiv -b_i^t \left(\overline{g}, r_i^t\right) + b_i^t \left(\underline{g}, r_i^t\right) + s_i^t h_i^t c \left(r_i^t\right) \left(\overline{g} - \underline{g}\right) + \frac{\delta\left(v_i^{t+1} - v_i^{b,t+1}\right)}{1-\delta} \geq 0. \tag{CC$_i^{g,t}$}$$

(i) Since $v_i^t \geq v_i^{b,t}$ at $r_i^{*,t}$, both constraints hold if δ is close to 1. At $r_i^{*,t}$, u_i^t and (CC$_i^{r,t}$) do not change when δ falls, but (CC$_i^{g,t}$) will eventually bind because $b_i^t \left(\overline{g}, r_i^t\right) - b_i^t \left(\underline{g}, r_i^t\right) - s_i^t h_i^t c \left(r_i^t\right) \left(\overline{g} - \underline{g}\right) > 0$, which holds under the assumption that countries' emission decisions constitute a prisoner's dilemma. A binding (CC$_i^{g,t}$) defines $\widehat{\delta}_i^t \left(r_i^t\right)$ implicitly, and if $\delta \geq \overline{\delta}_i^t \equiv \widehat{\delta}_i^t \left(r_i^{*,t}\right)$, $r_i^{*,t}$ satisfies both constraints.

(ii) As soon as δ declines below the level $\overline{\delta}_i^t$, (CC$_i^{g,t}$) would be violated at $r_i^{*,t}$. However, for any given v_i^{t+1},

$$\Delta_{i,r}^t = -\left(\overline{g} - \underline{g}\right) \left[\frac{\partial}{\partial r_i^t} \frac{b_i^t \left(\overline{g}, r_i^t\right) - b_i^t \left(\underline{g}, r_i^t\right)}{\overline{g} - \underline{g}} - s_i^t h_i^t c' \left(r_i^t\right)\right],$$

which is positive under (G$_i$) and negative under (NG$_i$), implying that for (CC$_i^{g,t}$) to continue to hold when δ declines below the level $\overline{\delta}_i^t$, r_i^t must increase above $r_i^{*,t}$ under (G$_i$), but decrease below $r_i^{*,t}$ under (NG$_i$). Since the Pareto-optimal r_i^t is the investment level that is closest to $r_i^{*,t}$ satisfying (CC$_i^{g,t}$), we can write $r_i^t = \min \widehat{\delta}_i^{t-1} (\delta)$ under (G$_i$) and $r_i^t = \max \widehat{\delta}_i^{t-1} (\delta)$ under (NG$_i$). The operators min and max reflects the fact that there can be a range of r_i^t's satisfying (G$_i$), and there may be multiple r_i^t's such that (G$_i$) binds. As δ declines further, (CC$_i^{g,t}$) is satisfied only if the distortion $\left|r_i^t - r_i^{*,t}\right|$ increases more. When δ is smaller than some threshold $\underline{\delta}_i^t \in \left(0, \overline{\delta}_i^t\right)$, $\left|r_i^t - r_i^{*,t}\right|$ is so large that $v_i^t < v_i^{b,t}$. Thus, for $\delta < \underline{\delta}_i^t$, no SPE satisfies $g_i = \underline{g}$ for every $i \in N$.

Stationarity: The above proof took v_i^{t+1} as exogenously given, without taking into account that v_i^{t+1} is itself a function of the equilibrium investment level. For the case with constant parameters, we now show that the best equilibrium is stationary. Since equilibrium investments are closer to the first-best level when v_i^{t+1} is large, we can write $v_i^t = \varkappa\left(v_i^{t+1}\right)$ as an increasing function of v_i^{t+1}. If $\varkappa'(\cdot) < 1$, there is a unique fixed point which we converge to when iterating from any t and v_i^{t+1} to earlier dates. To see that $\varkappa'(\cdot) < 1$ when $\delta \in [\max_j \underline{\delta}_j, \overline{\delta}_i^t)$, note that

$$\varkappa'\left(v_i^{t+1}\right) = (1-\delta)\frac{\partial u_i^t}{\partial r_i^t}\frac{\partial r_i^t}{\partial v_i^{t+1}} + \delta\frac{\partial v_i^{t+1}}{\partial v_i^{t+1}} \approx \delta < 1$$

when $r_i^t \approx r_i^{*,t}$, implying $\partial u_i^t/\partial r_i^t \approx 0$. *QED*

Proof of Proposition 3.

If $\delta < \overline{\delta}_i^t$, the Pareto-optimal SPE satisfying $g_i = \underline{g}$ for each $i \in N$ ensures that r_i^t solves $\widehat{\delta}_i^t(r_i^t) = \delta$, so that ($\mathrm{CC}_i^{g,t}$) binds. As long as ($\mathrm{CC}_i^{g,t}$) binds, we can differentiate the left-hand side of ($\mathrm{CC}_i^{g,t}$) to learn how r_i^t must change with the other parameters at time t. Formally,

$$\begin{aligned}
\frac{\partial r_i^t}{\partial h_i^t} &= -\frac{\Delta_{i,h}^t}{\Delta_{i,r}^t} = -\frac{s_i^t c(r_i^t)\left(\overline{g}-\underline{g}\right)}{\Delta_{i,r}^t} \text{ and} \\
\frac{\partial r_i^t}{\partial s_i^t} &= -\frac{\Delta_{i,s}^t}{\Delta_{i,r}^t} = -\frac{h_i^t c(r_i^t)\left(\overline{g}-\underline{g}\right)}{\Delta_{i,r}^t}, \text{ where} \\
\Delta_{i,r}^t &= \partial\left[-b_i^t\left(\overline{g}, r_i^t\right) + b_i^t\left(\underline{g}, r_i^t\right)\right]/\partial r_i^t + s_i^t h_i^t\left(\overline{g}-\underline{g}\right)c'\left(r_i^t\right).
\end{aligned} \tag{7}$$

Thus, $\partial r_i^t/\partial h_i^t$ and $\partial r_i^t/\partial s_i^t$ are both negative if $\Delta_{i,r}^t > 0$, and positive otherwise. If we write $b_i^t(\cdot) = \theta_i^t b(\cdot)$, where θ_i^t is a parameter that takes a high value, for example, in recession, then we can show that at such time countries ought to invest more in clean technologies, and less in adaptation or brown technologies, for compliance to be credible:

$$\frac{\partial r_i^t}{\partial \theta_i^t} = -\frac{b\left(\overline{g}, r_i^t\right) - b\left(\underline{g}, r_i^t\right)}{\Delta_{i,r}^t}.$$

If we differentiate Δ_i^t with respect to k_i^t, n^t, and r_i^t, we clearly get $\partial r_i^t/\partial k_i^t = \partial r_i^t/\partial n^t = 0$. The explanation for $\partial r_i^t/\partial k_i^t = 0$, for example, is that r_i^t must be set to satisfy ($\mathrm{CC}_i^{g,t}$). But, the investment cost at that time is sunk and thus irrelevant at the emission stage in period t. Of course, a larger k_i^t changes $v_i^{t'}$ and $v_i^{b,t'}$ with $t' \leq t$, and therefore the compliance constraints at the earlier periods. Similarly, changes in h_i^t or s_i^t will influence earlier investments as well as r_i^t. To illustrate the total effects without unnecessary notations we henceforth assume that all parameters are time-invariant, as in the model, and we thus skip the subscripts t.

Stationary parameters: We must now take into account the effects on the continuation value. Differentiating the left-hand side of (CC_i^g) with respect to r_i yields:

$$\Delta_{i,r} = \frac{\partial}{\partial r_i}\left[-b_i\left(\overline{g}, r_i\right) + b_i\left(\underline{g}, r_i\right)\right] + s_i h_i\left(\overline{g} - \underline{g}\right) c'\left(r_i\right) + \frac{\delta}{1-\delta}\frac{\partial u_i}{\partial r_i}.$$

Under (G), $\Delta_{i,r} > 0$ at r_i^* and at any $r_i > r_i^*$ that weakens (CC_i^g), i.e., for any $\delta > \underline{\delta}_i$. Similarly, under (NG), $\Delta_{i,r} < 0$ at r_i^* and at any $r_i < r_i^*$ that weakens (CC_i^g), i.e., for any $\delta > \underline{\delta}_i$.

(i) *Effect of* k_i: The compliance constraint (CC_i^g) depends on k_i because:

$$v_i - v_i^b = -\left[b_i\left(\overline{g}, r_i^b\right) - b_i\left(\underline{g}, r_i\right)\right] + nh_i\left[c\left(r_i^b\right)\overline{g} - c\left(r_i\right)\underline{g}\right] - k_i\left[r_i - r_i^b\right].$$

Suppose, by contradiction, that r_i does not change in k_i. Then, $\partial u_i/\partial k_i = -r_i$. Furthermore, from the Envelope theorem, $\partial u_i^b/\partial k_i = -r_i^b$. Thus:

$$\frac{dr_i}{dk_i} = -\frac{\Delta_{i,k}}{\Delta_{i,r}} = \frac{\delta}{1-\delta}\frac{r_i - r_i^b}{\Delta_{i,r}}. \tag{8}$$

To see the sign of $r_i - r_i^b$, assume, to prove the claim by contradiction, that r_i remains at r_i^*. If so, all investment levels are given by the first-order condition:

$$\frac{\partial b_i\left(g, r_i\right)}{\partial r_i} - h_i c'\left(r_i\right) ng = k_i,$$

which we can differentiate to get

$$\frac{dr_i}{dg} = \frac{-\frac{\partial^2 b_i(g,r_i)}{\partial r_i \partial g} + h_i c'\left(r_i\right) n}{\frac{\partial^2 b_i(g,r_i)}{\left(\partial r_i\right)^2} - h_i c''\left(r_i\right) gn}, \tag{9}$$

where the denominator is simply the second-order condition with respect to r_i, which must be negative. Since g is discrete, we have:

$$r_i^* - r_i^b = \int_{\underline{g}}^{\overline{g}} \frac{-\frac{\partial^2 b_i(g,r_i)}{\partial r_i \partial g} + h_i c'\left(r_i\right) n}{\frac{\partial^2 b_i(g,r_i)}{\left(\partial r_i\right)^2} - h_i c''\left(r_i\right) gn} dg. \tag{10}$$

Thus, for adaptation and brown technologies, $r_i^* < r_i^b$. For such technologies we also have $r_i \leq r_i^* < r_i^b$ and $\Delta_{i,r} < 0$, so from (8) we have that $dr_i/dk_i > 0$. For clean technologies, Eq. (10) gives $r_i^* > r_i^b$. For such technologies, we also have $r_i \geq r_i^* > r_i^b$ and $\Delta_{i,r} > 0$, so from (8) we again have that $dr_i/dk_i > 0$.

(ii) *Effect of* s_i *and* δ: The effect of s_i on r_i is exactly as Eq. (7) for the case with time-varying parameters. It is trivial to see that Δ_i in (CC_i^g) increases in δ. Since r_i^* does not depend on s_i or δ, the distortion $|r_i - r_i^*|$ increases when s_i and δ decrease.

(iii) *Effect of* n: Consider first the case of brown or clean technologies, where $c'(r_i) = 0$. In this case, $\partial\left(v_i - v_i^b\right)/\partial n = \left(\overline{g} - \underline{g}\right) h_i c > 0$, so

$$\frac{dr_i}{dn} = -\frac{\Delta_{i,n}}{\Delta_{i,r}} = -\frac{\delta}{1-\delta}\frac{\left(\overline{g} - \underline{g}\right) h_i c}{\Delta_{i,r}},$$

which has the opposite sign of $\Delta_{i,r}$. Consider next adaptation technologies, where $c'(\cdot) < 0$ but $\partial b_i(\cdot)/\partial r_i = 0$. Suppose, by contradiction, that r_i were invariant in n. Then, we would have $\partial\left(v_i - v_i^b\right)/\partial n = h_i\left[c\left(r_i^b\right)\overline{g} - c(r_i)\underline{g}\right]$, but r_i and r_i^b may differ since $r_i^*(g)$ depends on g. If $c(r_i^*(g))\,g$ increased in g, we would have $c\left(r_i^b\right)\overline{g} > c(r_i^*)\,\underline{g}$. Using Eq. (9), we can show that $c(r_i^*(g))\,g$ increases in g if:

$$c\left(r_i^*(g)\right) - \frac{\left[c'\left(r_i^*(g)\right)\right]^2}{c''\left(r_i^*(g)\right)} > 0. \tag{11}$$

So, under (11), $\partial\left(v_i - v_i^b\right)/\partial n$ and $\Delta_{i,n}$ would be positive for r_i close to r_i^* and, then, $dr_i/dn > 0$.

Effect of h_i: This effect is derived in a similar way. For adaptation technologies, if r_i were invariant in h_i, we would have $\partial\left(v_i - v_i^b\right)/\partial h_i = n\left[c\left(r_i^b\right)\overline{g} - c(r_i)\underline{g}\right]$, which is, as before, positive under condition (11). Then, $\Delta_{i,h} > 0$ and, therefore, $dr_i/dh_i > 0$ for adaptation technologies. For brown technologies we have $\Delta_{i,h} > 0$ and $dr_i/dh_i > 0$ while for clean technologies we have $\Delta_{i,h} < 0$ and $dr_i/dh_i < 0$. *QED*

Proof of Proposition 4.

In cooperation phase, each country i's continuation value is:

$$v_i = (1-\delta)\left[b\left(\underline{g}, r_i\right) - hc(r_i)\,n\underline{g} - kr_i\right] + \delta\left[(1-q)\,v_i + q\left(\left(1-\delta^T\right)v^b + \delta^T v_i\right)\right]. \tag{12}$$

The country cooperates at the emission stage if

$$v_i \geq (1-\delta)\left[b(\bar{g}, r_i) - hc(r_i)\left((n-1)\,\underline{g} + \bar{g}\right) - kr_i\right] + \delta\left[(1-p)\,v_i + p\left(\left(1-\delta^T\right)v^b + \delta^T v_i\right)\right],$$

which gives equation (CC_F^g) in the text. Inserting the expression for v_i obtained from a binding (CC_F^g) into Eq. (12) and simplifying yields Eq. (5).

(i) Let $\tilde{r}$ be the solution of the first-order condition when maximizing (5) with respect to r:

$$0 = \frac{\partial b\left(\underline{g}, \tilde{r}\right)}{\partial r_i} - hc'(\tilde{r})\,n\underline{g} - k - \frac{q}{p-q}\left(\frac{\partial}{\partial r}\left[b(\bar{g}, \tilde{r}) - b\left(\underline{g}, \tilde{r}\right)\right] - hc'(\tilde{r})\left(\bar{g} - \underline{g}\right)\right). \tag{13}$$

We assume that the following second-order condition holds:

$$\frac{\partial^2 b\left(\underline{g},\tilde{r}\right)}{\left(\partial r_i\right)^2} - hc''\left(\tilde{r}\right) n\underline{g} - \frac{q}{p-q}\left(\frac{\partial^2}{\left(\partial r\right)^2}\left[b\left(\bar{g},\tilde{r}\right) - b\left(\underline{g},\tilde{r}\right)\right] - hc'\left(\tilde{r}\right)\left(\bar{g}-\underline{g}\right)\right) < 0.$$

Eq. (13) gives $\tilde{r} < r^*$ under (NG) and $\tilde{r} > r^*$ under (G) if $q > 0$. With $r_i = \tilde{r}$, a binding (CC_F^g) implicitly defines $T(\delta)$. Differentiating $T(\delta)$ with respect to δ yields $T'(\delta) < 0$. Let $\overline{\delta}$ be the minimum δ satisfying (CC_F^g) when $r_i = \tilde{r}$ and requiring $T = \infty$. Then, for any $\delta \geq \overline{\delta}$, the Pareto-optimal PPE in which everyone emit little is characterized by $r_i = \tilde{r}$ and $T = T(\delta)$.

(ii) When $\delta < \overline{\delta}$, the punishment period is infinite and (CC_F^g) can be written as

$$\Delta \equiv b\left(\underline{g},r_i\right) - hc\left(r_i\right) n\underline{g} - kr_i - v^b - \frac{1-\delta\left(1-q\right)}{\delta\left(p-q\right)}\left(b\left(\bar{g},r_i\right) - b\left(\underline{g},r_i\right) - hc\left(r_i\right)\left(\bar{g}-\underline{g}\right)\right) \geq 0.$$

The optimal investment r is now pinned down from $\Delta = 0$. The derivatives of r with respect to δ and $\widehat{g}$ are

$$\begin{aligned}
\frac{dr}{d\delta} &= -\frac{\Delta_\delta}{\Delta_r} = -\frac{b\left(\bar{g},r\right) - b\left(\underline{g},r\right) - hc\left(r\right)\left(\bar{g}-\underline{g}\right)}{\Delta_r} \text{ and} \\
\frac{dr}{d\widehat{g}} &= -\frac{\Delta_{\widehat{g}}}{\Delta_r} = -\frac{-\frac{\partial}{\partial \widehat{g}}\left[\frac{1-\delta F\left(\widehat{g}-n\underline{g}\right)}{\delta\left(F\left(\widehat{g}-n\underline{g}\right) - F\left(\widehat{g}-n\underline{g}-\left(\bar{g}-\underline{g}\right)\right)\right)}\right]}{\Delta_r}, \text{ where} \\
\Delta_r &= \frac{\partial b\left(\underline{g},r\right)}{\partial r} - hc'\left(r\right) n\underline{g} - k - \frac{1-\delta\left(1-q\right)}{\delta\left(p-q\right)}\left(\frac{\partial}{\partial r}\left[b\left(\bar{g},r\right) - b\left(\underline{g},r\right)\right] - hc'\left(r\right)\left(\bar{g}-\underline{g}\right)\right).
\end{aligned}$$

Note that $p - q$ is nondecreasing in $\widehat{g}$ since the monotone likelihood ratio property holds, so $\Delta_{\widehat{g}}$ is positive. For r close to $\tilde{r}$, the term Δ_r is positive under (G) and negative under (NG). Hence, the derivatives $dr/d\delta$ and $dr/d\widehat{g}$ have the opposite sign of Δ_r. The optimal $\widehat{g}$ is determined by maximizing (5), namely

$$\begin{aligned}
&\frac{\partial}{\partial \widehat{g}}\left[\frac{1-F\left(\widehat{g}-n\underline{g}\right)}{F\left(\widehat{g}-n\underline{g}\right) - F\left(\widehat{g}-n\underline{g}-\left(\bar{g}-\underline{g}\right)\right)}\right] \qquad (14) \\
=&\frac{1}{b\left(\bar{g},r\right) - b\left(\underline{g},r\right) - hc\left(r\right)\left(\bar{g}-\underline{g}\right)}\left[\begin{array}{c}\left(\frac{\partial}{\partial r} b\left(\underline{g},r\right) - hc'\left(r\right) n\underline{g} - k\right) \\ -\frac{q}{p-q}\left(\frac{\partial}{\partial r}\left(b\left(\bar{g},r\right) - b\left(\underline{g},r\right)\right) - hc'\left(r\right)\left(\bar{g}-\underline{g}\right)\right)\end{array}\right]\frac{dr}{d\widehat{g}}.
\end{aligned}$$

For $\delta < \overline{\delta}$, $dr/d\widehat{g} < 0$ and $r > \tilde{r}$ under (G) and $dr/d\widehat{g} > 0$ and $r < \tilde{r}$ under (NG). Hence, the right-hand side of (14) is positive. As δ declines, (CC_F^g) can be satisfied only if the distortion $|r - r^*|$ and therefore the right-hand side of (14) increase. But the optimal $\widehat{g}$ must then increase as well, requiring a smaller increase in $|r - r^*|$. Eventually, δ becomes so small that either (i) the compliance constraint at the investment stage is violated,

(ii) v_i is reduced so much that (CC_F^g) is violated for any level of r, or (iii) δ reaches zero. We let $\underline{\delta}$ measure the maximum of these three thresholds. Clearly, $\underline{\delta} \in \left[0, \overline{\delta}\right)$. *QED*

Proof of Proposition 5.

This proof is in the text. Our working paper version (Harstad et al., 2015) includes proofs of other more detailed results for this extension.

Proof of Proposition 6.

With technological spillover, the Pareto-optimal SPE satisfying $g_i = \underline{g}$ for every $i \in N$ and sustained by a threat of reversion to BAU solves:

$$\max_{r_i} u_i = b\left(\underline{g}, z_i\right) - hc\left(z_i\right) n\underline{g} - kr_i,$$

where $z_i \equiv (1-e)\, r_i + \frac{e}{n-1}\sum_{j \neq i} r_j$, s.t.

$$\frac{v_i - v^b}{1-\delta} - \frac{e}{1-e} k\left(r_i - r^b\right) \geq 0, \tag{CC_e^r}$$

$$-b\left(\overline{g}, z_i\right) + b\left(\underline{g}, z_i\right) + shc\left(z_i\right)\left(\overline{g} - \underline{g}\right) + \frac{\delta\left(v_i - v^b\right)}{1-\delta} \geq 0. \tag{CC^g}$$

Constraint (CC_e^r) is clearly satisfied when $e = 0$ or $r_i \leq r^b$.

(i) Since $v_i > v^b$ at r^*, conditions (CC_e^r) and (CC^g) hold when δ is close to one. Binding (CC_e^r) and (CC^g) implicitly define the thresholds for discount factor $\widehat{\delta}^r\left(r_i\right)$ and $\widehat{\delta}^g\left(r_i\right)$ such that if $\delta \geq \max\left\{\overline{\delta}^g, \overline{\delta}^r\right\}$ with $\overline{\delta}^g \equiv \widehat{\delta}^g\left(r^*\right)$ and $\overline{\delta}^r \equiv \widehat{\delta}^r\left(r^*\right)$, r^* satisfies both compliance constraints.

(ii) Consider now the case with $\delta < \max\left\{\overline{\delta}^g, \overline{\delta}^r\right\}$. Under (G), $r^* > r^b$ for any e. So, (CC^g) binds first, i.e., $\overline{\delta}^g \geq \overline{\delta}^r$, if

$$b\left(\overline{g}, r^b\right) - hc\left(r^b\right) n\overline{g} - kr^b + k\frac{1}{1-e}\left(r^* - er^b\right) \leq b\left(\overline{g}, r^*\right) - hc\left(r^*\right)\left(\overline{g} + (n-1)\,\underline{g}\right). \tag{15}$$

The right-hand side of (15) does not vary with e, while the left-hand side is increasing in e. Hence, $\overline{\delta}^g \geq \overline{\delta}^r$ if and only if $e \leq \overline{e}$, where $\overline{e}$ is pinned down from condition (15) holding with equality. Like in the basic model with $e = 0$, $r_i \geq r^g(\delta) \equiv \widehat{\delta}^{g-1}\left(r_i\right) > r^*$ with $r^g(\delta)$ decreasing in δ for $\delta < \overline{\delta}^g$. When $e > \overline{e}$, constraint (CC_e^r) binds first, i.e., $\overline{\delta}^r > \overline{\delta}^g$, and $r_i \leq r^r(\delta) \equiv \widehat{\delta}^{r-1}\left(r_i\right)$ with $r^r(\delta)$ increasing in δ for $\delta < \overline{\delta}^r$. The optimal investment r then solves

$$\max_{r_i} v_i + \eta\left(v_i - v^b - \frac{k\,(1-\delta)\, e}{1-e}\left(r_i - r_i^b\right)\right),$$

where $\eta > 0$ is the shadow value of satisfying a strictly binding (CC_e^r). The first-order condition for an interior r_i is:

$$0 = (1+\eta)\frac{\partial v_i}{\partial r_i} - \eta \frac{k(1-\delta)e}{1-e}. \tag{16}$$

The second-order condition is satisfied for r_i close to r^* since the countries' per capita utility is concave. Eq. (16) implies that $r^r(\delta) < r^*$. Moreover, as δ declines, (CC$_e^r$) can be satisfied only if the distortion $|r - r^*|$ increases. Under (NG), if e is smaller than $\overline{e}$ and $r^* > r^b$ or e is so small that $r^* \leq r^b$, (CC^g) binds first, i.e., $\overline{\delta}^g \geq \overline{\delta}^r$, and $r_i \leq r^g(\delta) < r^*$ with $r^g(\delta)$ increasing in δ for $\delta < \overline{\delta}^g$. When $r^* > r^b$ and $e > \overline{e}$, constraint (CC$_e^r$) binds first, i.e., $\overline{\delta}^r > \overline{\delta}^g$, and $r_i \leq r^r(\delta) \equiv \widehat{\delta}^{r-1}(r) < r^*$ with $r^r(\delta)$ increasing in δ for $\delta < \overline{\delta}^r$. In both cases, as δ declines further, the binding constraint can be satisfied only if the distortion $|r - r^*|$ increases. Finally, no r can enforce compliance with the agreement if δ is so small that either (i) (CC^g) is violated, (ii) v_i is reduced so much that (CC$_e^r$) is violated for any level of r, or (iii) δ reaches zero. We let $\underline{\delta}$ measure the maximum of these three thresholds. Clearly, $\underline{\delta} \in \left[0, \overline{\delta}\right)$. *QED*

Proof of Propositions 7.

Let $d_i \equiv \overline{y} - g_i - r_i$ be the decrease in consumption relative to the bliss level $\overline{y}$. The first-best emission (and consumption) level is then simply given by $d^* = cn/B$, while, in BAU, $d^b = c/B$. We can write a country's continuation value as

$$v_i = d_i(nc - Bd_i/2) + r_i(nc - Kr_i/2) - cn\overline{y}.$$

(i) The compliance constraint at the investment stage can be written as:

$$\tfrac{v_i}{1-\delta} \geq (n-1)c\left(r_i - r^b\right) + \tfrac{v^b}{1-\delta} \qquad \text{(CC}_c^r\text{)}$$

$$\Rightarrow \delta \geq \widehat{\delta}^r(g_i, r_i) \equiv 1 - \tfrac{-cn\overline{y} + d_i(nc - Bd_i/2) + r_i(nc - Kr_i/2) - v^b}{(n-1)c\left(r_i - r^b\right)}, \text{ so}$$

$$\overline{\delta}^r \equiv \widehat{\delta}^r(g^*, r^*) = 1 - \tfrac{(n-1)^2(1/2B + 1/2K)}{(n-1)^2/K} = \tfrac{B-K}{2B}.$$

The compliance constraint at the emission stage can be written as:

$$\tfrac{v_i}{1-\delta} \geq nc\left(r_i - r^b\right) - \tfrac{K}{2}\left(r_i^2 - \left(r^b\right)^2\right) + c(n-1)\left(d_i - d^b\right) + \tfrac{v^b}{1-\delta} \qquad \text{(CC}_c^g\text{)}$$

$$\Rightarrow \delta \geq \widehat{\delta}^g(g_i, r_i) \equiv 1 - \tfrac{-cn\overline{y} + d_i(nc - Bd_i/2) + r_i(nc - Kr_i/2) - v^b}{nc\left(r_i - r^b\right) - \frac{K}{2}\left(r_i^2 - \left(r^b\right)^2\right) + c(n-1)\left(d_i - d^b\right)}, \text{ so}$$

$$\overline{\delta}^g \equiv \widehat{\delta}^g(g^*, r^*) = 1 - \tfrac{(n-1)^2(1/2B + 1/2K)}{(n-1)^2/2K + (n-1)^2/B} = \tfrac{K}{B+2K}.$$

(ii) Clearly, the emission stage compliance constraint will bind first if $K/B > 1/2$, i.e., $\overline{\delta}^g > \overline{\delta}^r$. If only (CC$_c^g$) binds and, thus, $\delta \in \left(\widehat{\delta}^r(g_i, r_i), \overline{\delta}^g\right)$, $r_i = r^*$ is both maximizing v_i and weakening (CC_c^g). Given this r^*, the optimal d is the largest d_i satisfying (CC_c^g). Substituting for v_i and then solving (CC_c^g) for the largest d_i, we get:

$$\begin{aligned}
dB &= nc - \phi(\delta), \text{ where} \\
\phi(\delta) &\equiv c(n-1)\left[1-\delta-\sqrt{\delta^2+\delta B/K}\right] \\
&= c(n-1)\left[1-\delta-\sqrt{(1-\delta)^2-\left(\overline{\delta}^g-\delta\right)/\overline{\delta}^g}\right],
\end{aligned}$$

where $\phi(\delta)$ decreases from $c(n-1)$ to 0 as δ increases from 0 to $\overline{\delta}^g$. The threshold $\underline{\delta}$ is defined implicitly by requiring (CC_c^r) to hold with equality at $\underline{\delta}$.

(iii) If instead $\overline{\delta}^g < \overline{\delta}^r$ and $\delta \in \left(\widehat{\delta}^g(g_i, r_i), \overline{\delta}^r\right)$, the level $d^* = cn/B$ is both maximizing v_i and weakening (CC_c^r). The optimal r is then given by the largest r_i satisfying (CC_c^r). If we substitute for v_i in (CC_c^r) and solve for the largest r_i, we get from $\delta = \widehat{\delta}^r(g_i, r_i)$:

$$\begin{aligned}
r &= \frac{nc}{K} - \frac{\psi(\delta)}{K}, \text{ so} \\
g &= \overline{y} - d^* - r = \overline{y} - \frac{cn}{B} - \frac{nc}{K} + \frac{\psi(\delta)}{K}, \text{ where} \\
\psi(\delta) &\equiv c(n-1)\left[1-\delta-\sqrt{\delta^2+K/B}\right].
\end{aligned}$$

QED

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