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COOPERATION BETWEEN POLITICIANS AND ECONOMETRICIANS ON THE FORMALIZATION OF POLITICAL PREFERENCES

By

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Preface

Celebrating its 300th anniversary in 1968 the Bank of Sweden (Sveriges Riksbank) decided to set up a prize in Economics.

This prize was established in recognition of the status enjoyed by the Nobel Prizes, and in memory of Alfred Nobel, who instituted them. As a result, Economics is now equated with the Natural Sciences and Literature in the annual awarding of the Nobel Prizes. The first winners, Professor Ragnar Frisch, Oslo, and Professor Jan Tinbergen, Haag, received the prize on December 10, 1969.

The Federation of Swedish Industries intends to make it a tradition to invite the prize-winners to give a lecture on an optional subject. These lectures will subsequently be published.

The first in this planned series of lectures, Professor Tinbergen's "On the International Division of Labour", was published last year. The second one, Professor Frisch's "Cooperation between Politicians and Econometricians on the Formalization of Political Preferences" is presented in this booklet.

How can government rank priorities as to economic policy goals in a systematic and conscious way and choose among the most efficient tools to achieve these goals? Professor Frisch asks himself this question and develops a method of doing this ranking by numerically establishing a preference function and applying it to decision making situations.

The Federation of Swedish Industries hopes that the publication of professor Frisch's lecture will stimulate discussion about the methods of decision making in society.

Axel Iveroth
Director General
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Stockholm, September 1971

Introduction

The purpose of this paper is to make a plea for a new type of cooperation between politicians and econometricians. The new type of cooperation consists in formalizing the *preference function* which must underlie the very concept of an *optimal* economic policy. A preference function is simply a function of some of the variables that enter into a description of the economy, the function being such that the *maximazation* of it can be looked upon as the definition of the goal to be obtained by the economic policy.

How can we reach an expression for the numerical character of this function? And how can it be applied in practice?

It is my firm conviction that an approach to economic policy through a preference function contains the key to a *much needed reform* of the methods of decision making in society at large in the world of today.

On the one hand we are today facing crucial *environmental factors* which previously were—and could be—almost completely neglected. A whole spectrum of production processes, steered more or less exclusively by pecuniary gains, today create enormous quantities of *waste* in the form of toxious matter which is left for society to handle. The same applies to the *preservation of nature*, the relief of *city congestion* and a variety of other questions concerning *human welfare*.

On the other hand political discussions today come dangerously close to resembling a dog fight where the *global nature* of and the *interconnections* between the basic questions have a tendency to get lost, and only shouting about striking *partial* aspects of inefficiencies and injustices counts.

All this calls for radical and unconventional thinking about the decision-making machinery in society at large.

The preference function is a tool for defining the goal. Another important problem is to construct a model of the *conditions* (bounds and equations) according to which our striving towards the goal has to proceed. But this latter problem will not be considered in the present paper.

Since I am addressing two very different groups: Politicians and econometricians, the form of the presentation is a difficult question. Some parts of the sequel may perhaps be too technical for the liking of politicians, and other parts too trivial for the liking of econometricians. But this risk I shall have to take.

Before entering upon the technical aspects of this problem it has been found necessary to say something on its general aspects. This I have done by reproducing in section 1 below some passages from my article in "Les Prix Nobel en 1969". The technical aspects that make up the main parts of the present paper were not discussed in that article.

1. Misunderstandings and bacis ideas regarding the preference function

A common misunderstanding regarding the preference function is due to no distinction being made between *targets* (i.e. specific values of some selected variables which one will try to realize), and the use of a *preference function*, and also due to the free not being distinguished from the reduced form of the preference function. It is said that the decision maker at the national level (the responsible political authority) cannot understand the meaning of the core (the equations between and bounds on the variables that depict the economy). This, of course, is true, but it does not apply to the free form of the preference function.

Another misunderstanding we sometimes meet, is this: It is said that there are *many different* systems of preferences. It is impossible to choose between these systems. Therefore the concept of a preference function cannot be used in connection with national models. This is one of the biggest pitfalls in the discussion of this matter. Of course, there are differences of opinion. One social group may have one type of preference and another social group may have other preferences, and different persons may have different preferences, and even the same person may have different preferences at different points of time. All this is, of course, true. But the problem of settling differences of opinion *is not a special problem of econometrics*. It is a general problem of human behaviour and opinions. And there exists a machinery for settling such differences. This machinery is simply the political system of the country. This political system—whatever it may be—has been created for the very purpose of settling such differences. What we have to do as econometricians is to apply this very system for the *formalization* of the preferences to go with our models. Thus the preference function as it appears in our models is an expression of the preferences of the decision-making authority, whatever that authority may be. The preference function in the model must not be confused with a general "Welfare function" in the sense of welfare theory.

It is not our task as econometricians and social engineers to go into a detailed discussion of the political system. Somewhere in the hierarchy of sciences a line of demarcation has to be drawn. And here is where we find the line of demarcation for the econometric planner. As *citizens* we are, of course, allowed to work for any political system we think is just and effective. I, for one, would like to work for a system that really deserves the name democracy, but that is another story.

Still another point must be clarified. Sometimes we hear the suggestion that instead of going into the trouble of discussing preferences, we ought to leave it to the experts to present to the politicians a number of *alternatives* for the development course of the nation's economy, and ask the politicians to choose among these alternatives. This may be a defendable procedure if the number of meaningful alternatives is *very small* and if we can trust the experts not to smuggle their *own personal preferences into the choice of alternatives*. A bad case of such smuggling, is to be found, for instance, in the work of the expert committee on the location of the main airport to be built in southern Norway.

Even if we could trust the experts, the listing of alternatives would be impossible in an advanced form of planning. Indeed, in economic political discussions there is an almost infinite number of specific questions that may be asked. "Should we build a road between points A and B

in the country?", "Should we promote investments that will give employment to many people, or should we on the contrary promote investment that will save labour?", "Should we aim at a high growth rate of the gross national product, or should we put more emphasis on a socially justifiable distribution of it?", "Should we aim above all, at keeping the price level under control?", "Or should we sacrifice the stability of the price level and put more emphasis on the growth of the gross national product (in real terms)?", "Should we sacrifice a part of the growth of the total gross national product in order to be able to increase the living standards of one specific social group, say fishermen or industrial workers?", "Should we put more emphasis on factors so far excluded from the statistical concept of the gross national product? For instance, should we try to avoid air-pollution and all kinds of environmental contamination that may be caused by refuse and waste, a problem that must be studied in its *totality* as a problem of *circulation of matter* in society, much in the same way as we study inter-industry relations in an input-output table?" "Should we put an economic value to undisturbed nature?" etc.

If we should ask the experts to produce a list of feasible alternatives for the development course of the economy, a list that would be comprehensive enough to cover only very approximately all these various specific questions, the list of possible courses of development, would have to contain millions and millions of alternatives. The number of alternatives would multiply by cross classification.

Such a list is impossible, for the simple reason that the experts would be physically unable to analyse and present all these alternatives, and even if all the alternatives could be analysed and put before the politicians, they would be absolutely drowned in information. They would not know where to start and where to end in discussing which alternative to choose. In the electronic computing there is a phenomenon known as "information death", which occurs if the mistake is made of letting the computer print out too many intermediate results. The unfortunate politicians would suffer a similar information death if they were presented with a hypothetical list of the millions and millions of feasible courses of development.

In rational economic planning the only way is to have enough patience to start with a discussion of the preference function. To begin with the model would have to be heavily aggregated, but as experience is gained, more details can be included.

Finally, a warning should be given about one very simple (and therefore very popular) procedure. There are many examples of such simple procedures. One of these is as follows: You start by *guessing* at the probable growth rate of gross national product in future years. And from this guess you try to estimate by using input-output analyses, national accounts etc. what the development of the various production sectors, consumption etc. will be. This is unsatisfactory for at least three reasons: (I) The growth rate depends essentially on what *decisions* are made regarding the control of the economy. Guessing at the growth rate, therefore, implies a guess regarding the economic policy to be pursued in the years to come. (II) Even if the growth rate is given, it does *not* necessarily indicate what the development of the various sectors of production or consumption etc. will be. The economy has many more degrees of freedom than just one. (III) How can you assert that the growth rate guessed at is the *optimal* one? The growth rate is indeed not a datum but a *consequence* of an optimal solution, with all the intricacies connected with the determination of that optimum.

A preparatory phase of the expert's work on the preference function would simply consist in his making a systematic use of his general knowledge of the political atmosphere in the country, and in particular the political atmosphere in the party in question to which a constructed preference function would apply. The expert will have formed

an opinion, *a tentative opinion*, about what the preferences of this party would look like if they were formalized in a way that fits in with the expert's model, and is expressed in a language understandable to his electronic computer.

In a subsequent phase the expert — on the basis of this tentative formalization — will work out a system of interview questions by which he will get *closer* to the formalization of the preferences in question.

It is well known that people will not always behave in a given situation exactly in the way they *said* in an interview question that they *would* behave in such and such a situation. But still, I think, it remains that valuable information may be obtained by means of interview questions, provided the questions are wisely formulated in a *conversational manner*, and not simply carried out by some youngster in the opinion poll trade. I have worked out a rather elaborate technique for such conversational interviews to be carried out by econometric experts. And I have had the good fortune of testing this out in conversations with high-ranking politicians both in developing countries and in industrially developed countries. I have found that it is surprising how far one can get in this field when the conversation is wisely steered. Details are discussed in he subsequent sections.

Essential points in this connection are: (I) To use the free form — the "Santa Claus" form — of the preference function. (II) To assure that the interviewed person rids his mind completely of any pre-conceived (and in many cases erroneous) ideas he might have on the nature of the core, and thus disregards whether it is actually possible to *realize* the alternatives involved in the interview questions. (III) To assure that the interviewed person has rid his mind completely of any possibility of trading in the market any of the alternative situations which are hypothetically offered to him in the interview questions. This is the *earmarking* principle. Cf. (8.1) — (8.7) below.

We may take the following as a simple example of an interview question: What would you, politician, choose *if you had the choice* between two packages of economic results, for instance, one package with, say 3% unemployment and an annual inflation rate of 5%, and another package with, say, 10% unemployment and an inflation rate of 1%? By repeating this question, but with different figures involved, it will be relatively easy to reach a situation where the interviewed person says: "It is all the same to me which one of the two packages I receive." This point of indifference is precisely what the expert is driving at.

Similarly for other kinds of comparisons. There will be a whole series of such partial "package questions". From answers to a complete system of such partial questions the expert will be able to build up a preference function in its free form. If he finds it convenient, the expert may subsequently transform this preference function to a reduced form. But this is only a secondary matter.

In a third phase the expert will go back to his electronic computer in which he had already entered the data regarding the core of the economy. To this he will now add the formalization of the preferences in the quantitative form as he now sees it. This will give him a solution, in the form of an optimal development course for the economy. Optimality being defined through the preferences of this party and in the preference formalization which the expert has *now* reached.

When the expert comes back to the politicians with his solution, the politicians will perhaps say: "No, this was not really what we wanted . . . We have to change these particular aspects of your solution."

The expert will understand more or less precisely what sort of changes are needed in the formulation of the preference function in order to produce a solution that comes closer to what the politicians now say they want. This leads to a discussion back and forth. In this way one will work step by step towards a preference formulation such that the politicians can say about the resulting solu-

tion: "All right, this is what we would like to see." Or perhaps the expert will have to end by saying politely: "Your Excellencies, I am sorry but you cannot, at the same time, have all these things on which you insist." Their excellencies, being intelligent persons, will understand the philosophy of the preference questions and the expert's study of the core, and will therefore acquiesce in a solution which is not quite what they like, but at least something better than other alternative lines of the development course which have emerged from the previous tentative formulations of the preference function.

Even if we did not go any further with the formalization of the system of preferences than to work out such an analysis *separately for each political party*, an enormous gain would be achieved in elucidating the economic political discussions.

But we should not stop at this point. We should proceed to a discussion of what sort of *political compromise* which might be reached in the formulation of a unified system of preferences. And then having reached this compromise formulation, there would appear a compromise optimal solution. Here, too, an iteration between politicians and experts would take place.

The principal political authority – in a democratic country it would be the elected Parliament – ought to concentrate *most of its time and efforts* on a discussion of this compromise on the formulation of the system of preferences, instead of using practically all of its time on discussing *one by one* the specific economic measures that may have been proposed, and in each case deciding whether to accept the measure or not. In the way suggested, the Parliament would concentrate its time and energies on *the most important things*, on the really vital issues. If this were done, many details could safely be left to the experts. *Big* issues would of course, finally be checked one by one by means of parliamentary decisions.

2. Some simple examples illustrating the concepts of a preference function, an optimal solution, and the optimal price of a bound

Example 1. Consider the two variables x and y. No need to specify what these two variables may stand for concretely.

Suppose that the magnitudes of these two variables are to be determined by the requirement that the preference function

$$P = 2x + 3y \tag{2.1}$$

is to be made as *large as possible*; with the provisio, however, that x is not to be larger than 50, and y not larger than 70, i.e.

$$x \leq 50 \qquad y \leq 70. \tag{2.2}$$

The preference function (2.1) obviously becomes all the larger, the larger x and y are. The only thing that can stop the increase in x and that in y, is (2.2). Hence the *optimal solution is*:

$$\hat{x} = 50 \qquad \hat{y} = 70 \qquad \hat{P} = 310. \tag{2.3}$$

In this case both the upper bound on x and that on y have been hit in the optimum.

Interesting concepts are the *optimal prices* of the bounds (sometimes called the shadow prices).

The optimal price of the bound on x is simply the *ratio* between the increase in the optimal value of the preference function which would be produced by a small increase in the bound on x, and the corresponding magnitude of this increase in the bound on x. If the small increase in the bound on x is denoted δ, the optimal value of the preference function would be increased by 2δ. Dividing this by δ we find that the optimal price of the bound on x is 2. Similarly for y. Hence, if the optimal prices of the bounds on x and y respectively are denoted p_x and p_y, we get in this case:

$$\hat{p}_x = 2 \qquad \hat{p}_y = 3 \tag{2.4}$$

It is important to note that an optimal price is not a price *of a variable*, but the price of a *bound*.

Example 2. Same as in example 1 but with the supplementary condition that x and y are to be connected by the equation

$$y = \frac{1}{5}x \tag{2.5}$$

The solution in this case is easily illustrated graphically as follows.

The *admissible region* as defined by the *bounds* (2.2) is the non-shaded area in Fig. (2.6). If we add the condition (2.5) the admissible region is reduced to *that* part of the straight line AB in fig. (2.6) that passes through the non-shaded part of the figure.

Since we will increase the value of the preference function by moving North-East along the straight line AB, the optimum is characterized by:

$$\hat{x} = 50 \qquad \hat{y} = 10 \tag{2.7}$$

$$\hat{P} = 2 \text{ times } 50 + 3 \text{ times } 10 = 130.$$

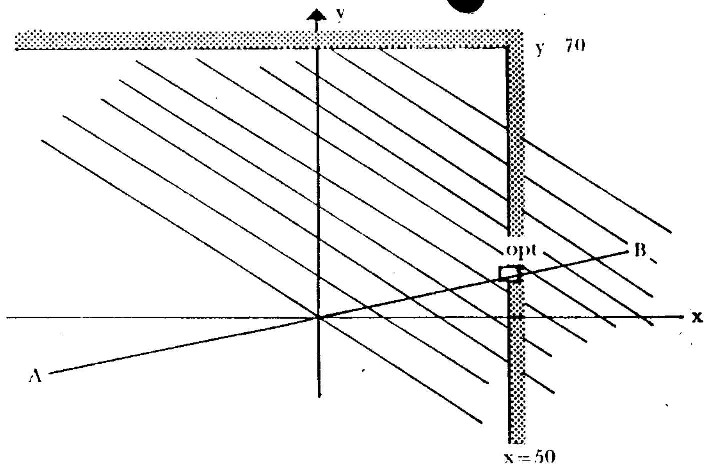

Fig. (2.6).

It is seen that the imposition of the equation (2.5) has seriously reduced the optimal value of the preference function.

In the present case the bound on y, namely 70, *has not been hit* in the optimum. The optimal solution will consequently not be changed if the upper bound on y is *lowered* to, say, 60 or 30 or 20. Only if this bound becomes less than 10, will the optimal solution be changed. It will then be the bound on x that has not been hit in the optimum.

If in Example 2 we add the small magnitude δ to the bound on x, the optimum will become

$$(2.8) \qquad \hat{x} = 50 + \delta \qquad \hat{y} = 10 + \frac{\delta}{5}$$

$$\hat{P} = 130 + 2\delta + \frac{3}{5}\delta = 130 + \frac{13}{5}\delta.$$

The *increase* in the optimal value of the preference function obtained by changing the bound on x is consequently $\frac{13}{5}\delta$,

which, reckoned per unit of δ, is $\frac{13}{5}$.

Since in this case *nothing* is gained by increasing the bound on y, we now have:

$$(2.9) \qquad \hat{p}_x = \frac{13}{5} \qquad \hat{p}_y = 0.$$

It will be seen that in the present case it pays *better dividends* to increase the bound on x than it did in example 1, $\left(\frac{13}{5} > 2\right)$.

The optimal prices are important concepts. The general meaning is that they indicate how much will be gained (in units of the preference function) by *increasing* the various bounds one at a time. The optimal price of an upper bound that has been hit in the optimum is *positive*, the optimal price of a lower bound that has been hit in the optimum is *negative* (increasing a lower bound that has been hit in the optimum means reducing the admissible region). The optimal price on a bound that has not been hit in the optimum is *zero*.

The above extremely simple examples already give a good illustration of what is meant by a preference function, an optimal solution and the optimal price of a bound.

In an actual case with many variables the optimal prices are not actually computed by the above procedure of considering the change produced in the preference function by a change of a bound for each particular variable involved. The actual computations are made in a much more condensed way.

3. Quantification of the characteristics of the economy

Some of the characteristics of the economy are represented by well defined variables in current statistics, such as the Gross National Product, the Visible Trade Balance, the Number of Unemployed etc.

But in other cases a preparatory work may have to be done in order to construct an *index*, by which to measure the phenomena in question. Let me give one example.

An important characteristic of the economy is the *regional inequality of the income distribution.* Many politicians are today greatly concerned about this.

There are many ways in which this kind of inequality can be measured. To make practical progress we have to settle on an index which may not be our ideal, but for which data can be obtained without a prohibitive amount of work. The following is a plausible procedure.

In each of the localities considered we compute the total income which emerges in the form of wages and salaries and income of small independent entrepreneurs (they will nearly always have a manifestly local orientation). All of these income elements should be reckoned after taxes and subsidies. This net income sum should then be expressed as a figure per capita in the locality concerned.

Next we will have to get an index for the *local cost of living* for this locality seen in relation to the cost of living in the country as a whole. This too is a question of considerable concern in economic political discussions today.

This construction of a local cost of living index is a difficult problem (perhaps more difficult than most people believe), but it is essential. In this local cost of living one will have to take account of such things as the disadvantages of living in a crowded area. And on the positive side account must be taken of such advantages as a plentiful supply of special kinds of food (e.g. fresh fish in coastal areas).

The local net income per capita will have to be divided by the local cost of living in order to obtain an expression of the net *real* income per capita in the locality concerned. Let it be denoted $R_i^{\text{net real}}$, where "i" denotes the locality in question. A subscript "o" may be used to indicate the country as a whole.

An index of the inequality discussed might then be taken as the following weighted arithmetic average of deviations, extended over all the localities and with the number of inhabitants N_i (of the three categories considered) used as weights, i.e.

$$(3.0) \qquad \frac{\sum_i N_i \left| R_i^{\text{net real}} - R_o^{\text{net real}} \right|}{N_o \cdot R_o^{\text{net real}}}$$

Here the vertical bars indicate "absolute value of". We could, of couse, instead have used the mean square deviation. This would have put more emphasis on those deviations ($R_i^{\text{net real}} - R_o^{\text{net real}}$) that are *great* in absolute value.

4. Preference variables and ranges

Let x_1, x_2, x_3, . . . etc. be the variables to which we want to apply a preference analysis. We call them the preference variables. And the set of these variables we call the preference set. We may speak of preferencial variables as synonymous with preference variable.

In the type of preference analysis here considered, there is only a *special kind* of variable which will be allowed into the preference set.

(4.1) The main principle for including a variable in the preference set, is that it is associated with an ethical, humanitarian, social, consumptional or justice *evaluation* which people in general can make without being experts on economic model building.

(4.2) As an *exception* to this rule we may in some cases also include in the preference set a variable for which the evaluation depends to some extent on an expert knowledge of the *consequences*, which the variable in question may have on the whole constellation of the economy. The trade balance is an example in point. We may have to acquiesce in the inclusion of such a variable simply in order to avoid making the preference analysis too *complicated.*

But the ideal is *not* to include such variables in the preference set, but leave their preferential aspects

to be taken care of *indirectly* through the effects which their magnitudes may have on the magnitudes of other varibles in the economy (or on the prospects for further development), and hence also on some variables that *are* included directly in the preference set according to the main principle (4.1). Only in this way can we ensure the *comparability* of the preference structure of the man in the street with that of the expert. In a truly democratic society this comparability is very much to be desired.

(4.3) Variables that are *parameters of action* for Government (tax rates, interest rates, rules of various sorts used in controlling the economy) are as a rule *not* to be included into the preference set. Whatever positive or negative opinion one may have about them will emerge *indirectly* through the effects which their varying magnitudes will have on the constellation of other variables depicting the economy.

Only if *the mere act of applying* a Government parameter of action (quite apart from the effects which it may have on the constellation of the other variables) is strongly *disliked*, because of the extra work and trouble involved in its use and the control of evasion may a (negative) "application preference" be attached to it.

If a certain Government parameter is, in itself, only a measurement of *an act of consumption*, it will, of course, have to be included by virtue of the main principle (4.1.). Examples in point are decisions on how much to spend on public health services, or social security measures, on old age pensions, on defence, on general education and research etc. Only the consumption aspect of these measures are to be included as preference variables.

(4.4) Variables of *minor importance* will, of course be excluded for practical reasons. Their inclusion or exclusion would not appreciably influence the location of the optimum in a mathematical programming analysis.

In a brief exposition it is impossible to give a detailed list of the variables that should be included in the preference set, or exact rules for handling limiting cases. But the above rules will be sufficient to indicate the general character of the preferential variables.

For each of the preferential variables a decision should be made about a *range* to be applied to this variable. The range may be larger or smaller according to how *locally specific* one wants to make the preference analysis (using the word "locally" in its geometric and mathematical sense.) Cf. (7.7), (10.6) and (10.7).

The upper and the lower bound of the range should both be sufficiently different to make these endpoints *perceptible* from the standpoint of preferences. But the differences should not diverge to such a degree as to deprive the concept of a bound of realistic meaning. For instance, a gross national product (in constant prices) one hundred times the size of that attained in recent years would be meaningless.

Instead of speaking about the upper and the lower bound we will speak of the *most preferred* and the *most deferred* bound (deferred is the opposite of preferred). For the preferential variable x_ν these two bounds will be denoted x_ν^{pref} and x_ν^{def} respectively. This change in terminology assumes that the preference changes monotonically from the lower to the upper bound.

Of course, x_ν^{pref} may be *smaller* than x_ν^{def} (as for instance in the case x_ν = the unemployment rate) or x_ν^{pref} may be *larger* than x_ν^{def} (as for instance in the case x_ν = total consumption at given price). In all cases it must be remembered that we take all preferences in the Santa Claus sense, without considering the question of how the magni-

tude of x_ν would have to be implemented in practice. Cf. (8.1).

For each preferential variable x_ν the two magnitudes x_ν^{def} and x_ν^{pref} for this variable are to be recorded in a list of the form (4.5)

Table (4.5) *List of preferential variables and ranges*

Ordinal number of the variable.	Person or group interviewed:	Interviewer:		Date:
	Description of the variable whose ordinal number is printed in the first column.	Range of the variable		Central Magnitude
		Most preferred endpoint of the range x_ν^{pref}	Most deferred (i.e. least preferred) endpoint x_ν^{def}	Cf. (4.6) x_ν^{centr}
$\nu = 1$				
2				
3				
etc.				

In the last column in tab. (4.5) we record the *central magnitude* x_ν^{center} defined by:

$$x_\nu^{centr} = \frac{1}{2}(x_\nu^{pref} + x_\nu^{def}) \tag{4.6}$$

5. Cardinality vs. Ordinality

The question of cardinality vs. ordinality is a question that pertains to the *manner* in which the choice indicator for different objects is *expressed*.

This question about the manner of expression may be raised no matter whether the distinctive mark that permits us to distinguish one object from another, is a *measurement*, as it is, for instance, in the case where the "different objects" simply means different magnitudes of a given variable, or if it is any other kind of distinctive mark that permits us to distinguish among the objects, for instance names such as Tom, Dick, Harry etc.

In the following I shall only be concerned with the case where the objects are different magnitudes of a single variable or different sets, each characterized by several magnitudes, as it is when several variables are considered simultaneously. Even in this case of measurable objects, it may, for the sake of brevity, be convenient to speak of "the objects".

Suppose that to each set of magnitude of the variables $(x_1, x_2 \ldots)$ we have a way of associating a *uniquely* determined *number* P. In other words suppose we have defined a single valued function $P(x_1, x_2 \ldots)$ of the variables $(x_1, x_2 \ldots)$.

Suppose that the function P can be taken as a choice indicator, in the sense that any object $(x_1', x_2' \ldots)$ will be preferred for, deferred for or be indifferent to some other object $(x_1'', x_2'' \ldots)$, accordingly as

$$
(5.1) \qquad \begin{array}{l} P(x_1', x_2' \ldots) > P(x_1'', x_2'' \ldots) \text{ or} \\ P(x_1', x_2' \ldots) < P(x_1'', x_2'' \ldots) \text{ or} \\ P(x_1', x_2' \ldots) = P(x_1'', x_2'' \ldots) \end{array}
$$

Then we have a clear example of what is called a *cardinal* preference indication.

This would be the case even if the *zero point* on the P scale is chosen arbitrarily and also if the *unit* of measurement along the P scale is chosen arbitrarily. Take, for instance, a Celsius (Centigrade) thermometer and a Fahrenheit thermometer. They are, in principle, equally good for measuring temperatures. One of the scales is simply a change in the zero point and a change in the unit of measurement as compared to the other. Or, we may say, one of the scales is simply a *linear transformation* of the other, with a *positive* coefficient. The insistence on the *positive* coefficient means that the reading on one of the scales should *increase* if the reading on the other scale increaes.

(5.2) In the general case we will say that the choice indicator P in (5.1) is *cardinal* when and only when it is *uniquely* determined apart from an arbitrary *linear* and *increasing* transformation.

Obviously if the variables $x_1, x_2 \ldots$ are variables in the core of a macro economic model, and if a choice indicator satisfying (5.2) exists, we have all we need in order to define an optimal economic policy.

In fact, we have *more* than we need for this particular purpose of defining an optimal economic policy.

Indeed, suppose we consider a *transformation* Q of the scale for P, a transformation which is not necessarily linear but only has the following property:

(5.3) If a point $(x_1', x_2' \ldots)$ lies preferentially above another point $(x_1'', x_2'' \ldots)$ on the P scale, it should do so also on the Q scale, and vice versa.

(5.4) In this case we say that the Q scale is uniquely determined apart from an arbitrary (not necessarily linear) increasing transformation. This means that any scale Q^{**}, which is obtained from another scale Q^* by an arbitrary (not necessarily linear) increasing transformation, is equally good. In this case we say that a scale of the Q type is an *ordinal* choice indicator.

(5.5) A scale of the Q type is all we need to arrange all the objects in a uniquely defined *order* of preferences. Any scale Q^{**} will define the same preference order of all the objects $(x_1, x_2 \ldots)$ as any other scale Q^*.

(5.6) It is obvious that if our purpose is only to define an optimal economic policy, it is *sufficient* that we possess a choice indicator of the Q type, i.e. an ordinal choice indicator. Indeed the preference order of all the objects $(x_1, x_2 \ldots)$ will be the same no matter which one of the infinite number of Q-type indicators we have used, and the theoretical definition of an optimal economic policy is just to pick the *best* of the objects $(x_1, x_2 \ldots)$.

(5.7) *But if we raise the further question of knowing if any given object* $(x_1, x_2 \ldots)$ *is far from or near to optimalicy, then we need a choice indicator which is of a more specific type than the ordinal indicator. A cardinal indicator will permit us to answer questions of this type.*

Since questions of the type (5.7) are *of great importance* in many practical applications, it is *very desirable* to have a cardinal choice indicator. Furthermore the interview technique defined in the sequel can in practice be worked out much more *easily* if it is built on a cardinal indicator. I have indeed great difficulty in perceiving how such a

technique could be worked out if it were to be based only on an ordinal choice indicator.

I shall, therefore, confine myself in the following to cardinal choice indicators.

6. Transitivity

We may consider the choice among measurable objects, such as the sets $(x_1, x_2 \ldots)$ from an even more general viewpoint. Let us for a moment forget everything about the choice indicator, and for the time being only assume that when any *two* objects $(x'_1, x'_2 \ldots)$ and $(x''_1, x''_2 \ldots)$ are given, it is always possible to decide whether the former is preferred to, deferred to or indifferent to the latter. In this case we say that the choice stucture has the property of *determinateness*.

This being so, we may further ask the following *triangular* question: Suppose that *three* objects are given $(x'_1, x'_2 \ldots)$, $(x''_1, x''_2 \ldots)$ and $(x'''_1, x'''_2 \ldots)$. If the first object has a certain preference relation to the second object (for instance the relation "preferred to"); and if the second object has the *same* preference relation to the third object, will the first object always have the *same* preference relation to the third object? If so we say that the preference structure is *transitive*. This is obviously a special case of a choice structure that only has the property of determinateness. In the transitivity case it is possible to arrange all the objects in a uniquely determined preference order.

Obviously in the case where a choice indicator—whether cardinal or ordinal—exists, the preference structure is certainly transitive. Inversely: If the choice structure is transitive and the objects have been arranged in a well defined

order, we can number these objects. And this numbering may be taken as one example of an *ordinal* choice indicator. Hence *transitivity* and the existence of an *ordinal* choice indicator are equivalent properties.

The concept of transitivity will not be utilized in the sequel. The only thing we should note is that if a *cardina'* choice indicator exists, the preference structure is certainly transitive.

7. Possible assumptions on the mathematical form of the preference function

(7.1) *The general form:* $P(x_1, x_2 \ldots)$ (P = "preference")
Usually one would assume the function P to have continuous partial derivatives of the first and second order. And to be single valued.

(7.2) *The quadratic preference function:*

$$P = \sum_{\nu} A_{\nu} x_{\nu} + \frac{1}{2} \sum_{\mu} \sum_{\nu} B_{\mu\nu} x_{\mu} x_{\nu}$$

where the coefficients A and B are *constants.* Usually one would assume the quadratic form to be negative semi-definite, i.e. such as *not* able to assume *positive* values, regardless of what (real) magnitudes are attributed to the variables. No need to introduce a constant term in the expression (7.2).

(7.3) *The independence form:*

$$P = \sum_{\nu} C_{\nu}(x_{\nu})$$

where the C_{ν} are functions of a single variable.

(7.4) *The general scale function form:*

$$s(x_1 x_2 \ldots) \cdot P(x_1, x_2 \ldots) = \sum_{\nu} P_{\nu} (x_1, x_2 \ldots) \cdot x_{\nu}$$

where s, the scale function, in the general case, may be a function of all the variables. The functions P_{ν} are the partial derivatives $\frac{\delta P}{\delta x_{\nu}}$ of the general form (7.1).

Any function P may be written in the form (7.4) (a property which is also utilized in the theory of production, hence the name "scale" function for s).

(7.5) *The scale constant form*
If the scale function s is a *constant,* one can just as well take the right member of (7.4) as a choise indicator, and consequently study the form:

$$\sum_{\nu} P_{\nu} (x_1, x_2 \ldots) \cdot x_{\nu}$$

instead of the general form (7.1)

This is a very interesting expression from several viewpoints. In this form the *preference coefficients* $P_{\nu}(x_1, x_2 \ldots)$, i.e. the partial derivatives of the general preference function (7.1), appear very plausibly as *coefficients* by which to multiply the magnitudes of the various variables in order to obtain a choice indicator for the complex $(x_1, x_2 \ldots)$.

(7.6) *The quasi linear form*
This is the special case of the form (7.5) where each preference coefficient P_{ν} is assumed to be a *polynomial* in the particular variable x_{ν} to which the preference coefficient P_{ν} applies i.e.

$$P_{\nu} = \sum_{\mu=0,1,2\ldots} P_{\nu}^{(\mu)} x_{\nu}^{\mu}$$

where the $P_{\nu}^{(\mu)}$ are *constants*; and x_{ν}^{μ} is x_{ν} raised to the power μ. This, of couse, is a more restrictive assumption, but it may be used as an approximation.

This form can also be regarded as a special case of (7.3).

(7.7) *The exactly linear form*
This is the special case of (7.6) where only the power $\mu = 0$ is present, i.e.

$$P_\nu = P_\nu^{(0)}$$

where the $P_\nu^{(0)}$ are *constants*.

In this case the preference coefficients P_ν are constants. As a choice indicator we may in this case simply take the linear form

$$\sum_\nu P_\nu^{(0)} x_\nu$$

This is an approximation which is admissible only in the case where the interview questions pertain to a fairly *small* range. Cf. (10.6), (10.7) and sections 12 and 14 below.

For certain more specific kinds of assumptions see (10.8.2)—(10.8.3).

8. Comparisons through pairs of variables

We will now start on a discussion on how the preference relations among the variables are to be pinned down through an interview approach.

As an instrument in our preferential analysis the preferential variables will be considered *in pairs*. Say the pair $(x_\alpha x_\beta)$, abbreviated as $(\alpha\beta)$. And for each pair $(\alpha\beta)$ introduced, a *dichotomic run* of interview questions will be made.

Take any pair $(\alpha\beta)$ consisting of *two* of the preferential variables that were defined in section 4. And consider *two packages* in this particular pair of two variables. The packages may be termed "The package to the left" and "The package to the right" respectively.

Each package will consist of one specific magnitude of one of the two variables and one specific magnitude of the other variable. Hence *four* magnitudes will be involved in each question namely

(8.0) $$x_\alpha^{\text{left}},\ x_\beta^{\text{left}},\ x_\alpha^{\text{right}},\ x_\beta^{\text{right}}$$

And the question itself will be: "Do you prefer the package to the left or that to the right, or are you indifferent to the two packages?"

For each pair $(\alpha\beta)$ a series of questions of this sort is worked out according to a special system. But in each question the *only information used* will be the answer in the form of one of the three alternatives: "left", "right", "indifferent".

Such an interview run is a *completely general procedure* which does not assume anything beyond the general preference form (7.1). Only when it comes to an *interpretation* of the results obtained from an interview run, will the special assumptions (7.5)—(7.7) come into the picture. This will be explained in detail in the sections 10 and 11.

For all the questions the following assumptions must be clearly explained to the interviewed person or group:

(8.1) The most important assumption which must be explained *most emphatically* to the person or group interviewed, is that all the interview questions are to be understood in the *Santa Claus* sence. That is to say the questions pertain to what the interviewed person or group *would* choose *if* he actually *had a free choice* between the package to the left and that to the right. The question of *implementation* of these packages is not raised at all.

(8.2) In this connection it is important to explain that the interviewed person or group must rid his mind completely of any idea he might have of *what sort of total economic policy* that would be required in order to produce a constellation of the economy where a given package can be realized. Such ideas would lead the politician into an extremely complicated reasoning involving the whole structure of the model. The politician would not be in a position to disentagle all this. That is the job of the economic and econometric experts. Even if the politician might have *some ideas of his own* in this field, these ideas will often be *erroneous* and will only tend to lead the questioning completely astray. This is why the Santa Claus type of questioning is so essential.

(8.3) *The earmarking principle.* Further, it must be explained that *if* the politician chooses one of the packages, he will *not* have an opportunity afterwards of arranging the *other*

variables in the whole model in some specific way that might be to his liking. He will not be allowed to "trade his package in the market" so as to obtain something else he might like.

(8.4) All the other preferential variables which the politician may happen to think of as something he feels he must take into account when choosing one of the two packages presented, should be assumed to have the *central magnitudes* specified in Table. (4.5).

(8.5) The Government's parameters of action to which no "application preference" is attached, cf. (4.3), do *not* concern the interviewed person or group.

But there may perhaps be some other non preferential variables, the magnitudes of which the interviewed person or group feels he must have specified before he can make his choice. These variables, too, should be assumed to have some sort of central magnitude.

It will not be a convenient procedure for the interviewer to specify to the interviewed person or group the magnitudes of all these non-preferential variables before the questioning starts. This would lead to an over-complication, which would be detrimental to the smooth running of the interview. But if and when the interviewed person or group at any time in the course of the interviews feels the need for having the magnitude of any such variable specified, the interviewer has to supply it.

For this purpose the interviewer should have a complete and consistent list of all the non preferential variables in the model ready for his own use, with appropriate central magnitudes indicated. If needed these magnitudes would—in the expert's rough estimate—have to be made consistent with the central magnitudes of Table (4.5).

(8.6) If any of the variables entered in a package question is an *aggregate* of some more specifically broken-down variables which the interviewed person or groups may happen to think of, he or they may assume that these broken-down variables can be arranged in the way they prefer, but with the proviso that the *aggregate* retains the magnitude it has been given in the package question.

(8.7) In addition to the above points the conversation with the interviewed person or group would, of course, also include the points discussed in section 4.

9. A recommended form of a run of package questions

Interviews may be organized in many different ways. Here I shall only be concerned with the form I now recommend on the basis of my experience.

The questions and the corresponding answers may be thought of as recorded in the form of a table like (9.1).

As the four magnitudes to be entered in the *first* question we choose those indicated on the first line in Table (9.1). Cf. section 4 on the ranges.

To facilitate understanding the questions, we will only change *one* of these magnitudes through all the questions. It will be one of the two *outer* magnitudes, i.e. x_α^{left} or x_β^{right} that is to be changed.

In the first question the choice may fall to the left or to the right. The side to which it falls will decide which one of the two magnitudes x_α^{left} or x_β^{right} that will be the one actually changing through the questions.

(9.2) If the choice in the first question falls to the left, it is x_α^{left} that is to be changed. And if it falls to the *right*, it is x_β^{right} that is to be changed. The first line of Table (9.1) illustrates the case where the choice on the first line fell to the left. And hence the *V* mark was entered to the left.

The three magnitudes *other* than the one, that is to be changed, will remain constantly equal to the magnitudes they had on the first line in Table (9.1)

Table (9.1) *A dichotomic run*

Date:

Person or group interviewed:				Interviewer:				
Description of x_α :								
Description of x_β :								
	Package to the left				Package to the right			
Question No.	Enter mark V here if the package to the left is preferred	Magnitudes of the variables x_α^{left}	x_β^{left}	Enter mark Ind here	Magnitudes of the variables x_α^{right}	x_β^{right}	Enter mark V here if the package to the right is preferred	Space for the expert's quick notes
1	V	x_α^{pref}	x_β^{def}		x_α^{def}	x_β^{pref}		
2		x_α^{def}	x_β^{def}		x_α^{def}	x_β^{pref}	V	
3		$x_\alpha^{pref} + x_\alpha^{def} / 2$	x_β^{def}		x_α^{def}	x_β^{pref}		
4								
5								
etc.								

through-out all the questions. Since in Table (9.1) we have assumed that the choice on the first line fell to the left it will be x_α^{left} that is to change in Table (9.1).

But the case where the choice in the first line fell to the right—and consequently x_β^{right} had to be changed—is quite similar, apart from the fact that x_β^{right} is now the actually changing magnitude. Hence it is sufficient to explain the case illustrated in Table (9.1), where the choice on the first line fell to the left.

The reason for this particular construction of the questions is that if all the magnitudes are to lie *in their prescribed ranges* between x^{pref} and x^{def}, in our search for a

point of indifference, it must in Table (9.1)—where we assumed that the choice on the first line fell to the left—be x_α^{left} that had to be the changing variable.

For the sake of principle we enter as the second question in Table (9.1) the one indicated in the table. This second question is only a formal one because it is *obvious* that on this line the choice must fall to the right, i.e. to the side opposite to that on which it fell on the first line.

In the first two lines of the case treated in tab. (9.1) we now have two opposite answers: On the first line, one that fell to the left, and on the second line one that fell to the right.

If the choice on the first line had fallen to the right, the four magnitudes on the second (formal) line would have been entered as x_α^{pref}, x_β^{def}, x_α^{def}, x_β^{def}, so that now we would be certain that the choice on the second line would fall to the left.

On the third line we simply enter in each of the four columns the *arithmetic average* of the figures we find in this column on the first and the second lines. Doing this in Table (9.1) the last three figures will be *unchanged* while the first will change. All the four figures will lie in the prescribed ranges.

On the third line the choice is not obvious. Whether it falls to the left or to the right will give a new piece of information on the choice structure of the interviewed person or group.

In all the subsequent questions the following applies:

(9.3) Whenever a question has been answered (after the first question), the situation will have the following property: In the series of questions answered *before* the last one, there will be at least one question where the answer falls to the side *opposite* to that on which it fell in the last question.

(9.4) *General rule for formulating a new question:* When the last question has been answered, look *backwards.*

Scrutinize the answers previously recorded one by one. Settle on the *first* line (going backwards) where we find that the choice fell to the side *opposite* to that on which it fell in the last question answered. Call this the *counterline*.

In each of the four columns take the *arithmetic average* of the magnitude that occurred in the last question answered, and the magnitude (in the same column) that occurred in the counterline. This will give the new question.

The following two examples will illustrate the rule.

If the choice fell to the *left* on the third line of Table (9.1), the first magnitude on the fourth line will become

(9.5) $$x_\alpha^{\text{left}} = \frac{1}{2}\left(x_\alpha^{\text{def}} + \frac{x_\alpha^{\text{pref}} + x_\alpha^{\text{def}}}{2}\right) = \frac{3x_\alpha^{\text{def}} + x_\alpha^{\text{pref}}}{4}$$

If on the third line the choice fell to the *right*, the first magnitude on the fourth line will become

(9.6) $$x_\alpha^{\text{left}} = \frac{1}{2}\left(x_\alpha^{\text{pref}} + \frac{x_\alpha^{\text{pref}} + x_\alpha^{\text{def}}}{2}\right) = \frac{3x_\alpha^{\text{pref}} + x_\alpha^{\text{def}}}{4}$$

(9.7) In any case all the four magnitudes on the fourth line, being *arithmetic averages* with *positive* weights of the endpoints of the respective ranges, will lie in the range in question.

(9.8) And this property will also *be maintained* in all further applications of the general rule (9.4).

In this way we can continue until one of the following two alternatives occurs:

(9.9) Either the interviewed person or group gives the answer that they are *indifferent* to the choice between the two packages, or

(9.10) The interviewer will conclude that there is no use continuing the sequence of questions because the observed change in x_α^{left} from one question to the next, *becomes so small* that he may simply take the four magnitudes which *would* be obtained by a a further application of the general rule (9.4), as a *sufficiently close approximation* to a situation where the choice between the two packages is indifferent. If he is not sure about this, he can simply pose the question obtained by this further application of the general rule (9.4) as a supplementary question. This would then in all probability lead to the answer "indifferent". In any case, a few more questions would lead to this.

(9.11) The sequence of questions here considered—with one variable and three constant elements—is constructed with a view to producing a *rapid convergence*. It is interesting to note that the convergence from one question to the next will manifest itself by *the time it takes to obtain the answer*. This time will *increase* from one question to the next, indicating that the interviewed person or group must make a more and more careful *weighing* before he can reach his answer.

By watching the time needed to get an answer to each question, the interviewer can usually *guess* fairly early in the sequence of questions approximately where the indifference point will finally lie.

Since we do not know beforehand what the number of questions in a given dichotomic run will be—sometimes only a few questions are needed—and since it is desirable to have all the recordings for several runs made in a *compact* form, the working sheet used in practice will be somewhat different from that in Table (9.1) which was used for explanatory purposes. A convenient form of the com-

Table (9.12) *Continuous working sheet for dichotomic runs*

Person or group interviewed:					Interviewer:			Date:		
Run No. Var. Nos.	Q. No.	Package to the left			ind.	Package to the right			Space for computations and results	
		Mark	x_α^{left}	x_β^{left}		x_α^{right}	x_β^{right}	Mark		
	etc.									

pact table is given in Table (9.12). Here the different runs are separated simply by a heavy line across the paper. See also Table (14.2).

10. Interpretation of two indifferent packages

When the indifference point has been reached and we are to interpret the result of the questions, we will have to make *some assumption* regarding the mathematical form of the preference function. It would in principle be possible to use an assumption that is *very general*, but then the subsequent steps in the analysis would become rather laborious. I believe that in many practical cases we will get an approach that is sufficiently realistic by using the form (7.6), and retaining here only *two terms* in the expansion of the preference coefficients. I.e. we may put

$$(10.1) \qquad P_\nu = P_\nu^{(0)} + P_\nu^{(1)} x_\nu \qquad (\nu = 1, 2, \ldots)$$

Instead of the cumbersome superscripts "ind.left" and "ind.right" we abbreviate them to il and ir.

In the indifference point we then get—using (7.6) and (10.1)—:

$$(10.2) \qquad \begin{aligned} &(P_\alpha^{(0)} + P_\alpha^{(1)} x_\alpha^{il}) \cdot x_\alpha^{il} + (P_\beta^{(0)} + P_\beta^{(1)} x_\beta^{il}) \cdot x_\beta^{il} \\ &= (P_\alpha^{(0)} + P_\alpha^{(1)} x_\alpha^{ir}) \cdot x_\alpha^{ir} + (P_\beta^{(0)} + P_\beta^{(1)} x_\beta^{ir}) \cdot x_\beta^{ir} \end{aligned}$$

where $P_\alpha^{(0)}$, $P_\alpha^{(1)}$, $P_\beta^{(0)}$, $P_\beta^{(1)}$ are contants, so far unknown.

The terms $\sum_{\nu \neq \alpha, \beta} (P_\nu^{(0)} + P_\nu^{(1)} x_\nu^*) x_\nu^*$ where x_ν^* are the *fixed* values of the preference variables *other* than x_α and x_β, will cancel because they are the same on both sides of the equals sign in (10.2).

If we even go so far as to assume the preference coefficients as *constants*, i.e. if we assume $P_\alpha^{(1)} = P_\beta^{(1)} = 0$, (10.2) reduces to

(10.3) $$P_\alpha^{(0)} x_\alpha^{il} + P_\beta^{(0)} x_\beta^{il} = P_\alpha^{(0)} x_\alpha^{ir} + P_\beta^{(0)} x_\beta^{ir}$$

and hence

(10.4) $$\frac{P_\alpha^{(0)}}{P_\beta^{(0)}} = \frac{x_\beta^{ir} - x_\beta^{il}}{x_\alpha^{il} - x_\alpha^{ir}}$$

(if the preference coefficients are constant).

In the series of questions defined in section 9 neither the nominator nor the denominator in the right member of (10.4) will be zero. The formula applies regardless of whether the first choice fell to the right or left in Table (9.1).

The last formula shows that if the preference coefficients are constants, *a single run* of package questions is enough to determine the ratio between the two preference coefficients.

The following two shorthand rules makes it easy to remember the formula (10.4):

(10.5.1) "Upstairs and downstairs are opposite in the two members" (α upstairs and β downstairs in the first member, β upstairs and α downstairs in the second member).

(10.5.2) "Right and left are opposite in the two differences" (Either right minus left upstairs and left minus right downstairs—as in (10.4)—or, which amounts to the same, left minus right upstairs and right minus left downstairs).

If the interview questions are arranged as in tab. (9.1)

Table (10.7) *Combination of possible ranges in (α β) runs*

Range	Combination	For x_α	For x_β
Reduced ranges	I	$(x_\alpha^{pref}, x_\alpha^{centr})$	$(x_\beta^{centr}, x_\beta^{def})$
	II	$(x_\alpha^{centr}, x_\alpha^{def})$	$(x_\beta^{pref}, x_\beta^{centr})$
	III	$(x_\alpha^{pref}, x_\alpha^{centr})$	$(x_\beta^{pref}, x_\beta^{centr})$
	IV	$(x_\alpha^{centr}, x_\alpha^{def})$	$(x_\beta^{centr}, x_\beta^{def})$
Total range	V	$(x_\alpha^{pref}, x_\alpha^{def})$	$(x_\beta^{pref}, x_\beta^{def})$

and the choice in the first question fell to the left, all the magnitudes in the right member of (10.4) will be equal to the deferred magnitudes except x_α^{il}. In practice it is just as convenient to use the general form (10.4).

If we do not assume that the preference coefficients are constant, but use (10.2), *one single* interview run is not sufficient to define the constants we are looking for. But *three* interview runs in ($\alpha\beta$) with different ranges are sufficient. Since the equation (10.2) is *homogeneous* it is only the *relative* sizes if the constants that are determined, as was also the case in (10.4). The normalization in (10.2) can be performed for instance by dividing through by $P_\beta^{(0)}$, which gives the following three unknowns:

(10.6) $$\frac{P_\alpha^{(0)}}{P_\beta^{(0)}}, \quad \frac{P_\alpha^{(1)}}{P_\beta^{(0)}}, \quad \frac{P_\beta^{(1)}}{P_\beta^{(0)}},$$

Three ($\alpha\beta$) runs with different ranges will give us three linear equations to determine the three ratios (10.6). We choose the ranges in such a way that the matrix of the equations become non-singular.

A plausible splitting of the ranges is obtained by using the *midpoint* x^{centr} in each of the previously considered total ranges (x^{pref}, x^{def}). This gives the five ranges in Table (10.7) to choose from.

The range combination I and II should always be used. They are symmetric in α and β and will be numerically effective in bringing to light any non-constancy which may exist in the preference coefficients.

In addition it is natural to use the range combination V. This will not disturb the symmetry between α and β.

If instead of V either III or IV are used, an asymmetry between α and β would be introduced. The symmetry would be re-established if we determined the three unknowns (10.6) by first using the range combinations (I, II, III) and then using the range combinations (I, II, IV), and, if need be, making a compromise between the two solutions obtained. Non-singularity must always be arranged for.

(10.8). *Extensions.*

(10.8.1) The set up (10.6) can be generalized by also taking account of the two unknowns $P_\alpha^{(2)}/P_\beta^{(0)}$ and $P_\beta^{(2)}/P_\beta^{(0)}$. In this case the five equations needed can be obtained by using all the five ranges in Table (10.7) if they give non-singularity.

(10.8.2) The case of interdependence *inside* the set $(\alpha\beta)$ can be handled in a similar way. For instance, if $P_\alpha = P_\alpha^{(0)} + P_\alpha^{(1)}x_\alpha + R_{\alpha\beta}x_\beta$ and $P_\beta = P_\beta^{(0)} + P_\beta^{(1)}x_\beta + R_{\beta\alpha}x_\alpha$ where $R_{\alpha\beta}$ and $R_{\beta\alpha}$ are two constants, $(\alpha\beta)$ runs with all the five ranges in Table (10.7) will give the equations needed, provided independence is maintained for all the variables *other* than x_α and x_β.

(10.8.3) Even more terms in x_α and x_β could be admitted in P_α and P_β if still other ranges for the $(\alpha\beta)$ runs were used than those listed in Table (10.7).

(10.8.4) But if we consider inter-dependence between any of the two variables (x_α, x_β) and some variables *outside* the set $(\alpha\beta)$, we will either have to use runs with alternative magnitudes of some of the other variables, or packages consisting of more than two variables. Cf. Section 12.

Table (10.10)

Person interviewed: The choice indicator $P = x_\alpha + 2x_\beta$

Question No.	Package to the left: Mark	x_α^{left}	x_β^{left}	ind	Package to the right: x_α^{right}	x_β^{right}	Mark	Approximation to $P_\alpha^{(0)}/P_\beta^{(0)}$	Note
1		120	40		80	70	V	0.750	
2	V	120	40		80	40		0.000	
3	V	120	40		80	55		0.375	
4		120	40		80	62.5	V	0.562	(av. = 0.509)
5	V	120	40		80	58.25		0.456	
6		120	40		80	60.375	V	0.509	(av. = 0.496)
7	V	120	40		80	59.3125		0.483	
8	V	120	40		80	59.84375		0.496	(av. = 0.500)
9		120	40		80	60.109375	V	0.504	

As a numerical illustration of a *package run* of the form defined in Section 9 and the *interpretation* of the results obtained by such a run, consider a fictitious interview run in the set $(\alpha\beta)$ where left-side or right-side choice was determined by the following choice indicator with constant preference coefficients:

(10.9) $P = P_\alpha^{(0)}x_\alpha + P_\beta^{(0)}x_\beta$ where $P_\alpha^{(0)} = 1$ and $P_\beta^{(0)} = 2$,

hence $P_\alpha^{(0)}/P_\beta^{(0)} = 0.5$.

Nothing but the left side or right side answers were assumed to be observable.

The ranges were chosen as:

$$x_\alpha^{pref} = 120,\; x_\alpha^{def} = 80,\; x_\beta^{pref} = 70,\; x_\beta^{def} = 40.$$

The result was as indicated in Table (10.10)

On the first line in Table (10.10) the choice fell to the right because here $P^{\text{left}} = 200$ and $P^{\text{right}} = 220$. The other choices are determined in the same way.

Even at an early stage in the questioning in Table (10.10) the $P_\alpha^{(0)}/P_\beta^{(0)}$ magnitude of 0.5 is reached with a fair degree of accuracy. For instance the average magnitude in the questions 4 and 5 is 0.509. In questions 6 and 7 it is 0.496, and in questions 8 and 9 0.500, i.e. correct to three decimal places.

(10.11) Incidentally: The approximation to $P_\alpha^{(0)}/P_\beta^{(0)}$ obtained by taking the average between the magnitude on any line and that on its counterline, is identical with forming the *next* question and recording the result that would follow from this question. This is illustrated in Table (10.10).

It is possible to introduce a refinement based on the concept of a *least perceptible difference*, but this I shall not discuss here.

11. Triangular relations. Minimal sets and checking set

On the assumption (10.1) the result of the run $(\alpha\beta)$ and the result of the run $(\beta\gamma)$ permits us to draw definite conclusions about the result that *would be* obtained by a run $(\alpha\gamma)$, i.e. a run where x_α and x_γ are compared directly. This is seen as follows.

Take first the case of constant preference coefficients:

Suppose that *by some method or other*,—assuming constant preference coefficients—we have determined the ratio

$$(11.1) \qquad \left(\frac{P_\alpha^{(0)}}{P_\beta^{(0)}}\right)^*$$

The asterisk * indicates that this magnitude has been determined. Cf. the numerical illustration in tab. (10.10).

If (11.1) is known, we can also easily compute the ratio $P_\beta^{(0)}/P_\alpha^{(0)}$, because we obviously have:

$$(11.2) \qquad \frac{P_\beta^{(0)}}{P_\alpha^{(0)}} = \frac{1}{\left(\frac{P_\alpha^{(0)}}{P_\beta^{(0)}}\right)^*}$$

Furthermore if, by some method or other,—assuming constant preference coefficients—we have determined the *two* ratios $(P_\alpha^{(0)}/P_\beta^{(0)})^*$ and $(P_\beta^{(0)}/P_\gamma^{(0)})^*$, it is easy also to compute the ratio $P_\alpha^{(0)}/P_\gamma^{(0)}$, because we obviously have:

$$(11.3) \qquad \frac{P_\alpha^{(0)}}{P_\gamma^{(0)}} = \left(\frac{P_\alpha^{(0)}}{P_\beta^{(0)}}\right)^* \cdot \left(\frac{P_\beta^{(0)}}{P_\gamma^{(0)}}\right)^*$$

The formula (11.3) may be called the *triangular* relation in the case of constant preference coefficients.

Similar relations may be derived also in the case (10.1). Indeed, suppose that by some method or other we have determined the three ratios (10.6). Also suppose that we have determined the three ratios

$$(11.4) \qquad \frac{P_\beta^{(0)}}{P_\gamma^{(0)}}, \quad \frac{P_\beta^{(1)}}{P_\gamma^{(0)}}, \quad \frac{P_\gamma^{(1)}}{P_\gamma^{(0)}}.$$

All these six magnitudes could be marked with the superscript * to indicate that they are known. We are, however, now only interested in the first two magnitudes in (10.6) and the first magnitudes in (11.4).

From these three magnitudes we can compute

$$(11.5) \qquad \frac{P_\alpha^{(0)}}{P_\gamma^{(0)}} = \left(\frac{P_\alpha^{(0)}}{P_\beta^{(0)}}\right)^* \cdot \left(\frac{P_\beta^{(0)}}{P_\gamma^{(0)}}\right)^* \quad \text{(similar to (11.3))}$$

$$(11.6) \qquad \frac{P_\alpha^{(1)}}{P_\gamma^{(0)}} = \left(\frac{P_\alpha^{(1)}}{P_\beta^{(0)}}\right)^* \cdot \left(\frac{P_\beta^{(0)}}{P_\gamma^{(0)}}\right)^*$$

(a new relation, now involving $P_\alpha^{(1)}$)

In other words we have by (11.5) and (11.6) computed the two ratios that characterize the $(\alpha\gamma)$ comparison and which would have emerged in runs performed directly in the set $(\alpha\gamma)$ provided we have (10.1).

There is no need to bother about the ratio $P_\gamma^{(1)}/P_\gamma^{(0)}$ because this ratio is already contained in (11.4), which is presumed to be known.

Similar triangular relations can be obtained also in the more general case (7.6), but there is no need here to write these formulae out.

Consequently, it will — in the case (7.6) — not be necessary to make runs for each of the preferential variables compared with every other preferential variable. It will be sufficient to pick out *certain pairs*, make *direct runs* for each of the pairs thus picked, and indirectly draw inferences for the other pairs.

(11.7) For instance if we have four preferential variables x_1,x_2,x_3,x_4 it would — in the case (7.6) — be sufficient to make direct runs for (12), (13), (14). That is to say comparing each of the three variables x_2, x_3, x_4 with x_1, taken as a common standard. Similarly it would be sufficient to make direct runs for (12), (23), (34). Should we have made all comparisons directly, we would have had to make all the runs for (12), (13), (14), (23), (24), (34).

As it is desirable to reduce the interview work involved in making runs as much as possible, we are interested in picking a *minimal set* of pairs. This means such a set of pairs that *direct runs* for each of the pairs in the minimal set are sufficient — in the case (7.6) — to get a complete picture of all the preferences.

Such a minimal set of pairs may be picked in different ways. Incidentally I may mention my conjecture that the *number* of different minimal sets that exist, is equal to:

$$(11.8) \qquad \text{Number of different minimal sets} = N^{N-2}.$$

I.e. N to be power (N-2), where N is the number of preferential variables included. An actual scrutiny shows that this formula is correct for $N = 2, 3, 4, 5$ but I have not proved the formula in general.[1]) Example (11.7) above gives $4^2 = 16$ minimal sets. The set (12), (13), (14) is one of them. Another is (12), (23), (34).

If these are only 2 preferential variables x_1 and x_2, (11.8) gives $2^0 = 1$, i.e. only one minimal set exists. This is ob-

[1]) Memorandum of 19 February 1956 from the Institute of Economics at the University of Oslo.

viously correct. It indicates the only existing set, namely the one consisting of the pair (12). Each minimal set will consist of $(N-1)$ pairs.

(11.9) By using only a single minimal set we are—in the case (7.6)—able to reach a complete picture of all the preferences within the set of preferential variables. But we do not get any possibility of *checking* the results. Such a check is desirable for many reasons, for instance because we will want to check for inaccuracies in the answers, and also because we may want to pin down any systematic deviation from the assumption (7.6).

(11.10) In practice we will therefore plan a preference analysis in such a way that the list of runs contains at least some runs in addition to those contained in a minimal set.

Usually the interviewer will use runs for the following *recommended* sets:

(11.11) The minimal set of pairs: (12), (23), (34) . . . $(N-1, N)$

(11.12) The checking set of pairs: (13), (24) . . . $(N-2, N)$.

The interviewer should not make a complete list of all these pairs known beforehand to the interviewed person or group. This may only cause confusion. He should take the pairs up, one after the other, for instance in the order (12), (23), (13), (34), (24) . . . etc. Here we get frequent opportunities of using the triangular checks (11.3) and (11.5)—(11.6). In practice any overdeterminateness may be smoother by least squares.

12. Non linearity

I shall make a few theoretical remarks on the non-linear case, assuming the general form (7.1).

In the non linear case when the choice on the first line of Table (9.1) fell to the left, and $x_\alpha^{pref} > x_\alpha^{def}$, $x_\beta^{pref} > x_\beta^{def}$, the situation is illustrated in fig. (12.1). All the variables other than x_α and x_β are assumed constant.

The slightly curved lines in Fig (12.1) are the actually

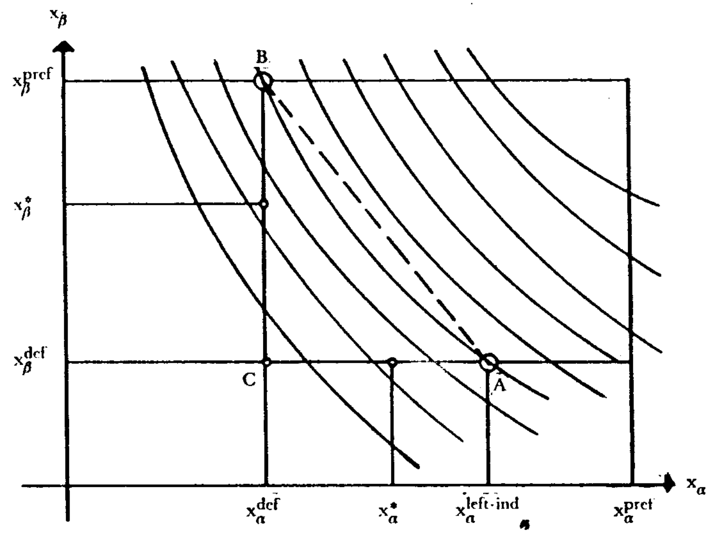

Fig. (12.1).

existing contour lines (all the points are indifferent along any of them). The point A represents the package to the *left* when the questioning has reached the indifference point. And B represents the package to the *right* when the indifference point has been reached. Hence the two points A and B are *in fact* indifferent in the general case (7.1), and are consequently connected by one of the curved countour lines. This particular contour line is indicated by the heavy line in Fig. (12.1). The straight dotted line between A and B is the linear approximation to the contour line between A and B. This linear approximation represents the case where it is assumed that there exist two *constant* preference coefficients $P_\alpha^{(0)}$, $P_\beta^{(0)}$. Their *ratio* will be determined by the empirically determined second member in (10.4).

Now compare this with what is actually true in the non linear case. Let $P = P(x_\alpha, x_\beta)$ be the actually existing preference function, whose contour lines are given by the curved lines of Fig. (12.1), and let $P_\alpha = \partial P/\partial x_\alpha$, $P_\beta = \partial P/\partial x_\beta$. Both P_α and P_β will be functions of the point (x_α, x_β). Let us consider the variation of P_α along the line CA and the variation of P_β along CB.

By the mathematical mean value theorem—assuming continuous variations of P_α and P_β—we know that along CA there exists at least one point x_α^* where we have

$$(12.2) \qquad P_\alpha^* = \frac{P^A - P^C}{x_\alpha^{\text{ind. left}} - x_\alpha^{\text{def}}}$$

Similarly by considering the variation of P_β along CB we see that there exists at least one point x_β^* on this line where we have

$$(12.3) \qquad P_\beta^* = \frac{P^B - P^C}{x_\beta^{\text{pref}} - x_\beta^{\text{def}}}$$

Hence

$$(12.4) \qquad P^A = P_\alpha^* (x_\alpha^{\text{ind. left}} - x_\alpha^{\text{def}}) + P^C$$

$$(12.5) \qquad P^B = P_\beta^* (x_\beta^{\text{pref}} - x_\beta^{\text{def}}) + P^C$$

Since $P^A = P^B$, we get by subtracting (12.5) from (12.4)

$$(12.6) \qquad \frac{P_\alpha^*}{P_\beta^*} = \frac{x_\beta^{\text{pref}} - x_\beta^{\text{def}}}{x_\alpha^{\text{ind. left}} - x_\alpha^{\text{def}}}$$

The second member here is identical with the second member of the empirically determined ratio (10.4).

If the *ranges* $(x_\alpha^{\text{pref}}, x_\alpha^{\text{def}})$ and $(x_\beta^{\text{pref}}, x_\beta^{\text{def}})$ are *reasonably small*, the magnitudes P_α^* and P_β^* of the *actually existing* partial derivatives will not "have time" to become much different from the magnitudes which these partial derivatives had in some *centrally defined point* whose abscissae x_α^{centr} and x_β^{centr} lie in the ranges considered. Hence the empirically determined second member in (10.4) can be looked upon as a fair approximation to the magnitude which the ratio between the partial derivatives assumed in the centrally defined point. Therefore the interview technique based on (7.7) is sufficiently accurate.

Quite similar conclusions can be drawn if the choice on the first line in Table (9.1) falls to the right and $x_\alpha^{\text{pref}} < x_\alpha^{\text{def}}$ or $x_\beta^{\text{pref}} < x_\beta^{\text{def}}$.

But if the ranges are *large* and we have the non linear case, there may be an appreciable difference between the second member of (12.6) and the ratio between the partial derivatives that actually exists in some centrally defined point. And this we might have to take into account if accuracy is wanted. The procedure (10.6)—(10.7) is one way of doing this.

Another and much more general approach would be the following:

(12.7) We first make up a list of multi-dimensional central points around which preferences are to be analysed. Multidimensionally means that in each of these central points we should specify the magnitudes of *all* the preferential variables $x_1, x_2, \ldots, x_N$.

(12.8) Around each of these central points we make an interview approach to the preferences *by means of very small ranges for all the variables* assuming (7.7). Around each central point triangularity should be tested by (11.3). If this is verified with sufficient accuracy, we may take it as a criterion that the ranges are small enough.

(12.9) If we find a *significant change* in the empirically determined ratios $P_2^{(0)}/P_1^{(0)}, P_3^{(0)}/P_1^{(0)} \ldots$ etc. as we go from one multi-dimensional central point to another, the non-linearity of the actually existing preference function $P(x_1, x_2, \ldots, x_N)$ has been manifested.

(12.10) Furthermore around any of the central points selected the *relative* sizes of the partial derivatives $P_1, P_2, \ldots, P_N$ have been measured. In other words we have been able to measure how the *direction* of the gradient on the preference function $P(x_1, x_2, \ldots, x_N)$ changes as we go from one point in the $(x_1, x_2, \ldots, x_N)$ space to another.

(12.11) In order to get a complete description of a *cardinal* choice indicator we need to make certain comparisons from one to another of the multi-dimensional central points that are involved. To do this we need to formulate questions of a type which I call *interlocal*.

13. Compatibility-smoothing of an inter-preference table

In the following the constancy of preference coefficients is assumed.

The available empirical observations $P_{\alpha\beta} = \frac{P_\alpha^{(0)}}{P_\beta^{(0)}}$ may be recorded in a two dimensional table with rows $\alpha = 1, 2, \ldots, N$ and columns $\beta = 1, 2, \ldots, N$; N being the number of preferential variables included. Many of the cells in this table may be empty. But it is assumed that in the set of the non empty cells it will be possible to pick at least one *minimal* set. Cf. the definition after (11.7). It is assumed that no observations occur in the diagonal of the matrix $P_{\alpha\beta}$.

If we make the linearity assumption (7.7) the problem is to find a set of N numbers $P_1^{(0)}, P_2^{(0)}, \ldots, P_N^{(0)}$ that can be taken as the preference coefficients.

These numbers should be uniquely determined, possibly apart from an arbitrary positive common multiplier.

In practice the empirical observations $P_{\alpha\beta}$ will as a rule not be *exactly compatible* with the assumption that there exists a set of preference coefficients $P_1^{(0)}, P_2^{(0)}, \ldots, P_N^{(0)}$, with the property just mentioned. But the deviations from this assumption may not be great.

Therefore, a statistical problem arises of *smoothing* the empirical observations $P_{\alpha\beta}$ in such a way that they can be produced by a set at N figures $P_1^{(0)}, P_2^{(0)}, \ldots, P_N^{(0)}$.

The smoothing should *utilize* all the information contained in the actual observations $P_{\alpha\beta}$ and distort these observations "as little as possible". The smoothed value of $P_{\alpha\beta}$ ($\alpha \neq \beta$), will be $P^{(0)}_{\alpha}/P^{(0)}_{\beta}$. The following is a suggestion for such a smoothing method.[1])

Instead of the actual observations $P_{\alpha\beta}$ we introduce the logarithmically transformed observations:

(13.1) $$Q_{\alpha\beta} = \log |P_{\alpha\beta}|$$

where the vertical bars indicate "absolute value of". All the $Q_{\alpha\beta}$ are real numbers.

We introduce the *occurence* number $A_{\alpha\beta}$ defined by (13.2) —(13.3):

(13.2) $A_{\alpha\beta}$ (for $\alpha \neq \beta$)
- = 0 if there is *neither* a $P_{\alpha\beta}$ observation in the cell $(\alpha\beta)$ *nor* a $P_{\beta\alpha}$ observation in the cell $(\beta\alpha)$.
- = 1 if there is an observation *either* in the cell $(\alpha\beta)$ *or* in the cell $(\beta\alpha)$ (but not in both these cells).
- = 2 if there is an observation in the cell $(\alpha\beta)$ and *also* an observation in the cell $(\beta\alpha)$.

(13.3) $$A_{\nu\nu} = 0$$

The matrix A is symmetric, i.e.

(13.4) $$A_{\alpha\beta} = A_{\beta\alpha}$$

The column sums in A are denoted

(13.5) $$A_{.\nu} = \sum_{\varkappa} A_{\varkappa\nu}$$

And the row sums are denoted

[1]) The method is built on my paper "The Smoothing of an Interpreference Table", in *Methods of Operations Research III*, edited by Rudolf Henn, 1967 (Volume in honour of Wilhelm Krelle). But the presentation in the present paper is more streamlined.

(13.6) $$A_{\nu.} = \sum_{\varkappa} A_{\nu\varkappa}$$

Obviously

(13.7) $A_{\nu.} = A_{.\nu}$

The formula (13.7) can be used as a check on the counting. It can be assumed that all the figures $A_{\nu.} = A_{.\nu}$ are positive, not zero.

We introduce the *difference* D_{ν} between the sum in column ν and the sum in row ν in the matrix $Q_{\alpha\beta}$ defined by (13.1). More precisely:

(13.8) $$D_{\nu} = \sum_{\varkappa} Q^{*}_{\varkappa\nu} - \sum_{\varkappa} Q^{*}_{\nu\varkappa}$$

where

(13.9) $Q^{*}_{\varkappa\nu}$
- = the observation $Q_{\varkappa\nu}$ if there actually exists an observation in the cell $(\varkappa\nu)$.
- = 0 otherwise

We form the system of N linear equations in the N unknown $Q_1, Q_2, \ldots, Q_N$:

(13.10)
$$\sum_{\varkappa} (A_{\nu\varkappa} - e_{\nu\varkappa} A_{.\varkappa}) Q_{\varkappa} = D_{\nu} \text{ where } e_{\nu\varkappa} \begin{cases} = 1 \text{ if } \nu = \varkappa \\ = 0 \text{ otherwise} \end{cases}$$

Here all the magnitudes are empirically observed, except the N unknowns $Q_{\varkappa}$. The second term in the parenthesis in (13.10) could, of course, be written with $A_{.\nu}$ since the term only exists when $\varkappa = \nu$. And by (13.7) it could also be written with $A_{\nu.}$, or it could be written with any *arithmetic average* with constant weights of the two figures $A_{\nu.}$ and $A_{.\nu}$. For simplicity we consider equation (13.10) as it stands.

The matrix in (13.10) is singular (because of (13.11)). But I believe it is not of rank lower than $(\mathcal{N}-1)$ provided there is at least one minimal set included in the cells $(\alpha\beta)$ for which observations actually exist.

Another aspect of this singularity is that if $Q_\varkappa$ is a solution, then also $(Q_\varkappa + C)$ will be a solution, where C is a constant independent of $\varkappa$. This simply follows from the fact that

(13.11) $$\sum_\varkappa (A_{\nu\varkappa} - e_{\nu\varkappa} A_{.\varkappa}) = A_{\nu.} - A_{.\nu} = 0$$

In other words:

(13.12) The numbers $Q_\varkappa$ are by (13.10) only determined apart from an arbitrary additive constant.

This arbitrariness can be *removed* for instance by replacing one of the equations in (13.10) by an equation that fixes the *sum* of the $Q_\varkappa$; i.e.

(13.13) $$\sum_\varkappa Q_\varkappa = S$$

where S is an arbitrarily given number, for instance $S = 0$. (Instead of the left member of (13.13) we could also have introduced a positively weighted average of the $Q_\varkappa$.)

It does not matter which one of the equations in (13.10) we replace by (13.13). The solution $Q_\varkappa$ will be the same.

The solution of the system of linear equations as now considered, can be performed by any standard elimination method. Since $\mathcal{N}$ will not be a very great number, there is no need to have recourse to an iteration method for solving the equations.

The preference coefficients $P_\nu^{(0)}$ $(\nu = 1, 2 \ldots \mathcal{N})$ looked for, will now be given by the two conditions:

(13.14) $$|P_\nu^{(0)}| = \text{antilog } Q_\nu$$

(13.15) $$\text{sgn. } P_\nu^{(0)} = \text{sgn. } (x_\nu^{\text{pref}} - x_\nu^{\text{def}})$$

sgn = signum = the sign of

(13.16) If S in (13.13) is left as an arbitrarily chosen magnitude, the preference coefficients $P_\nu^{(0)}$ as determined by (13.14)—(13.15) will be effected with an arbitrary common multiplier. But if S is fixed, for instance as $S = 0$, then the preference coefficients $P_\nu^{(0)}$ are uniquely determined, provided the rank of the matrix in (13.10) is $(\mathcal{N} - 1)$.

(13.17) As a check on the determination of the $P_\nu^{(0)}$, we should form the linear function written as the formula in (7.7) with these preference coefficients, and verify that the left side and right side choices according to the preference function thus obtained, will *coincide* with the choices that we actually observed.

(13.18) The reasoning behind the above method can very briefly be described as follows: If the observations $Q_{\alpha\beta}$ are such as to be exactly compatible with the linearity assumption, numbers Q_ν will exist such that

(13.18.1) $$c_{\alpha\beta} Q_{\alpha\beta} = c_{\alpha\beta}(Q_\alpha - Q_\beta)$$

where $c_{\alpha\beta}$ is a number that is equal to 1 if there actually exists an observation in the cell $\alpha\beta$, but otherwise is equal to zero.

Performing a summation of (13.18.1) over α and β respectively and subtracting the two equations obtained, we are led to an equation of the form (13.10).

14. An example of a concrete application of the interview runs

A high ranking civil servant in the Norwegian Ministry of Finance has kindly consented to answer interview questions on preferences regarding the development of the Norwegian economy in the next year. The construction of the set of dichotomic runs, and the answers given would, of course have been very different had the development of the economy been considered over a longer range of years. This longer perspective could also have been interview analyzed.

Traits of the economy that would come in as preferential traits in the next-year analysis were listed as follows by the interviewed person:

1. The number of unemployed (the average for next year).
2. The annual growth rate of the GNP (gross national product) from this year to the next.
3. Next year's inequality in the net real income per capita in the form of wages and salaries and incomes of small private enterpreneurs. Cf. the remarks in Section 3.
4. The relative change in the consumer price index from this year to the next.
5. Next year's visible trade balance.
6. Next year's breakdown of GNP by economic activities.

Table (14.1)

Description	Variable	Most preferred bound	Most deferred bound
Number of unemployed	x_1	10 thousand	23 thousand
Growth rate of GNP	x_2	+6 per cent	+ 2 per cent
Regional skewness if income	x_3	0 per cent	+ 40 per cent
Consumer price change	x_4	+ 2 per cent	+ 7 per cent
Visible trade balance	x_5	− 3 milliards N.kr.	− 11 milliards N.kr.

7. Next year's breakdown of GNP by age groups.
8. An index of next year's breakdown of land utilization.
9. Next year's geographical population pattern.
10. Undesirable effects of the disposal of waste.
11. Next year's government expenditure on health services.
12. Next year's government expenditure on education.
13. Next year's government expenditure on research.
14. An index for next year's standard of housing. (Food and clothing are assumed to have already reached such a standard in Norway that they constitute an item of little interest as regards next year's preference analysis).
15. Next year's net flow of resources to developing countries.
16. An index of next year's maldistribution of traffic (roads vs. railways, watertransport etc.)
17. Next year's total government expenditure on nature conservancy.

In a first approach the variables 1—5 were selected for interviewing. The ranges chosen are indicated in Table (14.1)

The runs (12), (23), (13), (24), (25) were made assuming a provisionally constant preference coefficients. As soon as time permits other runs in the complete set of the 17 preferential traits will be made on the assumption (10.1). Table (14.2) also indicates the magnitude of $P_{\alpha\beta} = \frac{P^{(0)}_{\alpha}}{P^{(0)}_{\beta}}$ as it

Table (14.2) *Concrete interview runs with a high ranking civil servant in the Norwegian Ministry of Finance*

Interviewer: RF Date: 21 October 1970

Run	Quest. No.	Mark	x_α^{left}	x_β^{left}		x_α^{right}	x_β^{right}	Mark	$P_{\alpha\beta}$ (slide rule accuracy)
(12)	1		10	2		23	6	V	
	2	V	10	2		23	2		
	3		10	2		23	4	V	
	4	V	10	2		23	3		
	5		10	2	ind	23	3½		—0.1153
(23)	1		6	40		2	0	V	
	2	V	6	40		2	40		
	3	V	6	40		2	20		
	4		6	40		2	10	V	
	5		6	40	ind	2	15		—6.25
(13)	1		10	40		23	0	V	
	2	V	10	40		23	40		
	3		10	40		23	20	V	
	4		10	40		23	30	V	
	5	V	10	40		23	35		
	6	V	10	40		23	32½		
	7		10	40	ind	23	31¼		+0.675
(24)	1	V	6	7		2	2		
	2		2	7		2	2	V	
	3		4	7		2	2	V	
	4	V	5	7		2	2		
	5		4½	7	ind	2	2		—0.500
(25)	1		6	—11		2	— 3	V	
	2	V	6	—11		2	—11		
	3		6	—11		2	— 7	V	
	4		6	—11		2	— 9	V	
	5		6	—11		2	—10	V	
	6		6	—11	ind	2	—10½		+0.125

emerged in the indifference points ("β upstairs and α downstairs. Right minus left upstairs and left minus right downstairs") Cf. the comments to (10.4).

The figures in Table (14.2) tell their own and very interesting story.

Take for instance the run (25). It shows that the interviewed person is here *very much concerned about the visible trade balance.* It shows that he would be willing to sacrifice *a whole percent* of the GNP growth rate in order to obtain *as little as a 125 millions N.kr.* improvement in the visible trade balance. And this is not an accidental mistake in the answer at the indifference point, but it is fully consistent with all the answers to the previous questions in the run (25). This can be verified numerically by considering the preference function,

$$P = x_2 + 8x_5 \tag{14.3}$$

This function is obtained from the run (25) if we conventionally put $P_2 = 1$. (The preference coefficients—from the viewpoint of left-right-indifference choice—may be multiplied by an arbitrary positive constant).

In the package to the left in run (25), the formula (14.3) gives $P = -82$ throughout. But in the package to the right it gives the following P values: -22, -86, -54, -70, -78, -82 which is completely compatible with the answers given to all the questions in the run (25).

There is nothing implausible in this result. The run (25) is on the contrary highly suggestive: The result obtained depends heavily on the *range* chosen for x_5, viz. $(-3, -11)$. This range leads inevitably to the result that all the x_5^{right} magnitudes (after question 1) lie in the *danger region* of the trade balance. In particular the indifference magnitude $-10½$ is a very dangerous magnitude. Here a red light has been lit for x_5. But the magnitudes for x_2 are still reasonable. Consequently the *marginal* preference for increasing the trade balance as compared to the marginal preference for GNP must be very high in run (25) in Table (14.2).

If we had made some other (25) run with a range covering *more desired* values of x_5, say the range (+3, −4), a (25) run would undoubtedly have given a much *lower* preference coefficient for x_5. In other words we have here a clear case of non linearity, which ought to be handled by the ranges (I, II, V) in Table (10.7).

Another comment to Table (14.2): By the results of the first three runs (12), (23), (13) we get an opportunity of applying the *triangular* test (11.3).

In Table (14.2) the directly observed P_{13} is +0.675 while the product $P_{12} \cdot P_{23}$ is equal to $(-0.1153) \cdot (-6.25) = +0.721$. In view of the fact that so few questions were used here, the triangular test is satisfied rather well.

If greater care and time had been used (i.e. longer series of questions) to reach greater accuracy in the determination of the ratios, more precise information on the triangular tests could have been obtained.

THE END