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Thesis for Master of Science degree.

Quasi-dynamic forward-looking model for joint household retirement decision under AFP scheme

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Abstract[a]

Structural forward-looking quasi-dynamic behavioural model is estimated for the Norwegian households where husband is eligible for early retirement between 1993 and 1996. Random utility approach is applied to rationalize the observed shifts between four main stages on the labour market. Specific attention is paid to the retirement decisions. Based on the finest model policy simulation is run to register the impact of altering taxation on the labour force participation.

Acknowledgements

I would very much like to thank the supervisor Prof. Steinar Strøm for priceless support in the breakthrough periods of work and wise suggestions in all the times, Prof. Erik Hernæs for bringing me to the project, useful advices and fruitful discussions since the very start of my work, Prof. John Dagsvik for outstanding theoretical support, Maria Kalvarskaia for major contribution in AFP data preparation and the researchers at the Frisch Centre for helpful cooperation.

Preface

The following thesis was prepared within the aging population and pension project carried out by Erik Hernæs and Steinar Strøm at the Frisch Centre for Economic Research. The project is financed by the Norwegian Research Council.

The project has resulted in a whole series of papers which date back to 1997 ([Hernæs et al. 2001, 2002a, 2002b], [Dagsvik, Strøm 1997], [Brinch et al., 2002]). As the data quality increases and more data is collected with years, it is necessary to come back to some previous results to improve and correct them. The modelling details also change with time although the major path has been laid out solid. In the current research the data is substantially improved with additional occupational pension included in the dataset for the very first time. Besides, new three periods model is developed to make predictions more accurate and less restrictions are introduced on the sample selecting stage to make the models more general.

[a] The current master thesis was written within the Frisch Center project 1132 (Yrkesaktivitet blant eldre og finansiering av pensjonssystemet). We acknowledge financing from the Research Council of Norway.

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Introduction

Retirement behaviour is the theme of increasing interest in the recent labour economic studies due to its growing importance. In most countries which adopted pay-as-you-go public pension system the relative number of pensioners is increasing together with the level of life. This puts additional fiscal weight on the taxpayers who are financing current pensions. In order to compensate this demographic dynamics the governments have to take certain steps to improve labour force participation and stimulate additional supplementary pensions. To test possible policies in this field different models can be estimated to describe people's behaviour on the labour market.

Current study presents several structural forward-looking quasi-dynamic models of households to serve the purpose of policy simulations. The models are set up in the random utility framework used by Thurstone (1927), McFadden (1973), Ben-Akiva, Lerman (1985) and spread widely in the discrete choice analysis. This approach suggests that the rational decision maker maximizes over the set of choices an utility function with determined and stochastic components. Stochastic part of the utility takes care of unobserved by the researcher factors and is assumed to be perfectly known to the agent. The determined part depends on the characteristics of either the agent or the alternatives, or both, and allows any specifications. Several utility specifications make up several models to be estimated and compared.

Since the utility is specified as dependent on the fundamental attributes of the available options such as disposable income and leisure, the model appears structural other than reduced form. Structural models yield much better, more accurate results in simulations since they reflect all direct and indirect effects the policy may have on the taken choices.

In the current study two revisions of the standard multinomial logit model are developed. It is taken into account that not only the current period utility is considered by the choice maker, but also the discounted sum of future periods utility. In other words, the agent plans the utility flow for several years ahead. Correspondingly, the two models reflect two and tree period setups – in the first one the agent is allowed to consider the next period consequences of the current choice, and in the second one the planning horizon rises to two periods. The future periods considerations incorporated into the models make them forward-looking. These forward-looking issues are quite substantial when studying the retirement behaviour due to the fundamental property of being retired – once a person has retired it is hardly possible to start working again. In the models we assume the retirement state to be strictly absorbing. Thus, once people choose retirement, they can not go back to any other state.

Keeping this in mind, they will correct their retirement decisions compared to simple one period logic.

The models are quasi-dynamic due to the fact that even though the time scale is present, only one static choice is modelled. Namely, at the eligibility point new option (to retire) becomes available to an individual and the choice is made whether to retire or to stay in the labour force.

Finally, the sample is organized by households to take care of probable coordination between husband and wives as to retire simultaneously. Even if wife for example does not have retirement option available for her, she may change her labour force participation at the point when husband retires to take advantage of common leisure time. Thus, we assume some coordination within the household with respect to behaviour on the labour market and concentrate on studying coordinated choices of spouses.

Once the models are estimated and the best one chosen, some simple simulation is performed. We try altering taxation by rising the total amount of tax paid by households by 10%. These policy results in the change of distribution of households among states.

The thesis is organized as follows. Chapter 1 briefly describes the pension system in Norway and gives overview of the occurring rules. More detailed description with the mathematical formulas can be found in Appendix A. Chapter 2 is devoted to developing and presenting the models, but also a lot of attention is paid to describing the possible choice sets. One of the objectives of the current study was to keep the sample as vast as possible and not to exclude households with unusual wife's state. This resulted is several case divisions which are also discussed in chapter 2. Chapter 3 contains the sample data description used for estimating the models and concludes model specifications by introducing the deterministic part of the utility function. In chapter 4 the sequence of estimated models is followed, the models are compared and the best one is picked for simulations. Finally, chapter 5 presents the results of the simulation procedures. Appendix A contains accurate definitions of the procedures used for data construction while Appendix B has a set of table and graphs to visualize the sample.

Chapter 1. Institutional settings

1.1 Public pension (NIS)

Public pension in Norway has taken its modern form in 1967 when earning based system came into the place of old flat rate pension. All permanent residents are covered with the scheme with the general eligibility at age of 70. However, the pension can be taken out at 67 without reduction apart from the loss of opportunity to earn extra pension rights if the upper limit was not yet reached.

The pension consists of three major component. The first one is the basic pension which is paid to everybody with at least 3 years of working life. The level of basic pension is corrected every year and is referred to as G. From 36 500 in 1992 the basic pension has risen up to 45 370 in 1998. It is paid in full to the individuals who have worked at least 40 years, otherwise it is reduced proportionally.

Second main component is an earnings based pension. Its level rests on so called pension points, which are calculated from annual salaries. For the earnings up to $6G$ ($8G$ before 1992) they simply equal to the excess of the salary over the basic pension expressed in G. For the earnings up to $12G$ the points are reduced by one third while bigger salaries result in flat point of 7 (8,33 before 1992). The average of 20 best points is then multiplied by 0.42 (0.45 if they are earned before 1992) and by G to give the level of the earnings based component.

The third component is formed of special supplementary terms aimed on preventing the pension to go lower certain minimum. The values of these terms are also corrected on the yearly bases and enter the pension equation under the maximum sign to kick in place if the pension is falling too much down. Thus, low salary may not contribute at all to the pension level.

Besides these principle regulations there are some minor rules dealing with particularities. For example, pensions differ for married and single people. In families, pensions are generally reduced for both spouses compared to the sum of their individual pensions were they separated. This rule does not work in families having one person not working or with income less than minimum pension. Besides, for those not having completed 40 years of working time the earnings based component is as well reduced proportionally. There are some exceptions, however. The pension system is still on the phasing-in stage since there are people who started to work before the modern scheme was introduced. The phasing in will finish in 2013 (assuming the youngest working age equal to 19). For such individuals regulations of minimum working history are significantly relaxed. Moreover, the sector in which an individual is occupied also matters. Those working in public companies have an alternative pension with calculating technique which differs a little from the general one. The pensions levels from different calculations are coordinated so that the actually received amount is the maximum between differently calculated pensions.

All these features (except the sector division) were taken into account in the calculation of potential pensions.

Regarding the financing of the public pension system it can be mentioned that contributions to the system come both from employers and employees (also self-employed) in a form of percentage deduction from their earnings as part of the general taxation. Even though there is a central pension fund, it is not required to meet its net future obligations and the system is supported by yearly transfer payments from the government.

1.2 Occupational pension (OP)

So-called occupational or employer based pensions was introduced together and in addition to the public pension provided by the state. This product of the insurance market gave employers opportunity to deduct the payments paid to pre-funded occupational pensions from the tax base according to the tax-code from 1922.

The coverage by occupational pension was gradually expanding until it was forced to play a minor role after the introduction of the earnings based public pension. However, the schemes continued to be used as a pathway to favorable tax regime.

The tax treatment of private occupational pension plans has the following traditional pattern. Contributions both by employer and employee and returns on the accumulated funds are tax-deductible, while the benefits from the scheme are subject to income tax (as a pension) when paid out to the pensioner. In order to qualify for this favorable tax regime private company plans must obey certain rules.

First, an occupational pension plan must be insured with a life insurance company or established as a separate pension fund. Second, if a company chooses to establish a pension plan, all standard, full-time employees of the company must be included. However, a waiting period of one year is allowed (five years for the workers below 25) and part-time workers with less than 50 percent of full time, temporary and seasonal workers can be excluded. Third, even though there are no limits on the replacement ratios, the principle of proportionality must be satisfied. This principle states that private pensions can compensate for the fairly redistributive profile of the NIS pension, but only up to the point where they aim at perfectly proportional total replacement ratios. The total gross replacement ratios can not be higher for employees with higher earning levels than for the employees with lower earning levels. Finally, old age private pensions generally cannot start before age 67.

Although these rules have to be complied with in order to obtain tax deductions, any company is of course free to operate pension arrangements without a tax break. In a company survey , about one quarter of the companies answered that they give such provisions, but there is no information available on the type or amounts of benefit [Pedersen, 2000].

A full pension is usually accrued after 30 years of work. However, all decisions about establishing and design of occupation pension plan are made within a company itself. Therefore the above age and tenure limitations can not be taken as strict.

Thus, the occupation based pensions are only regulated by general principles which do not say anything about the levels of the pension or the accruing mechanisms. In the current study, the occupation based pensions are calculated according to the model estimated in [Iskhakov, Kalvarskaia 2003]. Simple regression model is estimated to relate the last salary received by worker to his or her occupational pension. All people working full-time in the OP companies are said to be OP-eligible. In turn, OP companied are traced down by observing any worker to receive occupational pension.

1.3 Early retirement (AFP)

An early retirement scheme[b] was introduced in 1989 as a result of negotiations between trade unions and major employers. People covered with it received an opportunity to retire earlier than regular NIS retirement time with no loss in their pension benefits.

The scheme covers the whole public sector and part of the private sector. In order to be eligible an individual must be employed in a company covered and meet certain individual requirements. These include:

- Having been employed in the AFP-company the last 3 years or having been covered by AFP scheme during last 5 years;
- Having earnings no less than G the year AFP is taken up and the year before;

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[b] In Norwegian notation AFP, Avtalefestet Pensjonsordning.

- Not receiving pensions or similar payments from employer without work effort in return;
- Having at least 10 years after the age of 50 with earnings no less than *G*;
- Having the average earnings in 10 best years since 1967 no less than 2*G*.

The age of early retirement has been gradually lowered from 66 when it was initially introduced on January 1st, 1989 to 65 from January 1st, 1990, 64 on October 1st, 1993, 63 on October 1st, 1997 and finally to 62 on March 1st, 1998. With the eligibility age going down and more and more companies participating in the scheme, the AFP coverage has grown constantly; now covering about 65-70% of the labour force (whole public sector and part of the private sector without self-employed).

The pension level calculations under AFP scheme are aimed to provide the same pension benefit as if person would continue until the ordinary retirement age instead of retiring early. This implies that the pension points in the years between the AFP eligibility age and 67 should be forecasted with some mechanism. The one agreed on uses the maximum between the average of the last three earned points and the average of ten best points from whole working history to substitute unrealized points from the 'future' years. Once they are substituted, the AFP pension is calculated with regular NIS calculation technique.

So, the AFP pension is exactly the regular public pension under the assumption that person earns the last points according to the described forecasting procedure. Since the calculations rules are spelled out explicitly, this type of pension is not hard to calculate as well as general public pension. The only difficulty is to identify the AFP participating companies. As in OP case, these companies are traced down by observing AFP recipients and finding their last jobs.

To conclude the institutional settings section, it is worth mentioning that the tax levels are generally lower for pensioners compared to the working people and differ for single and married individuals. Moreover, AFP pensions are taxed a little differently from other types of pensions. Haugen (2000) gives all details in describing tax functions, they were calculated to the full scale in the current study.

Chapter 2. The models

2.1 Objectives

The model described below serves the purpose of studying the joint decisions made inside households which determine the behaviour of the couple on the labour market. Primary attention is paid to retirement decision of the husband under the early retirement (AFP) scheme.

For each person a sequence of shifts among some states such as working full-time or part-time, being unemployed or retired, is observed. These shifts may result from some rational choice performed by the agents. Appearing on the time line they form an ordered flow which is referred to as labour market behaviour.

In real life the choice may be thought over continuously and the shifts appearing when utility of one alternative becomes greater than the utility of the other. The changes in the choice set or the states available to the agent also greatly influence the decision. When modelling we very much simplify the reality. The model is set up in discrete time allowing the agent to solve the choice problem annually. The focus of the study is retirement decision by husband. The first time is can be taken is right after reaching certain eligibility age. That is why it is natural to assume that the decision problem is solved every birthday. Finally, husbands are given primary importance in the study (due to their leading role in traditional families), that is why the household is modelled to make behavioural decisions on husbands' birthdays.

After estimating the structural model to describe this decision making process several policy simulations can be performed to understand how this decisions can be influenced by the government.

2.2 Sample overview

The basic unit in the sample is household with both husband and wife. Corresponding to Norwegian reality not only registered married couples, but also *samboer* couples are included. Household construction details are given in appendix A.

Considered are only households where husband becomes AFP-eligible during the period from 1.01.1993 to 31.12.1996 and is registered as working in the previous calendar year. General time layout is shown on Fig. 1. Here the flag marks the moment T1 when husband becomes AFP-eligible and the decision to retire or to continue working is first taken. This decision influences the choice set at times T2 and T3, the moments one and two years after retirement through AFP became available (T3-T2 = T2-T1 = 1 year).

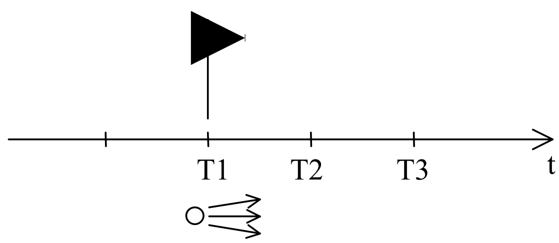


Fig. 1. Timeline for husband and household.

In the period from 1.01.1993 to 31.12.1996 the minimum AFP age was lowered from 65 to 64 years (on 1.10.1993) [Haugen, 2000], which implies that households with husband born from 1.01.1928 to 31.12.1932 come into the consideration. The cohort has 85 283 men with 64 636 of them married (registered as married or samboer). Out of this number 21 549 are AFP-eligible and 21 437 are also working in the calendar year before date T1 thus forming the target sample of 21 437 households.

2.3 Available states and choice sets

In the year before T1 husband is required to work by the sample definition. At time T1 retirement becomes available and the decision is made whether to retire or stay in the labour force. We distinguish among the following sates available for husband:

1. Full time work. Husband stays in the labour force and works full time.
2. Part time work. Husband stays in the labour force but works only half of the regular working time.
3. Immediate retirement. Husband retires within 2 month from T1. Option is available only for the first year after becoming AFP-eligible.
4. Retirement. Husband retires later than 2 month after T1 or in the beginning of any other year.

Accurate definitions of the states are given in Appendix A.

States 3 and 4 are absorbing – if one of them is chosen at T1 than only state 4 is available at T2. The same applies for the choice in T2 which effects the set of options in T3. By taking this property into account the model becomes forward-looking. Making their decisions at T1 households realize that if husband retires then, no other option except retirement is available at T2 and T3.

One difficulty faced at this point was many husbands not observed in any of the defined states. There was performed an additional investigation with some more states temporary added to the choice set. Table 1 presents the results while details are given in the Appendix A.

States occupied by husband after T1	Number of observations	Percentage, %
Full-time work	6 406	29.88
Part-time work	2 467	11.51
Immediate retirement	4 096	19.11
Retirement	6 205	28.95
Unemployed (SOFA listed)	761	3.55
Receiving disability pension	929	4.33
Not in any mentioned group	573	2.67
Total	21 437	100.00

Table 1. Observations of husband after T1.

The households in which husband occupies irrelevant states were filtered out thus bringing the sample power down to 19 174 observations. The final dataset used in the estimation has 19 027 households after 147 of them were also filtered out with fundamental variables essential for estimation missing.

By general setup wife's choices are made together with the husband's ones – that implies that they are made not exactly at the time when new element appears on her choice set (or some option disappears from her choice set), but on the nearest husband's birthday after such change. This seems quite natural in the households where husband possesses the leading role. However, the retirement state is assumed to appear on wife's choice set one year before. That is wife is considered eligible if she can retire within next year. This is done in order to allow husbands and wives make simultaneous retirement decisions in the same modelling year and retire together right after wife becomes eligible.

States available for wives are as following:

1. Full time work. Wife works full time.
2. Part time work. Wife works only half of the regular working time.
3. Out of labour force. Wife does not work, though she may have some income in the form of for example disability benefits.
4. Retirement. Wife is retired.

Contrary to husband, whose choice set at T1 contains all four defined states, wife's choice set neither at T1, T2 nor at T3 is clear. It depends on wife's age and AFP-eligibility and should be investigated. Denote W1 the time when wife becomes AFP-eligible (if she does) and W2 the time when she reaches general retirement age of 67. Since between these two points there must be at least two years, they can not fall into the interval (T1; T3) and all possible combinations of allocations of T1, T2, T3, W1 and W2 can be summarized in 12 cases. These cases and the corresponding wife's choice sets are presented in Table 2.

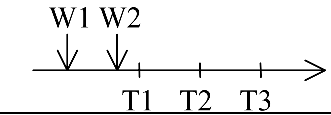
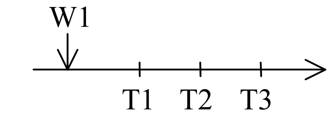
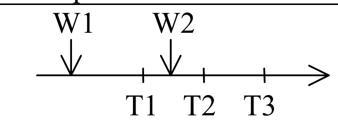
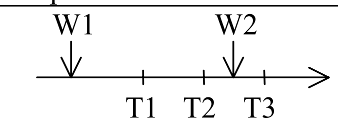
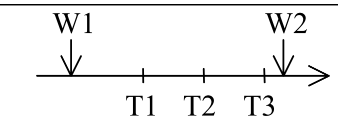
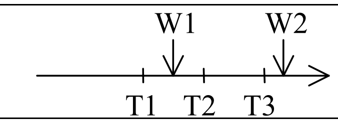
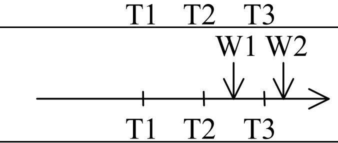
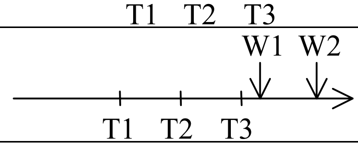
	Allocation of T1, T2, T3, W1, W2	Wife's choice set			Group
		T1	T2	T3	
AFP-eligible wives		4	4	4	1
	 AFP pension taken out at T1	4	4	4	1
	 AFP pension not taken out at T1	1234	4	4	2
	 AFP not taken out at T1	1234	4 or 1234	4	3
	 AFP not taken out at T1	1234	4 or 1234	4 or 1234	4
		1234	4 or 1234	4 or 1234	4
		123	1234	4 or 1234	5
		123	123	123[c]	6

Table 2. Possible choice sets for wife.

[c] For simplicity we do not consider here the case when AFP-eligibility comes within a year after T3 and AFP retirement option is technically available for wife at T3.

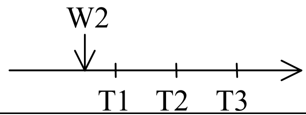
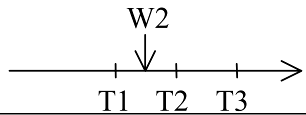
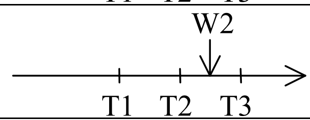
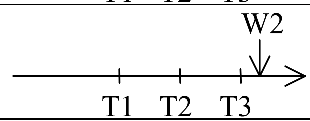
Not AFP-eligible wives		4	4	4	1
		123	4	4	7
		123	123	4	8
		123	123	123	6

Table 2. Continued.

All the cases can be separated in 8 groups with the same combinations of options available at each of three points. The same grouping done on the bases of only first two time points gives similar groups – the correspondence is presented in Table 3. Here *GroupB* is the same as in the last column of Table 2 and *GroupA* refers to two periods logic.

GroupA	GroupB	Wife's choice sets		
		T1	T2	T3
1	1	4	4	4
2	2	1234	4	4
3	3	1234	4 or 1234	4
	4	1234	4 or 1234	4 or 1234
4	5	123	1234	4 or 1234
5	6	123	123	123
	8	123	123	4
6	7	123	4	4

Table 3. GroupA and GroupB correspondence.

It is quite natural that including a third period into consideration results in splitting some of the two-period-logic groups, given in Table 3. Both of the grouping will be used further.

To conclude the choice sets discussion we note that retirement state is absorbing for wives as well. This is reflected in the previous Tables: for groups 3, 4 and 5 (groups of the type B) the choice sets faced at T2 and T3 depend on the state chosen previous year. In groups 3 and 4 wife has an option to retire through AFP scheme at the same time as husband. Here we have the case of possible joint AFP retirement.

Zhiyang (2000) argues that the most common situation is the one when husband does have and wife does not have AFP retirement option in their choice sets, Hernæs (2001) also considers only such case. Table 4 gives the distribution of households in the target population by groups and does justify this statement, but it seems quite interesting also to study the rest of the groups and especially families with joint AFP retirement option (1 222 households are in such position). In whole, not to consider any groups other than the biggest one, excludes 3 571 households from the defined sample.

GroupA		Number of households	GroupB	
Number	Percentage		Number	Percentage
1	5.39	1 026	1	5.39
2	0.33	63	2	0.33
3	6.48	138	3	0.73
		1 094	4	5.75
4	4.43	842	5	4.43
5	81.23	14 859	6	78.09
		598	8	3.14
6	2.14	407	7	2.14
Total	100.00	19 027		100.00

Table 4. Division of the sample by wife's choice sets.

To conclude the section denoted to the agent's choice sets we should put down explicitly the rules of dependence for the set of options available at time t+1 after a certain state was chosen at time t. First, Table 5 contains the summary of modelled states husband and wife can occupy. Let indexes i and j denote husband's and wife's choice correspondingly so that household's choice is expressed in a vector (i,j).

State	Husband	Wife
	i	j
Full-time work	1	1
Part-time work	2	2
Immediate retirement	3	-
Retirement	4	4
Out of labour force	-	3

Table 5. Indexes for the states available for husband and wife.

To avoid double scripts later on let different indexes be used for choices at different moments of time. Let (i,j) correspond to the time point T1, (r,s) to the time point T2 and (k,l) to the time point T3. Still indexes i, r and k describe husband's choice while indexes j, s and l describe that of the wife. The relations between choice sets at different times are best presented in a table (see Table 6).

GroupA	GroupB	At T1		At T2		At T3
		Available options	Choice made	Available options	Choice made	Available options
Column number		1	2	3	4	5
1	1	$i\in\{1,2,3,4\}$ $j=4$	$i\in\{1,2\}$ $j=4$	$r\in\{1,2,3,4\}$ $s=4$	$r\in\{1,2\}$ $s=4$	$k\in\{1,2,3,4\}$ $l=4$
					$r\in\{3,4\}$ $s=4$	$k=4$ $l=4$
			$i\in\{3,4\}$ $j=4$	$r=4$ $s=4$	$r=4$ $s=4$	$k=4$ $l=4$
2	2	$i\in\{1,2,3,4\}$ $j\in\{1,2,3,4\}$	$i\in\{1,2\}$ $j\in\{1,2,3,4\}$	$r\in\{1,2,3,4\}$ $s=4$	$r\in\{1,2\}$ $s=4$	$k\in\{1,2,3,4\}$ $l=4$
					$r\in\{3,4\}$ $s=4$	$k=4$ $l=4$
			$i\in\{3,4\}$ $j\in\{1,2,3,4\}$	$r=4$ $s=4$	$r=4$ $s=4$	$k=4$ $l=4$
3	3	$i\in\{1,2,3,4\}$ $j\in\{1,2,3,4\}$	$i\in\{1,2\}$ $j\in\{1,2,3\}$	$r\in\{1,2,3,4\}$ $s\in\{1,2,3,4\}$	$r\in\{1,2\}$ $s\in\{1,2,3,4\}$	$k\in\{1,2,3,4\}$ $l=4$
					$r\in\{3,4\}$ $s\in\{1,2,3,4\}$	$k=4$ $l=4$
			$i\in\{3,4\}$ $j\in\{1,2,3\}$	$r=4$ $s\in\{1,2,3,4\}$	$r=4$ $s\in\{1,2,3,4\}$	$k=4$ $l=4$
			$i\in\{1,2\}$ $j=4$	$r\in\{1,2,3,4\}$ $s=4$	$r\in\{1,2\}$ $s=4$	$k\in\{1,2,3,4\}$ $l=4$
					$r\in\{3,4\}$ $s=4$	$k=4$ $l=4$
			$i\in\{3,4\}$ $j=4$	$r=4$ $s=4$	$r=4$ $s=4$	$k=4$ $l=4$
	4	$i\in\{1,2,3,4\}$ $j\in\{1,2,3,4\}$	$i\in\{1,2\}$ $j\in\{1,2,3\}$	$r\in\{1,2,3,4\}$ $s\in\{1,2,3,4\}$	$r\in\{1,2\}$ $s\in\{1,2,3\}$	$k\in\{1,2,3,4\}$ $l\in\{1,2,3,4\}$
					$r\in\{3,4\}$ $s\in\{1,2,3\}$	$k=4$ $l\in\{1,2,3,4\}$
					$r\in\{1,2\}$ $s=4$	$k\in\{1,2,3,4\}$ $l=4$
					$r\in\{3,4\}$ $s=4$	$k=4$ $l=4$
			$i\in\{3,4\}$ $j\in\{1,2,3\}$	$r=4$ $s\in\{1,2,3,4\}$	$r=4$ $s\in\{1,2,3\}$	$k=4$ $l\in\{1,2,3,4\}$
					$r=4$ $s=4$	$k=4$ $l=4$
			$i\in\{1,2\}$ $j=4$	$r\in\{1,2,3,4\}$ $s=4$	$r\in\{1,2\}$ $s=4$	$k\in\{1,2,3,4\}$ $l=4$
					$r\in\{3,4\}$ $s=4$	$k=4$ $l=4$
			$i\in\{3,4\}$ $j=4$	$r=4$ $s=4$	$r=4$ $s=4$	$k=4$ $l=4$

Table 6. Previous period choice and next period choice set relations.

<table>
<tr><th rowspan="2">GroupA</th><th rowspan="2">GroupB</th><th colspan="2">At T1</th><th colspan="2">At T2</th><th>At T3</th></tr>
<tr><th>Available options</th><th>Choice made</th><th>Available options</th><th>Choice made</th><th>Available options</th></tr>
<tr><td colspan="2">Column number</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr>
<tr><td rowspan="6">4</td><td rowspan="6">5</td><td rowspan="6">$i\in\{1,2,3,4\}$
$j\in\{1,2,3\}$</td><td rowspan="4">$i\in\{1,2\}$
$j\in\{1,2,3\}$</td><td rowspan="4">$r\in\{1,2,3,4\}$
$s\in\{1,2,3,4\}$</td><td>$r\in\{1,2\}$
$s\in\{1,2,3\}$</td><td>$k\in\{1,2,3,4\}$
$l\in\{1,2,3,4\}$</td></tr>
<tr><td>$r\in\{3,4\}$
$s\in\{1,2,3\}$</td><td>$k=4$
$l\in\{1,2,3,4\}$</td></tr>
<tr><td>$r\in\{1,2\}$
$s=4$</td><td>$k\in\{1,2,3,4\}$
$l=4$</td></tr>
<tr><td>$r\in\{3,4\}$
$s=4$</td><td>$k=4$
$l=4$</td></tr>
<tr><td rowspan="2">$i\in\{3,4\}$
$j\in\{1,2,3\}$</td><td rowspan="2">$r=4$
$s\in\{1,2,3,4\}$</td><td>$r=4$
$s\in\{1,2,3\}$</td><td>$k=4$
$l\in\{1,2,3,4\}$</td></tr>
<tr><td>$r=4$
$s=4$</td><td>$k=4$
$l=4$</td></tr>
<tr><td rowspan="6">5</td><td rowspan="3">6</td><td rowspan="3">$i\in\{1,2,3,4\}$
$j\in\{1,2,3\}$</td><td rowspan="2">$i\in\{1,2\}$
$j\in\{1,2,3\}$</td><td rowspan="2">$r\in\{1,2,3,4\}$
$s\in\{1,2,3\}$</td><td>$r\in\{1,2\}$
$s\in\{1,2,3\}$</td><td>$k\in\{1,2,3,4\}$
$l\in\{1,2,3\}$</td></tr>
<tr><td>$r\in\{3,4\}$
$s\in\{1,2,3\}$</td><td>$k=4$
$l\in\{1,2,3\}$</td></tr>
<tr><td>$i\in\{3,4\}$
$j\in\{1,2,3\}$</td><td>$r=4$
$s\in\{1,2,3\}$</td><td>$r=4$
$s\in\{1,2,3\}$</td><td>$k=4$
$l\in\{1,2,3\}$</td></tr>
<tr><td rowspan="3">8</td><td rowspan="3">$i\in\{1,2,3,4\}$
$j\in\{1,2,3\}$</td><td rowspan="2">$i\in\{1,2\}$
$j\in\{1,2,3\}$</td><td rowspan="2">$r\in\{1,2,3,4\}$
$s\in\{1,2,3\}$</td><td>$r\in\{1,2\}$
$s\in\{1,2,3\}$</td><td>$k\in\{1,2,3,4\}$
$l=4$</td></tr>
<tr><td>$r\in\{3,4\}$
$s\in\{1,2,3\}$</td><td>$k=4$
$l=4$</td></tr>
<tr><td>$i\in\{3,4\}$
$j\in\{1,2,3\}$</td><td>$r=4$
$s\in\{1,2,3\}$</td><td>$r=4$
$s\in\{1,2,3\}$</td><td>$k=4$
$l=4$</td></tr>
<tr><td rowspan="3">6</td><td rowspan="3">7</td><td rowspan="3">$i\in\{1,2,3,4\}$
$j\in\{1,2,3\}$</td><td rowspan="2">$i\in\{1,2\}$
$j\in\{1,2,3\}$</td><td rowspan="2">$r\in\{1,2,3,4\}$
$s=4$</td><td>$r\in\{1,2\}$
$s=4$</td><td>$k\in\{1,2,3,4\}$
$l=4$</td></tr>
<tr><td>$r\in\{3,4\}$
$s=4$</td><td>$k=4$
$l=4$</td></tr>
<tr><td>$i\in\{3,4\}$
$j\in\{1,2,3\}$</td><td>$r=4$
$s=4$</td><td>$r=4$
$s=4$</td><td>$k=4$
$l=4$</td></tr>
</table>

Table 6. Continued.

Read left to right Table 6 displays a tree structure which reflects the reduction of choice set once a retirement state was picked by husband or wife. The wider branching appears in the groups where the spouses have larger choice sets, specifically have retirement option and it matters whether they take it or not. It is also worth repeating that households' choices are described by a two-dimensional vector which implies that each cell in Table 6 contains the number of options equal to product of powers of the sets faced by spouses. Thus, the biggest number of options a household may have is 16.

2.4 Model presentation

Since in most cases households have more than two alternatives we will apply some multinomial discrete choice model. The model needs to be set up in more than one period of time since by our assumption households take into account the consequences of current choices on future choice sets.

To meet these requirements we make use of a revised version of multinomial logit model described in [Hernæs, Strøm 2001] with some further modifications to stretch it onto three time periods. For traditional multinomial logit see [Dagsvik, 2000], [Greene, 2000], [Maddala, 1983].

The starting point is the random utility framework (see [McFadden, 1973], [Ben-Akiva, Lerman 1985], [Stock, Wise, 1990]). We assume that the utility of each alternative consists of two components – a deterministic part which depends on the characteristics of the alternative as well as the decision making household and the stochastic part which randomly effects the choice. Common assumption in dynamic random utility settings is that the decision makers know the stochastic part of their utility only in present time and rely their judgements about the future on the expectation of random future utility. We add to this a discount factor γ to be able to compare the future and present utility. Denoting $U_{ij}(t)$ the random utility of household at time t when husband occupies state i and wife occupies state j (household subscript is suppresses for simplicity) we have a dynamic random forward-looking recursive utility function of the form

$$U_{ij}(t) = u_{ij}(t) + \varepsilon_{ij}(t) + \gamma E\{ \max_{(x,y)\in S(i,j,t)} U_{xy}(t+1) \}, \qquad (1)$$

where $u_{ij}(t)$ is the deterministic part of the utility and $\varepsilon_{ij}(t)$ is stochastic. We assume that $\varepsilon_{ij}(t)$ are independent and identically extreme value (Gumbel type I) distributed (IDD) with location parameter 0 and scale parameter σ for all i, j and t. The last term in (1) represents the forward-looking nature of the utility. Choice set $S(i,j,t)$ faced by household in time period t+1 depends on the choice (i,j) made in period t according to the rules outlined in Table 6.

Gumbel distribution has very convenient properties (see [Gumbel, 1958]) which allow transferring from the recursive expression (1) to simple multinomial logit model with some additional terms. We will do this separately for two models described below. The first one, model A is a two periods model – it repeats the one reported in [Zhiyang, 2000]. Second one is set up in three periods framework.

Despite their dynamic appearance both models are designed to only analyze the choice made at time T1. Future periods are taken into account in a sense that the households care about their future choice sets – in one year for model A and in two years for model B. Grouping the households by wife's choice sets of type A and B introduced above corresponds to the two models.

Model A

In the two periods setup the second period utility looses it's forward looking term and becomes just

$$U_{rs}(T2) = u_{rs}(T2) + \varepsilon_{rs}(T2). \tag{2}$$

Since the random term is extreme value distributed the whole utility expression also follows this distribution but with the location parameter equal to $u_{rs}(T2)$. Maximum of the extreme value distributed values as well follows it but with the location parameter equal to $\frac{1}{\sigma}\ln\sum_{(r,s)\in S(i,j,T1)}\exp[\sigma\cdot u_{rs}(T2)]$ (see [Gumbel, 1958]) and common scale parameter. Then it is not hard to find

$$E\{\max_{(r,s)\in S(i,j,T1)} U_{rs}(T2)\} = \frac{1}{\sigma}\ln\sum_{(r,s)\in S(i,j,T1)}\exp[\sigma\cdot u_{rs}(T2)] + \frac{\eta}{\sigma}, \tag{3}$$

where η is Euler constant (η ≈ 0.577). Utility function at T1 becomes

$$U_{ij}(T1) = u_{ij}(T1) + \varepsilon_{ij}(T1) + \gamma\frac{1}{\sigma}\ln\sum_{(r,s)\in S(i,j,T1)}\exp[\sigma\cdot u_{rs}(T2)] + \gamma\frac{\eta}{\sigma}. \tag{4}$$

Denote

$$v_{ij}(T1) = u_{ij}(T1) + \gamma\frac{1}{\sigma}\ln\sum_{(r,s)\in S(i,j,T1)}\exp[\sigma\cdot u_{rs}(T1+1)] + \gamma\frac{\eta}{\sigma}, \tag{5}$$

then

$$U_{ij}(T1) = v_{ij}(T1) + \varepsilon_{ij}(T1). \tag{6}$$

It is now clear that one period forward looking model allows for standard multinomial logit interpretations. Indeed, the probability of choosing a particular state (*i*,*j*) by household *h* at T1 can be evaluated as follows (S_0 is the choice set faced by the household at T1, the household script is still suppressed).

$$Pr(i,j,h) = Pr\{U_{ij}(T1) > \max_{(x,y)\in S_0\setminus(i,j)} U_{xy}(T1)\} = Pr\{U_{ij}(T1) - \max_{(x,y)\in S_0\setminus(i,j)} U_{xy}(T1) > 0\} =$$

$$= 1 - L(0) = 1 - \frac{1}{1+\exp[\sigma\cdot v_{ij}(T1) - \sigma\frac{1}{\sigma}\ln\sum_{(x,y)\in S_0\backslash(i,j)}\exp(\sigma v_{xy}(T1))]} =$$

$$= 1 - \frac{1}{1+\frac{\exp(\sigma\cdot v_{ij}(T1))}{\sum_{(x,y)\in S_0\backslash(i,j)}\exp(\sigma\cdot v_{xy}(T1))}} = 1 - \frac{\sum_{(x,y)\in S_0\backslash(i,j)}\exp(\sigma\cdot v_{xy}(T1))}{\sum_{(x,y)\in S_0}\exp(\sigma\cdot v_{xy}(T1))} =$$

$$= \frac{\exp(\sigma\cdot v_{ij}(T1))}{\sum_{(x,y)\in S_0(h)}\exp(\sigma\cdot v_{xy}(T1))}, \qquad (7)$$

where L(x) is the logistic cumulative distribution function. This argument relies on the property that the difference of two extreme value distributed random values has logistic distribution. Now define

$$Y(i,j,h) = \begin{cases} 1 \text{ if household } h \text{ is obsereved in state } (i,j) \text{ after } T1, \\ 0 \text{ otherwise.} \end{cases} \qquad (8)$$

The likelihood function can then be directly written as

$$LF = \prod_{h=1}^{H}\prod_{(i,j)\in S_0(h)} Pr(i,j,h)^{Y(i,j,h)}, \qquad (9)$$

where H is the total number of households. The log-likelihood function is

$$logLF = \sum_{h=1}^{H}\sum_{(i,j)\in S_0(h)} Y(i,j,h)\cdot\ln Pr(i,j,h). \qquad (10)$$

Maximizing likelihood function (or equivalently log-likelihood) one maximizes the probability of the obtained sample to actually be observed. Parameters of the model can be estimated through this procedure. Chapter 4 is devoted to estimation.

Model B

In the three periods model it is reasonable to trace the evolution of the utility function from the recursive form in all periods starting from the third one. At the last period the forward looking component is missing as in model A.

$$U_{kl}(T3) = u_{kl}(T3) + \varepsilon_{kl}(T3). \qquad (11)$$

The same logic as in model A helps to express the second period expectation of the third period best choice as

$$E\{\max_{(k,l)\in S(r,s,T2)} U_{kl}(T3)\} = \frac{1}{\sigma}\ln \sum_{(k,l)\in S(r,s,T2)} \exp[\sigma \cdot u_{kl}(T3)] + \frac{\eta}{\sigma}. \tag{12}$$

At T2 these two variables take form

$$U_{rs}(T2) = u_{rs}(T2) + \varepsilon_{rs}(T2) + \gamma E\{\max_{(k,l)\in S(r,s,T2)} U_{kl}(T3)\} =$$

$$= u_{rs}(T2) + \varepsilon_{rs}(T2) + \gamma \frac{1}{\sigma}\ln \sum_{(k,l)\in S(r,s,T2)} \exp[\sigma \cdot u_{kl}(T3)] + \gamma \frac{\eta}{\sigma}. \tag{13}$$

$$E\{\max_{(r,s)\in S(i,j,T1)} U_{rs}(T2)\} =$$

$$= \frac{1}{\sigma}\ln \sum_{(r,s)\in S(i,j,T1)} \exp\left(\sigma \cdot u_{rs}(T2) + \gamma \ln \sum_{(k,l)\in S(r,s,T2)} \exp[\sigma \cdot u_{kl}(T3)] + \gamma\eta\right) + \frac{\eta}{\sigma}. \tag{14}$$

And finally at T1 we have

$$U_{ij}(T1) = u_{ij}(T1) + \varepsilon_{ij}(T1) + \gamma E\{\max_{(r,s)\in S(i,j,T1)} U_{rs}(T2)\} =$$

$$= u_{ij}(T1) + \varepsilon_{ij}(T1) + \gamma \frac{1}{\sigma}\ln \sum_{(r,s)\in S(i,j,T1)} \exp\left(\sigma \cdot u_{rs}(T2) + \gamma \ln \sum_{(k,l)\in S(r,s,T2)} \exp[\sigma \cdot u_{kl}(T3)] + \gamma\eta\right)$$

$$+ \gamma \frac{\eta}{\sigma}. \tag{15}$$

Assuming

$$w_{ij}(T1) =$$

$$= u_{ij}(T1) + \gamma \frac{1}{\sigma}\ln \sum_{(r,s)\in S(i,j,T1)} \exp\left(\sigma \cdot u_{rs}(T2) + \gamma \ln \sum_{(k,l)\in S(r,s,T2)} \exp[\sigma \cdot u_{kl}(T3)] + \gamma\eta\right) + \gamma \frac{\eta}{\sigma}, \tag{16}$$

we again get simple expression

$$U_{ij}(T1) = w_{ij}(T1) + \varepsilon_{ij}(T1). \tag{17}$$

Once again it is clear that the model allows for simple multinomial logit interpretation with the choice probabilities

$$Pr(i,j,h) = \frac{\exp(\sigma \cdot w_{ij}(T1))}{\sum_{(x,y)\in S_0(h)} \exp(\sigma \cdot w_{xy}(T1))}, \tag{18}$$

and the same as in model A log-likelihood function

$$logLF = \sum_{h=1}^{H} \sum_{(i,j)\in S_0(h)} Y(i,j,h) \cdot \ln Pr(i,j,h)\,. \tag{19}$$

Thus, both models are simple modifications of the standard multinomial logit model. This is due to the special approach to dynamic modelling. Future choices are represented by terms describing the expected best option which will be chosen from the available set in the next periods. The only difficulty left to the estimation stage is the relationship between the choices being made and the choice sets available in the following periods. In calculating the values for $v_{ij}(T1)$ and $w_{ij}(T1)$ all the branches of the decision tree presented in Table 6 must be carefully followed. Columns 1-3 are used for model A and columns 1-5 for model B. Otherwise, the models are estimated with standard maximum likelihood procedure.

Chapter 3. Functional forms and data description

3.1 Data source

The sample used for estimation and described in section 2.2 was obtained from the databank of Frisch Centre for Economic Research. This databank covers the whole Norwegian population and contains information on demography, received wages, social benefits and pensions, capital income and other forms of income, paid taxes, unemployment, education and some other issues. Primarily the data comes from Statistics Norway, but also from some other institutions. The best covered time period is 1992 to 1997, though some files have records up to 2002. Some files are organized yearly, but other have monthly and even week-by-week data. Most of the time one record corresponds to an individual, each of whom have special code number (different from the social security number to guarantee anonymity). These individual codes are the same in all the files allowing linking different types of information and studying different aspects of individual life such as labour market behaviour.

Based on the data available the modelling time period was chosen so that at least one calendar year before T1 and one calendar year after T1 are covered by the data files so that initial earnings information can be obtained as well as both initial and chosen states. The variables describing future periods (from the point of view of a household making decision at T1) were predicted based on the last year figures. The wage income and all forms of non-labour income were predicted with constant linear extrapolation of the last year amounts. Pensions were calculated according to official rules used by the authorities (details are given in Appendix A). Full scale tax function was applied for the household income (the tax function details can be sound in [Haugen, 2000]). Also the following assumptions were made:

- Part-time work was assumed to result in half of the wage income from the full-time work. Similarly the full-time wage income was assumed to equal twice the part-time one.
- The previous year wage income for the wives who did not work in the initial period was estimated by the average of the last ten non-zero pension points plus one multiplied by the corresponding basic amount (G).
- Some individuals working in private sector have access to occupational pension which is calculated with the help of the model described in [Iskhakov, Kalvarskaia, 2003].
- Pensioners (with NIS, AFP, OP or disability pensions) in the initial period were assumed to receive the same pensions in the next periods.

Thus, the use was made of the predicted values for the variables describing available states. Instead, the actual observed values from the data files could be used for the states chosen (not occupied states characteristics would have to be predicted anyway). However, the latter approach has many drawbacks – the figures are disturbed by unobserved or unpredicted factors, less households can be included in the sample since three years after T1 must be covered by the data. But the main disadvantage of such technique is a big distortion in modelling the actual decision taking process – the households at T1 only observe figures from the previous years and have to predict future values by themselves. This is what is done in the former approach to characteristic data construction making it more accurate and realistic.

Appendix B contains summary statistics on the constructed sample. Table 18 displays the frequencies for states observed in the initial period and those chosen by the individuals in the following period, Figures 3 and 4 present the same information graphically. Figure 5 presents the distribution of waiting time (in days) before the AFP pension is taken out by husbands for the first 21 months. If continued the distribution shows the same structure – the kinks correspond to the beginning of the years, thus to the husbands' birthdays, when many people prefer to retire, whereas the beginnings of calendar years (which are also popular moments for shifts) are scattered along the time line and do not show up. Table 19 contains two dimensional (husbands and wives) distribution between public and private sectors. Finally, Tables 20 to 29 show the states occupied by households in the initial and the following periods in the form of transfer matrixes –correspondingly full matrix and separate matrixes for the groups (type B).

3.2 Utility functional forms

Utility function to describe labour market behaviour generally depends on disposable income and leisure term. Applying this design to constructing household utility rises the problem of comprising the variables of husband and wife together and reflecting their interplay as accurately as possible. We follow the path discovered by the previous studies in this field (see [Hernæs et al., 2001, 2002a, 2002b]) and use combined household disposable income together with three variables describing joint as well as individual leisure for the spouses.

Denote $I_{ij}(h,t)$ the household disposable income and $L_{ij}^{H}(h,t)$, $L_{ij}^{W}(h,t)$ correspondingly husband's and wife's leisure with $L_{ij}(h,t)$ being their common leisure. Indexes i and j correspond to the states spouses occupy, h is the index of household while t is the index of time.

Several functional forms for the utility can be applied. Although it seems that Box-Cox transformation is agreed to be the best in the behavioural models estimated on the labour market data [Hernæs et al., 2001, 2002a, 2002b] we still consider several functional forms for comparison.

We consider the following forms of the utility function:

- Linear utility is very convenient in computational sense and it is reasonable to start estimation with this functional form. With notations given above the deterministic part of the utility will look like (household index is suppressed)

$$u_{ij}(t) = a \cdot I_{ij}(t) + b_1 \cdot L_{ij}^H(t) + b_2 \cdot L_{ij}^W(t) + b_3 \cdot L_{ij}(t). \quad (20)$$

- Cobb-Douglas utility is very famous functional form, in line with linear utility they form two border cases between linear and logarithmic specifications. In case of Cobb-Douglas utility the formula used is

$$u_{ij}(t) = a \cdot \ln I_{ij}(t) + b_1 \cdot \ln L_{ij}^H(t) + b_2 \cdot \ln L_{ij}^W(t) + b_3 \cdot \ln L_{ij}(t). \quad (21)$$

- The middle case is Box-Cox utility – non-linear even in parameters and specified in such a way that with different values of parameters λ_i can be either linear, logarithmic or in between[d]. Utility in this case looks like

$$u_{ij}(t) = a\frac{[I_{ij}(t)]^{\lambda_1}-1}{\lambda_1} + b_1\frac{[L_{ij}^H(t)]^{\lambda_2}-1}{\lambda_2} + b_2\frac{[L_{ij}^W(t)]^{\lambda_3}-1}{\lambda_3} + b_3\frac{[L_{ij}(t)]^{\lambda_4}-1}{\lambda_4}. \quad (22)$$

It is also possible to combine different setups in one function making it for example linear with respect to income and logarithmic to the other variables. In Box-Cox transformation the covariates may have individual specific λ parameters or common one. The latter simplification may help a lot while estimating the model, see for example [Seaks, Layson 1985]. We shall try different utility specifications from the drown set and compare the results.

3.3 Components of disposable income

The household disposable income $I_{ij}(h,t)$ variable is quite complex – both husband's and wife's individual incomes as well as taxes paid by the household are taken into account resulting in the following formula.

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[d] Namely, with λ=1 the function is linear while with $\lambda \to 0$ the function approaches logarithmic form.

$$I_{ij}(h,t) = I^H(i,j) + I^W(i,j) - T^H[I^H(i,j), I^W(i,j)] - T^W[I^H(i,j), I^W(i,j)],$$

$$h \in \{1,..H\},\ t \in \{T1,T2,T3\}, \tag{23}$$

where $I^H(i,j)$ and $I^W(i,j)$ are correspondingly husband's and wife's individual incomes. $T^H[I^H(i,j), I^W(i,j)]$ and $T^W[I^H(i,j), I^W(i,j)]$ are their tax functions which depends not only on the individual incomes of the spouses, but also on their states *i* and *j*.

Salary and four types of pensions are taken into account to form income. Although this is a simplification, the major sources are accounted for. Table 7 summarizes the components of household's income for each state husband or wife are in.

	Type of income	States of the spouses				Extrapolation rules
	Husband's states	Full-time work	Part-time work		Immediate or delayed retirement	
	Wife's states	Full-time work	Part-time work	Out of labour force	Retirement	
1	Salary	+	+ (1/2 of full value)	-	-	Constant from initial value
2	NIS or AFP pension	-	-	-	+	Recalculated according to the rules[e]
3	OP pension	-	-	+	+	Constant, calculated from initial salary
4	Disability benefits	-	+	+	+	Constant from initial value

Table 7. Components of individual income and employed extrapolation rules.

The last column in Table 7 repeats the forecasting principles adopted in the study. We assume that households use the last known values to form their estimations about the future incomes. Also simple one half rule is used to relate full- and part-time jobs. These are perhaps the most simple techniques of predicting the future, but they are likely to be the most accurate in describing how forecasting is done by regular people. The only source of income that is calculated accurately is pension. Still, figures of the calendar year containing T1 are used for all future periods to simulate the forecasting inside of households done at T1.

Table 30 in Appendix B contains descriptive statistics on different income sources.

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[e] Except for those wives already observed in retirement state initially, their pension is not recalculated.

Different components are summed up to obtain individual incomes, then they are taxed separately in accordance to the spouse's income and state, and finally summed up together to result in household disposable income. Once calculated the income variable is scaled down by 100 000, this should be remembered when interpreting the results.

3.4 Leisure calculation

Both individual leisure terms are presented in the model and the common leisure is introduced to take care of the interrelation of the first two. This seems to be one of the best ways to solve the difficulty of comprising hardly compatible variables together to describe household and was adopted in many previous studies. Somewhat questionable is the way to calculate common leisure – we apply the minimum principle as most relevant.

$$L_{ij}(t) = \min(L_{ij}^{H} (t), L_{ij}^{W} (t)). \tag{24}$$

One other issue is the minimum sleeping time which may or may not be exogenously fixed. We adopt the point of view expressed in [Dagsvik, Strøm 1997] and introduce 8 hours sleeping time a day. Finally, arithmetically leisure terms are presented in a form of fraction of free time to all time available. Table 8 contains the calculation results for different states.

Husband's state	Wife's state	Leisure
Full-time work	Full-time work	$1 - \frac{37.5 \cdot 46 + 8 \cdot 365}{8760} = \frac{823}{1752} = 0.46975$
Part-time work	Part-time work	$1 - \frac{37.5 \cdot 23 + 8 \cdot 365}{8760} = \frac{1991}{3504} = 0.56821$
Delayed retirement	-	$1 - \frac{37.5 \cdot 23 + 8 \cdot 365}{8760} = \frac{1991}{3504} = 0.56821$
Immediate retirement	Retirement	$1 - \frac{8 \cdot 365}{8760} = \frac{2}{3} = 0.66667$
-	Out of labour force	$1 - \frac{8 \cdot 365}{8760} = \frac{2}{3} = 0.66667$

Table 8. Leisure terms.

3.5 Problem of identification

Before proceeding to estimation it is necessary to examine the model specified in the previous sections once more to answer the question – which parameters is it possible to estimate and which are not identifiable. Quite often it happens that some additional assumptions have to be taken in order to get just one vector of estimated parameters instead of the family of such vectors. For example, in ordinary multinomial logit model with additive utility function the parameters can only be estimated up to a linear shift, that's why usually the first one is assumed to be zero and the parameter vector then appears to be fixed. Since in the current study we deal with modification of multinomial logit, it is very likely that we get some unidentifiable parameters as well.

To investigate the issue we start with listing all the parameters of the models. These are the parameters of the utility functions (a, b_1, b_2, b_3, λ_1, λ_2, λ_3, λ_4) plus discount factor γ and scale parameter of the stochastic part of the utility σ.

After combining the formulas (5) and (7) we get

$$Pr(i,j,h) =$$

$$= \frac{\exp(\sigma \cdot u_{ij}(T1) + \gamma \ln \sum_{(r,s)\in S(i,j,T1)} \exp[\sigma \cdot u_{rs}(T2)] + \gamma\eta)}{\sum_{(x,y)\in S_0(h)} \exp(\sigma \cdot u_{xy}(T1) + \gamma \ln \sum_{(r,s)\in S(x,y,T1)} \exp[\sigma \cdot u_{rs}(T2)] + \gamma\eta)} =$$

$$= \frac{\exp\left(\sigma \cdot u_{ij}(T1) + \gamma \ln \sum_{(r,s)\in S(i,j,T1)} \exp[\sigma \cdot u_{rs}(T2)]\right)}{\sum_{(x,y)\in S_0(h)} \exp\left(\sigma \cdot u_{xy}(T1) + \gamma \ln \sum_{(r,s)\in S(x,y,T1)} \exp[\sigma \cdot u_{rs}(T2)]\right)}. \tag{25}$$

The constant term in the expressions under the exponential signs cancel away. The same would happen with any constant term in the utility function, they are introduced without one in (20-22). Similar things happen for model B. Plugging (16) into (18) we get

$$Pr(i,j,h) =$$

$$\frac{\exp\left\{\sigma \cdot u_{ij}(T1) + \gamma \ln \sum_{(r,s)\in S(i,j,T1)} \exp\left(\sigma \cdot u_{rs}(T2) + \gamma \ln \sum_{(k,l)\in S(r,s,T2)} \exp[\sigma \cdot u_{kl}(T3)] + \gamma\eta\right)\right\}}{\sum_{(x,y)\in S_0(h)} \exp\left\{\sigma \cdot u_{xy}(T1) + \gamma \ln \sum_{(r,s)\in S(x,y,T1)} \exp\left(\sigma \cdot u_{rs}(T2) + \gamma \ln \sum_{(k,l)\in S(r,s,T2)} \exp[\sigma \cdot u_{kl}(T3)] + \gamma\eta\right)\right\}} \tag{26}$$

Furthermore, we notice that parameter σ enters the right hand side in (25) and (26) only together with the utility function. This implies that it only changes the scale of the utility measure allowing for substitution by slightly different specification of the utility with the same resulting probabilities *Pr(i,j,h)*. Or in other words since the utility is linear in parameters a, b_1, b_2, b_3 we are only able to identify σa, σb_1, σb_2, σb_3 and not σ separately.

Thus, without loss of generality, it is possible to state

$$\sigma = 1. \tag{27}$$

This leaves the model with nine parameters (a, b_1, b_2, b_3, λ_1, λ_2, λ_3, λ_4, γ) which have to be estimated.

Chapter 4. Estimation

4.1 Estimation method

The models described in chapter two are estimated with maximum likelihood method (ML).

Maximum likelihood is the widely used method for estimating the models with discrete dependent variable like the ones we have developed here. It is based on the idea to maximise the probability of obtaining the very sample which was observed. This maximization is straightforward for the samples drawn from discrete random value distributions but it is a little less clear for the continues distributions. In the discrete dependent variable models, however, the choice making agent only has a finite set of options to choose from and his choice is very well represented by the discrete distribution. For each agent the probability of one particular option to be chosen is strictly defined and generally greater than zero, the joint density for all observations is obtained by multiplying the individual densities due to the independence of observations in a random sample. Thus the probability of obtaining the sample is simply the product of the individual probabilities of actually chosen states. The choice probabilities for each of the alternatives follow from the model specification and are dependent on some parameters. In turn the probability of obtaining the sample is dependent on them. According to the maximum likelihood method it is maximized with respect to these parameters. The probability of obtaining the sample is then called a likelihood function (of the parameters). Usually, it is computationally less difficult to maximize the logarithm of this function while this results in the same parameter values since logarithm is positive monotone transformation.

Formulas (7-10) and (18-19) show theoretical preparation for the utilization of maximum likelihood method for models A and B accordingly.

The maximum likelihood method presents the following properties (for details se for example [Greene, 2000]):

- Consistency. The ML estimators vector $\widehat{\theta}_{ML}$ converges (component wise) in probability to the true value of the parameter vector θ. In other words for any small positive number δ (n is number of observations)

$$\lim_{n\to\infty} \Pr(\left|\widehat{\theta}_{ML} - \theta\right| > \delta) = 0\,. \tag{28}$$

- Asymptotic normality. The ML estimators vector $\widehat{\theta}_{ML}$ converges in distribution to the multidimensional normal distribution with mean θ and covariance matrix $I(\theta)^{-1}$,

$$I(\theta)^{-1} = -E\{\frac{\partial^2 \ln L}{\partial\theta\,\partial\theta'}\}, \quad (29)$$

where L is likelihood function. In other words if G is the cumulative distribution function of the ML estimator and N is the normal cumulative distribution function with given parameters for every x holds

$$\lim_{n\to\infty}\left|G(x) - N(x)\right| = 0. \quad (30)$$

- Asymptotic efficiency. With the number of observations approaching infinity the variance of the ML estimator approaches the lower boundary for consistent estimators.

So, maximum likelihood estimators have nice properties in the asymptotic sense. This supports well-known point that this method is not the best one for the finite samples and may easily lead to biased estimation (ML estimation of the sample variance is one example). However, in the current study the usage of the maximum likelihood method is quite reasonable. The sample contains 19 027 observations and is built on the whole Norwegian population bases. It can hardly be considered small or finite and thus the estimates obtained in the paper are hardly biased. For this big sample asymptotic properties are very likely to be in full effect.

The second property gives very useful instrument to test hypothesis on parameters and build (asymptotic) confidence intervals for them. The hypothesis of a parameter to equal zero will look very much like ordinary t-tests but with critical values coming from the normal distribution tables.

4.2 Valuation criteria

In order to be able to make some judgements about the quality of the models we have to develop some valuation criteria. In general it is very hard to say that one model is better than the other, but in practice often one has to be chosen. This task can be very hard and demanding. Intuition and personal experience may take the leading role. There are some guidelines which highlight the important issues in choosing the best model, but these are only recommendations. Rephrasing [Gujarati, 1978] they are:

- Parsimony. A model can never be a complete and accurate description of the reality. A model should be kept as simple as possible but capture the effects which are to be described.

- Identifiably. There must be only one estimator for every parameter.

- Goodness of fit. The model should explain as much as possible from the variation of the dependent variable by the explanatory variables included.
- Theoretical consistency. Any model has to be in consistency with the theory. Without theoretical background any perfectly fitted model is irrelevant.
- Predictive power. The model has not only be well-fitted inside of the sample, but also outside of the sample with the further observations. If so, the model can be efficiently used for predictions outside of the sample.

From the first look, the first two points are already met during the set up of the model, and the last one is left out of the scope of current study, so the main attention will be paid to the goodness of fit and theoretical consistency issues. The signs of the parameters must be those expected, so that estimation results are in line with the theory of a rational consumer. And specifications with different utility functions will be compared by goodness of fit.

Although the judgements do not entirely rely on some formal tests, there are some numerical criteria which can be applied in order to support the decision. The first one is mentioned in the previous section – we already have an instrument to test hypothesis about the parameter estimates. Another one is likelihood ratio (LR) which enables us to test complex hypothesis. It will play the role of F-test in the traditional regression analysis to test the validity of the model in whole. The test is calculated according to

$$LR = \frac{L_R}{L_U}, \tag{31}$$

where L_R and L_U are correspondingly the values of the likelihood function maximized under constraint $f(\theta)=0$ and without such constraint (global maximum). If $f(\theta)=\sum_i \theta_i^2$ then the test can be used to test for H_0: $\theta_i=0$, $\forall i$. Under the null hypothesis $-2\ln LR$ is chi-square distributed with the number of degrees of freedom equal to the number of parameters.

Two other connected to LR tests (Wald and Lagrange multiplier tests) can also be used for approximately the same purpose, but all three are identical asymptotically, so only LR will be used as least demanding in the computational sense.

Connected to the LR test is the measure of goodness of fit called likelihood ratio index (LRI).

$$LRI = 1 - \frac{\ln L_U}{\ln L_R}. \tag{32}$$

The restriction in our case (as it is most of the times) will be equalizing all the parameters to zero. If the model as whole is relevant then the fraction on the right hand side will be small, the further parameter estimates are from zeros, the less, and the index value will grow. Besides, it is clear that index is zero if all the slopes are zero and never exceeds one. So, it can be considered a substitution for conventional R square, although the values between 0 and 1 do not have explicit interpretations.

We will use these three instruments in order to compare the estimation results below.

4.3 Estimation results

The models were estimated with the use of TSP 4.5 statistical package which allows one to perform general maximum likelihood estimation. The Berndt-Hall-Hall-Hausman (BHHH) convergence algorithm was applied to find the maximum of the log-likelihood function and corresponding Eicker-White procedure was used to calculate the approximation of the matrix of second derivatives in order to calculate standard errors according to (29). The standard errors for the estimators are calculated automatically while other two indicators of model quality had to be programmed manually.

In spite of the fact that powerful computer[f] was employed for calculations, the models and especially model B caused substantial difficulties for estimation. The origin of these difficulties is the fact that potential pension in future periods is dependent on the history of earnings, thus on the actions taken in current and previous periods. So, the potential pension at T2 has to be computed separately for all feasible states at T1 and further more, potential pension at T3 has to be computed separately for all feasible combinations of states at T1 and T2. Thus, generally, only for pension levels one needs 16 variables for T1, 256 for T2 and 4096 for T3. The available copy of TSP package, however, limits the number of variables in the model to just 4000. Fortunately, due to the absorbing property of retirement not all paths are possible, thus bringing the numbers down by about a half. But in the same time different pension levels imply different level of gross household income with different amounts of tax to be paid. Besides, about 2000 constants are needed to describe interrelation of the choice sets (see Table 6), they enter the log-likelihood function. Finally, the large number of observations also contributes to large processing time and memory usage.

The reasons stated above made it impossible to estimate model B within the current work, it will be done during the further development in the project.

[f] Dell server P4600 Intel XEO with N CPU 2.0GHz in the individual use.

So, we concentrate on the model A estimation results. This model did not cause severe problems although the time of estimation was considerable – about 15-20 minutes for the mentioned computer.

One of the most tricky parts in the ML estimation is the initial values of the parameters. Since the estimation procedure is based on the gradient method (with analytically evaluated derivatives) it is possible to confuse global and local maximums of the likelihood function, if there are any. With such a complex structure of the log-likelihood like we have here this is a particular concern. That is why in order to obtain initial values for the parameters a leading simple multinomial logit model was estimated.

Tables 9-13 contain the estimation results for different utility specifications. Besides the three given in section 3.2, two more – lin-log and Box-Cox-log – were estimated. On the top of each table the used utility function is shown and on the bottom – the values of the criteria discussed in the previous section. Standard errors for the estimates are computed from the covariance of the analytic first derivatives.

Utility specification:
$u_{ij}(t) = a \cdot I_{ij}(t) + b1 \cdot L_{ij}^{H}(t) + b2 \cdot L_{ij}^{W}(t) + b3 \cdot L_{ij}(t)$

Number of observations = 19027			Log-likelihood = -45 599.8	
Parameter	Estimate	Standard error	t-statistic	P-value
a	0.611284	0.017964	34.0291	0.000
b_1	2.34554	0.157116	14.9287	0.000
b_2	3.26555	0.135074	24.1759	0.000
b_3	-3.99905	0.186731	-21.4161	0.000
γ	.016509	0.010870	1.51881	0.129
-2logLR = 1851.63158			LRI = 0.019899	

Table 9. Model A. Estimation results with linear utility function.

First is the simplest linear utility (Table 9). This specification is usually the easiest for estimation and may serve as good starting point. In our case the model is estimated quite sharply but unfortunately, the time discount factor is not significantly different from zero. This literally brings the model back to the static multinomial logit model with some restrictions on the choice sets. It turns out that most of the models tend to be estimated with insignificant discount factor. Otherwise, the model is significant as whole (-2logLR value is much greater than the critical value from the Chi-squared distribution with five degrees of freedom), and all utility specifications lead to significant models. Another drawback for the linear specification is the low value of goodness of fit indicator. The LRI indicator correspond to the R-square in traditional regression analysis.

Utility specification: $u_{ij}(t) = a \cdot lnI_{ij}(t) + b_1 \cdot ln\, L_{ij}^{H}(t) + b_2 \cdot ln\, L_{ij}^{W}(t) + b_3 \cdot lnL_{ij}(t)$				
Number of observations = 19027			Log-likelihood = -45 620.7	
Parameter	Estimate	Standard error	t-statistic	P-value
a	1.20916	0.039061	30.9556	0.000
b_1	1.26028	0.088441	14.2499	0.000
b_2	1.68845	0.078638	21.4711	0.000
b_3	-2.19318	0.105151	-20.8573	0.000
γ	0.015589	0.997458E-02	1.56291	0.118
-2logLR = 1809.89310			LRI = 0.019450	

Table 10. Model A. Estimation results with Cobb-Douglas utility function.

Second is the Cobb-Douglas specification. Again, the model is estimated quite sharply and is significant as a whole, but the time discount component is insignificant and the goodness of fit measure is low.

Utility specification: $u_{ij}(t) = a \cdot I_{ij}(t) + b1 \cdot log[L_{ij}^{H}(t)] + b2 \cdot log[L_{ij}^{W}(t)] + b3 \cdot log[L_{ij}(t)]$				
Number of observations = 19027			Log-likelihood = -45 647.8	
Parameter	Estimate	Standard error	t-statistic	P-value
a	0.557795	0.018476	30.1900	0.000
b_1	1.10229	0.091923	11.9915	0.000
b_2	1.61018	0.077507	20.7748	0.000
b_3	-2.26082	0.104646	21.6046	0.000
γ	-0.856262E-02	-0.011466	0.746770	0.445
-2logLR = 175.71372			LRI = 0.018868	

Table 11. Model A. Estimation results with linear-logarithmic utility function.

Utility specification: $u_{ij}(t) = a\dfrac{[I_{ij}(t)]^{\lambda} - 1}{\lambda} + b_1 \cdot ln\, L_{ij}^{H}(t) + b_2 \cdot ln\, L_{ij}^{W}(t) + b_3 \cdot lnL_{ij}(t)$				
Number of observations = 19027			Log-likelihood = -45 600.6	
Parameter	Estimate	Standard error	t-statistic	P-value
a	.978830	.057756	16.9476	0.000
b_1	1.32216	.090446	14.6182	0.000
b_2	1.73522	.079265	21.8913	0.000
b_3	-2.14791	.104769	-20.5013	0.000
γ	.028669	.010806	2.65301	0.008
λ	0.395139	.069048	5.72265	0.000
-2logLR = 1850.1818			LRI = 0.0198834	

Table 12. Model A. Estimation results with Box-Cox - logarithmic utility function.

Next we try the combination of the first two specifications with logarithms of leisure terms and additive income. The model reveals all the same characteristics as the parent specifications. Again, there is a tendency to static behaviour and low goodness of fit criteria while the parameters are well identified and the model is significant as a whole.

The fourth model (Table 12) presents the middle case between the Cobb-Douglas and the linear-logarithmic specifications. Here Box-Cox transformation is applied to the income covariate, thus giving it freedom to move "between" logarithm and linear function. Though this specification does not allow resolving low goodness of fit measure, finally we get significant time discount factor while all other parameters are also sharply estimated.

Utility specification:

$$u_{ij}(t) = a\frac{[I_{ij}(t)]^{\lambda_1}-1}{\lambda_1} + b_1\frac{[L^H_{ij}(t)]^{\lambda_2}-1}{\lambda_2} + b_2\frac{[L^W_{ij}(t)]^{\lambda_3}-1}{\lambda_3} + b_3\frac{[L_{ij}(t)]^{\lambda_4}-1}{\lambda_4}$$

Number of observations = 19027			Log-likelihood = -45 600.6	
Parameter	Estimate	Standard error	t-statistic	P-value
a	1.14896	0.057311	20.0476	0.000
λ_1	0.067156	0.067693	0.992066	0.321
b_1	1.35948	0.752407	1.80685	0.071
λ_2	0.039542	0.955579	0.041381	0.967
b_2	10.9481	1.44931	7.55400	0.000
λ_3	3.80438	0.284234	13.3846	0.000
b_3	-0.022816	0.051383	-0.444038	0.657
λ_4	-5.53213	3.42491	-1.61526	0.106
γ	0.313956	0.081385	3.85765	0.000
-2logLR = 2439.9818			LRI = 0.026222	

Table 13. Model A. Estimation results with Box-Cox utility function.

To conclude the session of different utility specification tryouts we consider full Box-Cox case where all covariates are transformed with their own lambda parameters. Although this is the most general specification the estimation does not lead to any satisfactory result. As it is seen from Table 13, many of the parameters are insignificant. The increase in LRI indicator can be explained by the increased number of parameters compared to previous specifications.

4.4 The final model

All the specifications considered in the previous section result in the tendency to static rather than dynamic behaviour. People appear to be very myopic taking into account just small portion of their expected future utility. This result is quite outstanding – previous researches (see, for example, [Zhiyang, 2000]) indicated discount term around 30%. Definitely, this issue needs further investigation.

Among all specifications which fail to estimate the time discount term there are two which manages to do this. Box-Cox-logarithmic form however is the only one that also leads to nice estimations of all the rest of the parameters. Besides, it does not perform worse than the other models from the goodness of fit point of view. As a matter of fact, the goodness of fit indicator LRI from Table 12 is only below that of the linear model and full Box-Cox specification.

Still the chosen model possesses the same drawbacks as the rest of the models. The most serious of all is low fit. This may be due to the absence of agent characteristics being included in the model together with the choice of option attributes. Also different groups of households (by GroupB, for example) analysed together may cause this poor fit. Nevertheless, the coefficients estimated display logic signs, the model is significant. The issue of poor fit must be investigated further.

The best model from the five presented is the Box-Cox-logarithmic specification with the parameter estimates given in Table 12. This specification will be used in the next chapter for policy simulation.

Chapter 5. Policy simulation

5.1 Altering taxation rules proportionally

Policy simulation is usually the main objective for the estimation of a labour behavioural model. To confirm this we should simulate one possible policies aimed on giving the population better incentives to work longer. However, full scale policy simulation is out of scope of the current thesis and examples given below only serve for illustrative purposes.

The policy we try is to increase the tax burden of the household by 10 present. This is a very harsh version of the policy rising the tax level in some intelligent way. So, in formula (23) the added taxes of husband and wife are multiplied by 1.1. To illustrate the consequences of such policy Table 14 contains the predicted distribution of households among states under regular taxation together with the same distribution under alternated taxation.

State		Observed		Regular taxation		Alternated taxation	
H	W	Frequency	Percent	Frequency	Percent	Frequency	Percent
1	1	2944	15,473 %	22	0,116 %	1387	7,290 %
1	2	544	2,859 %	464	2,439 %	1937	10,180 %
1	3	2480	13,034 %	5867	30,835 %	8108	42,613 %
1	4	386	2,029 %	1935	10,170 %	2044	10,743 %
2	1	976	5,130 %	0	0,000 %	0	0,000 %
2	2	299	1,571 %	0	0,000 %	0	0,000 %
2	3	985	5,177 %	0	0,000 %	0	0,000 %
2	4	181	0,951 %	0	0,000 %	0	0,000 %
3	1	1633	8,583 %	0	0,000 %	0	0,000 %
3	2	379	1,992 %	0	0,000 %	0	0,000 %
3	3	1681	8,835 %	8	0,042 %	1	0,005 %
3	4	367	1,929 %	21	0,110 %	30	0,158 %
4	1	2813	14,784 %	10408	54,701 %	5350	28,118 %
4	2	656	3,448 %	302	1,587 %	170	0,893 %
4	3	2259	11,873 %	0	0,000 %	0	0,000 %
4	4	444	2,334 %	0	0,000 %	0	0,000 %
Total		19027		19027		19027	

Table 14. Distributions of household among states under increased tax burden.

Thus, when the taxes grow, people start working more and retire later. The total number of husbands who choose to stay at work after becoming AFP-eligible grew from 8 288 (43.56%) to 13 476 (70,83%) in the predicted terms. In the same time, the number of husbands who choose retirement declines.

The two columns in Table 14 under "Observed" name are presented for comparison of the observed and predicted distributions. This is again an indicator of insufficient fit.

5.2 Other possible policies

We tried to simulate just one policy which is very simple and straight forward. Other possible governmental actions to induce later retirement given the institutional rules which already exist may be very different. We briefly summarize those following from the model.

It is clear that the estimated model will reflect the effects of policies directly changing the utility of households in certain stages. So, the policies should address either leisure time or the household disposable income.

There is not much that can be done about the leisure time. Still, assuming that leisure is positively evaluated by individuals (positive signs for the leisure terms in the utility function), any action increasing the amount of leisure time in the full-time or half-time working states will make them more attractive keeping all other things constant. This will have greater effect on wives since their utility increase from the increased leisure is larger (according to Table 12).

Concerning the income side of the utility, a lot can be done. Most simple policy is increased taxation as we did show. Other may include altering taxation separately for groups of households or different sources of income. Components of the income, especially those depending on the government can be addresses. For example, slower growth of basic pension G will make the retirement options less attractive. Alternatively, policies stimulating wage income increase will also help. Finally, some public benefits can be set conditioned on the working state of at least one member of the household.

Conclusion

The current thesis was devoted to studying the labour market transitions of the households around the husband retirement age. Specific quasi-dynamic model was developed to describe the influence of he future opportunity set reduction from the choice currently made by households. Two modifications with two and three periods under consideration were developed. Both models are estimated by the maximum likelihood method, but while estimating the three periods model severe difficulties were discovered which made it impossible to estimate in the scope of the current research.

The difficulties resulted from the multiple branching of the likelihood calculation procedure due to different factors. First of all, the sample was divided into 8 groups according to the limitations in the wife's choice sets during the modelling period resulting from her age and AFP-eligibility. Secondly, since the both NIS and AFP calculation procedures require the history of earnings as input, and this history is not fixed during the modelling period, quite widespread tree had to be followed from the first period to the third. The number of variables and necessary constants describing each path for each of the group in the three period setup had exceeded the built-in limitations of the applied computer software.

In order to estimate the tree period model, the likelihood computing procedure has to be optimized from the point of view of variables and memory allocation, but still, maybe some additional software other than TSP is required. This model is left for future development.

The two periods model, however, was successfully estimated for different specifications of utility. The best specification was then chosen. It turned out to be the one with Box-Cox transformation of the income variable (with lambda close to 0.4) and logarithms of the leisure terms. It was applied in policy simulations to check the impact of alternative taxation rules on the labour market and retirement state. It was shown that general increase in tax bundle results in increase of labour force participation, while special regulation of pension taxation may have little effect depending on the structural part of the utility.

The time discount factor has been estimated quite low indicating myopic behaviour of the households. Only about 2% of the future expected utility is accounted for in the present utility.

Further analysis of the issues discussed in the thesis will be undertaken within the pension project at Frisch Centre for Economic Research.

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Appendix

A. Data construction procedures

The files from the Frisch Centre for Economic Research databank (see section 3.1) which were used in the data construction procedures are listed in Table 16.

Internal name	Files descriptions
ATmLTO	Employer registries linked with tax forms (*lonn-trekk opgave*). Used for identification of employer and finding employment characteristics such as salary and the number of days worked.
Demo_fam	Demography and family files. Contain demographical information of the individuals and their spouses. Used for household construction.
FD	Social benefit files. Contain information on NIS, AFP, disability, pensions, junior (*etterlatt*) benefits. Are used for retrieving these Figures.
Likning_Ppoeng	Pension points history from 1967 to 1995. Used for calculating AFP and NIS pensions and also for estimation of potential income for out-of-labour-force wives. Also tax files. Used for retrieving information of non-labour income.
SOFA	Unemployment database. Used only in 'missing husbands' investigation to retrieve information on unemployment

Table 15. Files used for data construction.

Household construction

The families were constructed on the bases of the family file from year 1996 according to the following principles. If a couple is married both spouses have corresponding marital status and references to each other. If a couple is not registered as married but has at least one child both spouses are assigned a common family number which repeats the identification number of female. One problem that arises here is that children are assigned the same family number as well. To avoid creating households of mother and son a 20 years age difference limit was introduced between wife and husband (but not wise versa – husbands are allowed to be older than wives by unlimited number of years). Both types of families were set to one dataset comprising 21 437 observations (see section 2.2).

Definitions of states

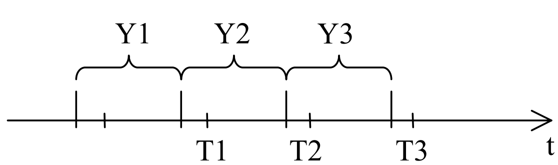

Fig. 2. Calendar year and modelling years.

	Husband			Wife		
Initial period (Y1)	Full-time work	Appear in employer register files in Y1 with more than 350 days worked		Full-time work	Appear in employer register files in Y1 with more than 350 days worked	◄
	Part-time work	Appear in employer register files in Y1 with not more than 350 days worked		Part-time work	Appear in employer register files in Y1 with not more than 350 days worked	◄
	Immediate retirement	Filtered out by sample definition		Out of labour force	Not in any other state	
	Delayed retirement	Filtered out by sample definition		Retirement	Have reached 67 or has taken AFP pension out before T1	●
First modelled year (T1,T2)	Full-time work	Appear in employer register files in Y2 with more than 350 days worked or appear in employer register files in both Y2 and Y3 with more than 700 of total number of days worked	◄	Full-time work	Appear in employer register files in Y2 with more than 350 days worked or appear in employer register files in both Y2 and Y3 with more than 700 of total number of days worked	◄
	Part-time work	Appear in employer register files in Y2 with not more than 350 days worked or appear in employer register files in both Y2 and Y3 with not more than 700 of total number of days worked	◄	Part-time work	Appear in employer register files in Y2 with not more than 350 days worked or appear in employer register files in both Y2 and Y3 with not more than 700 of total number of days worked	◄
	Immediate retirement	Have taken AFP pension out within 60 days after reaching AFP age (including those with negative number of days[g])	●	Out of labour force	Not in any other state	
	Delayed retirement	Have taken AFP pension out 60 or more days after reaching AFP age	●	Retirement	Have reached 67 before T1 or has taken AFP pension out before T2[h]	●

Table 16. Definitions of states. Arrows show overwriting rules.

[g] Small number of husbands was observed with negative waiting time before AFP pension was taken out. Apparently this is due to imperfect implementation of AFP eligibility rules and/or inaccuracies in recorded data.

[h] Retirement definitions for wife in the two periods are overlapping to guarantee the absorbing property.

The states occupied by husband and wives are strictly defined through the records from three calendar years Y1, Y2 and Y3 (see Fig. 2). Table 17 contains the definitions separately for husband and wives and for two periods. Retirement states overwrite the working states in the cases when an individual is observed in both of them.

'Missing husbands' problem

After the households were distributed among the states according to the definitions given above and irrelevant households were filtered out forming the sample of 21 437 observations a considerable difficulty was faced. In 2 263 (10,56% of the sample) households husbands were not assigned any state in the period (T1,T2). To investigate what states these husbands transferred to additional steps were undertaken.

First, disability pension recipients were found. A person was recognized as living on disability if no state was assigned to him previously and he was listed on the disability files (from FD) during the period (T1, T2). 929 husbands were given disability state.

Second, the unemployment database (SOFA) was checked. A person was given an unemployed status if he was listed in the unemployment registries as receiving unemployment benefits for at least one month between T1 and T2 given his state was not previously identified. It appeared that in 761 households instead of retiring through AFP husbands went to living off the unemployment benefits. This quite strange result indicates inaccuracy in AFP eligibility criteria implementation or some unobserved factors that actually lead to this behaviour.

Third, it was checked if any husbands died the next year after becoming AFP eligible. Demography files contain mortality information, but it turned out that none from the sample actually died in their T1 to T2 period.

Thus, 573 households were still in the unknown states in the first modelling year. It was agreed, however, that this is neglecTable number (just 2.67% of the sample) and these households can be simply dropped. These are likely to be the people receiving other types of pensions (left-along pension, etc.) and those who left the country or fell out of the official statistics due to some other reason.

The households with the husband observed in irrelevant state in period (T1, T2) were dropped out of the study forming the sample of 19 174 households. Later some households were also dropped due to the missing variables which were essential for income calculations bringing this number down to 19 027 households.

NIS pension

This and the following two sections give brief description of pension calculation rules applied in the pension data construction procedures for the current study. Details can be found in [Haugen, 2000], [Røgeberg, 2000] and [Iskhakov, Kalvarskaia, 2003].

Old age (NIS) pension is calculated according to the following formulas.

$$NIS = a \cdot G + \max(TP, ST_k), \tag{33}$$

$$TP = G \cdot SLP \cdot (0.45 \frac{x}{N} + 0.42 \frac{N-x}{N}),\ x \leq N \leq 40, \tag{34}$$

$$SLP = b \frac{1}{M} \sum_{t=1}^{M} PP_t,\ M \leq 20, \tag{35}$$

where G is the minimum pension in the particular year, PP_t are the pension points in different years which relate annual salary to G (ordered so that bigger values come first), N as well as M are the numbers of years with PP_t greater than zero, these two variables are truncated at 40 and 20 years correspondingly, x is the portion of the years with earnings before 1991 (inclusive). ST_k is the pension boost for those earning nothing or too little, its values come from the Tables (which can be found in [Haugen, 2000]) differently for five types of individuals:

- k=1. Single persons.
- k=2. Married persons, spouse is pensioner, who gets minimum pension (G).
- k=3. Married persons, spouse is pensioner, who gets pension bigger than G.
- k=4. Married persons, spouse is working and earning less than one G.
- k=5. Married persons, spouse is working and earning more than one G.

Coefficients a and b take the following values:

$$a = \begin{cases} 1 \text{ if } k \in \{1,4\}, \\ 0.75 \text{ if } k \in \{2,3,5\}, \end{cases} \tag{36}$$

$$b = \begin{cases} 1 \text{ if } Y \geq 40, \\ \frac{Y}{40} \text{ if } Y < 40. \end{cases} \tag{37}$$

Y is full number of years worked in the country. Low values are rarely observed here, correction coefficient *b* is mainly designed for immigrants. Since our sample is very unlikely to contain them, we assume *b*=1.

AFP pension

In AFP pension calculation some pension points, namely those supposedly earned between the ages 64 and 67, have to be forecasted. This is done by letting them equal to the maximum of the last three years average and the ordinary calculated *SLP*.

$$FPP = \max\left(\frac{PP_{t-1} + PP_{t-2} + PP_{t-3}}{3}; SLP\right), \qquad (38)$$

where *SLP* is calculated as in the case of NIS. After that the NIS calculation procedure is used in the same way to find the level of AFP pension.

If an individual's chose if to stay in the labour force in some of the modelling years, it is necessary to convert salary to pension points in order to apply formula (38). This is done according to the following conversion procedure.

$$PP = \begin{cases} \frac{S}{G} - 1 \text{ for } G \le S \le 6G, \\ 3 + \frac{S}{3G} \text{ for } 6G \le S \le 12G, \\ 7 \text{ for } 12G \le S. \end{cases} \qquad (39)$$

S denotes annual salary. The rule has changed twice in 1970 and 1991, but our whole sample falls into the latter period so one formula is applied (see [Haugen, 2000] for details).

The described way for AFP calculation is a little rough in a sense that it doesn't take into account several additional qualities (referred to as A_i in [Haugen, 2000]) which are added to the AFP pension according to the sector of economy and which are different in time. But we assume away these minor additions and use the formula provided above.

Most difficulties for the data construction result from the fact that according to the rules the value of the pension depends not only on the state the household occupies (to take care of the spouse occupation and income) and the previous state earnings, but in complex way to the whole previous history of earnings. So, it is impossible to construct just 16 variables for each time period to take care of pensions for each possible state of a household. Several cases must be considered.

First, depending on the spouse's state and earnings coefficient *a* and the value of ST_k can vary. This will give four cases for both husband and wife (since the sample does not contain single persons). Second, if one works in the lasts years before retirement and they come into considerations, the last pension points need to be calculated and the values of SLP and FFP may change. This will not apply for the first period (since the history is fixed then), but it will for the second and third giving 3 cases for each period for wife (zero earnings, full salary from full-time job or reduced salary from part-time job) and 2 cases for each period for husband (full-time and part-time salary). Moreover, chaining effect is in place – each case at T1 will result in two or three more cases next period. In total we get correspondingly for husband and wife 4 and 4 cases in the fist period, 8 and 12 cases in the second and 16 and 36 cases in the third. In total 28 different Figures were constructed to describe husband's pension and 52 Figures to describe that of wife. On the estimation stage the relevant Figures were used for each route household would choose on the decision tree.

Occupational pension

Contrary to the NIS and AFP pensions, occupational pension depends only on the previous position a household occupies, thus simplifying matters a lot. Namely, the pension depends on the last salary earned by OP-eligible person and is not influenced by his or her marriageable status. This simple approaches to specifying the occupational pension was developed with very little rules governing the accruing mechanism and large variety of schemes hosted by the companies.

The pension is calculated in NOK according to the following formulas for males and females correspondingly.

$$OP(males) = -13\,225 + 0.53895 \cdot S - 59\,890 \cdot ind1 - 15\,018 \cdot ind2, \quad (40)$$

$$OP(females) = 16\,099 + 0.34919 \cdot S - 28\,087 \cdot ind1, \quad (41)$$

where *S* is the last salary and industry dummies have unit values for the cases of individual working in the following industries:

- mining and manufacturing for *ind1*,
- agriculture, hunting, fishing, forestry, electricity, gas, water, construction, wholesale and retail trade, restaurants and hotels, transport for *ind2*.

See [Iskhakov, Kalvarskaia, 2003] for the OP data retrieving details.

B. Summary statistics

			Wife				
			1	2	3	4	Total
			Full-time work	Part-time work	Out of labour force	Retirement	
Husband	States observed in the initial period						
	1	Full-time work	9 852	685	6 775	976	18 288
	2	Part-time work	329	64	296	50	739
	Total		10 181	749	7 071	1 026	19 027
	Stated chosen in the following period						
	1	Full-time work	2 944	544	2 480	386	6 354
	2	Part-time work	976	299	985	181	2 441
	3	Immediate retirement	1633	379	1 681	367	4 060
	4	Delayed retirement	2 813	656	2 259	444	6 172
	Total		8 366	1 878	7 405	1 378	19 027

Table 17. Observed initial and chosen by households states (frequencies).

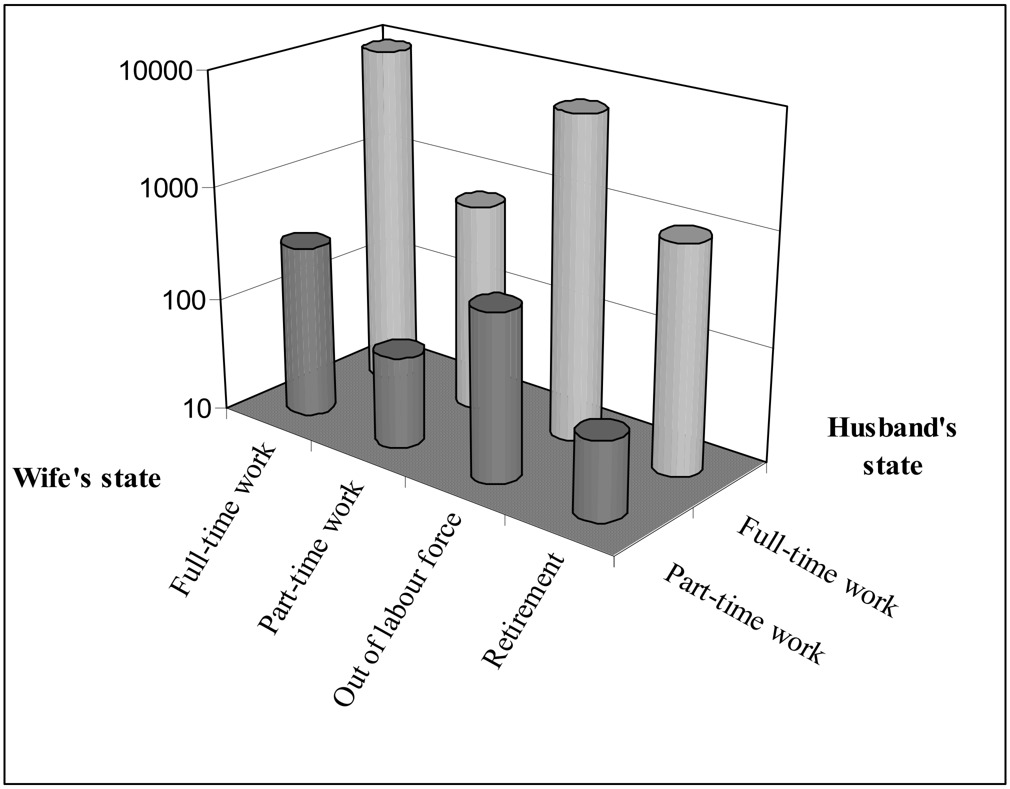

Fig. 3. Distribution of states in the initial period.

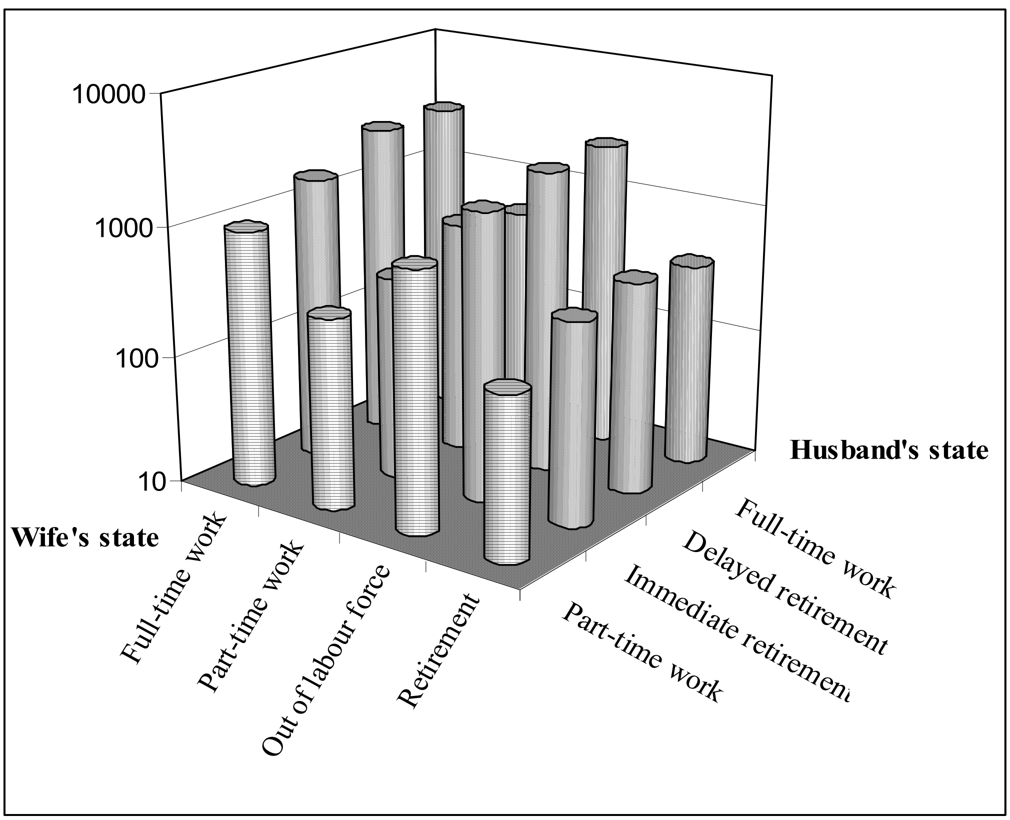

Fig. 4. Distribution of states in the following period.

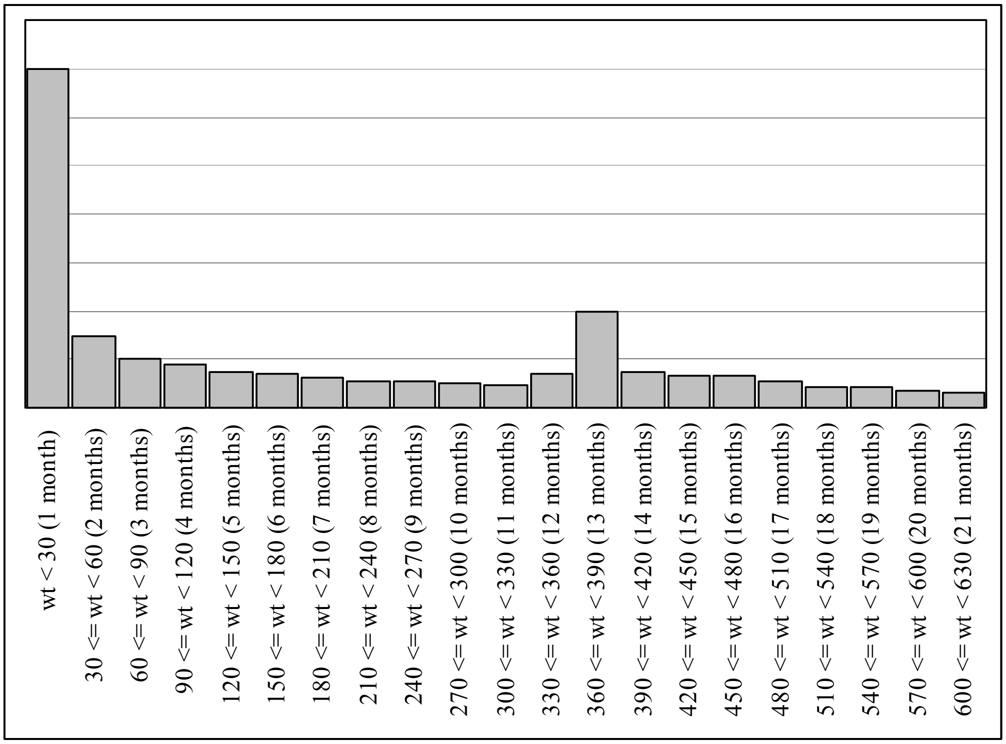

Fig. 5. Waiting time distribution before AFP takeout for husbands (first 21 months).

		Wife		
	Sector	Public	Private	Total
Husband	Public	1 906 15.21%	3 148 25.13%	5 054 40.34%
	Private	1 999 15.96%	5 475 43.70%	7 474 59.66%
	Total	3 905 31.17%	8 623 68.83%	12 528 100.00%
Missing = 6 499				

Table 18. Available information on employment sectors[i].

[i] Public/private sector division was performed on the bases of ATmLTO files with additional information for husbands gained from FD files.

		The following period (T1,T2)																
		H=1 W=1	H=1 W=2	H=1 W=3	H=1 W=4	H=2 W=1	H=2 W=2	H=2 W=3	H=2 W=4	H=3 W=1	H=3 W=2	H=3 W=3	H=3 W=4	H=4 W=1	H=4 W=2	H=4 W=3	H=4 W=4	Total
The initial period (Y1)	H=1 W=1	2 762	430	99	74	865	239	50	34	1 472	299	65	89	2 636	507	92	139	9 852
	H=1 W=2	87	38	90	1	31	18	38	0	45	22	67	1	88	51	108	0	685
	H=1 W=3	31	55	2 218	0	6	25	821	0	17	41	1 434	0	33	78	2 015	1	6 775
	H=1 W=4	0	0	0	302	0	0	0	131	0	0	0	250	0	0	0	293	976
	H=2 W=1	54	18	3	2	63	12	3	4	88	17	4	2	46	10	0	3	329
	H=2 W=2	10	1	3	1	9	1	2	0	9	0	9	1	10	6	2	0	64
	H=2 W=3	0	2	67	0	2	4	71	0	2	0	102	0	0	4	42	0	296
	H=2 W=4	0	0	0	6	0	0	0	12	0	0	0	24	0	0	0	8	50
	Total	2 944	544	2 480	386	976	299	985	181	1 633	379	1 681	367	2 813	656	2 259	444	19 027

Table 19. Full transfer matrix (H=*i* indicates husband's state, W=*j* indicates wife's state).

		(T1,T2)				
		H=1 W=4	H=2 W=4	H=3 W=4	H=4 W=4	Total
The initial period (Y1)	H=1 W=4	302	131	250	293	976
	H=2 W=4	6	12	24	8	50
	Total	308	143	274	301	1026

Table 20. Transfer matrix for GroupB=1 (zero columns and rows not shown, H=*i* indicates husband's state, W=*j* indicates wife's state).

		The following period (T1,T2)														
		H=1 W=1	H=1 W=2	H=1 W=3	H=1 W=4	H=2 W=1	H=2 W=2	H=2 W=3	H=3 W=1	H=3 W=2	H=3 W=3	H=4 W=1	H=4 W=2	H=4 W=3	H=4 W=4	Total
The initial period (Y1)	H=1 W=1	9	11	0	1	2	2	1	1	6	0	10	4	2	1	50
	H=1 W=2	1	0	3	0	0	0	3	0	0	1	0	1	2	0	11
	H=1 W=3	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
	H=2 W=1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1
	Total	10	11	3	1	2	2	4	2	6	1	10	5	4	2	63

Table 21. Transfer matrix for GroupB=2 (zero columns and rows not shown, H=*i* indicates husband's state, W=*j* indicates wife's state).

		The following period (T1,T2)																
		H=1 W=1	H=1 W=2	H=1 W=3	H=1 W=4	H=2 W=1	H=2 W=2	H=2 W=3	H=2 W=4	H=3 W=1	H=3 W=2	H=3 W=3	H=3 W=4	H=4 W=1	H=4 W=2	H=4 W=3	H=4 W=4	Total
The initial period	H=1 W=1	21	18	1	6	4	7	1	2	7	3	0	3	12	16	3	15	119
	H=1 W=2	1	0	6	0	1	0	1	0	0	0	3	0	1	0	5	0	18
	H=2 W=1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1
	Total	22	18	7	6	5	8	2	2	7	3	3	3	13	16	8	15	138

Table 22. Transfer matrix for GroupB=3 (zero columns and rows not shown, H=*i* indicates husband's state, W=*j* indicates wife's state).

		The following period (T1,T2)																
		H=1 W=1	H=1 W=2	H=1 W=3	H=1 W=4	H=2 W=1	H=2 W=2	H=2 W=3	H=2 W=4	H=3 W=1	H=3 W=2	H=3 W=3	H=3 W=4	H=4 W=1	H=4 W=2	H=4 W=3	H=4 W=4	Total
The initial period (Y1)	H=1 W=1	197	70	7	67	60	37	3	32	69	24	6	86	161	62	7	123	1011
	H=1 W=2	4	1	9	1	0	1	5	0	1	2	2	1	1	3	8	0	39
	H=1 W=3	0	1	0	0	0	0	1	0	0	0	0	0	0	0	2	0	4
	H=2 W=1	3	0	0	2	2	1	0	4	6	3	0	2	3	0	0	3	29
	H=2 W=2	1	0	1	1	2	0	0	0	0	0	2	1	1	1	1	0	11
	Total	205	72	17	71	64	39	9	36	76	29	10	90	166	66	18	126	1094

Table 23. Transfer matrix for GroupB=4 (zero columns and rows not shown, H=*i* indicates husband's state, W=*j* indicates wife's state).

		The following period (T1,T2)												
		H=1 W=1	H=1 W=2	H=1 W=3	H=2 W=1	H=2 W=2	H=2 W=3	H=3 W=1	H=3 W=2	H=3 W=3	H=4 W=1	H=4 W=2	H=4 W=3	Total
The initial period (Y1)	H=1 W=1	220	38	0	61	28	0	117	25	0	245	50	0	784
	H=1 W=2	3	2	1	0	1	2	2	2	0	6	2	1	22
	H=1 W=3	0	0	0	0	0	0	0	3	0	0	1	0	4
	H=2 W=1	3	2	0	5	2	0	7	1	0	7	0	0	27
	H=2 W=2	1	0	0	2	0	0	0	0	1	1	0	0	5
	Total	227	42	1	68	31	2	126	31	1	259	53	1	842

Table 24. Transfer matrix for GroupB=5 (zero columns and rows not shown, H=*i* indicates husband's state, W=*j* indicates wife's state).

		The following period (T1,T2)												
		H=1 W=1	H=1 W=2	H=1 W=3	H=2 W=1	H=2 W=2	H=2 W=3	H=3 W=1	H=3 W=2	H=3 W=3	H=4 W=1	H=4 W=2	H=4 W=3	Total
The initial period (Y1)	H=1 W=1	2 275	273	85	727	158	43	1 258	227	54	2 178	347	77	7 702
	H=1 W=2	74	34	66	30	15	24	42	18	53	80	42	86	564
	H=1 W=3	29	53	1 974	6	25	719	16	37	1 266	32	76	1 799	6 032
	H=2 W=1	48	13	3	56	7	3	73	12	2	36	10	0	263
	H=2 W=2	7	1	2	5	1	2	9	0	5	8	4	1	45
	H=2 W=3	0	2	58	2	4	63	2	0	81	0	4	37	253
	Total	2 433	376	2 188	826	210	854	1 400	294	1 461	2 334	483	2 000	14 859

Table 25. Transfer matrix for GroupB=6 (zero columns and rows not shown, H=*i* indicates husband's state, W=*j* indicates wife's state).

		The following period (T1,T2)												
		H=1 W=1	H=1 W=2	H=1 W=3	H=2 W=1	H=2 W=2	H=2 W=3	H=3 W=1	H=3 W=2	H=3 W=3	H=4 W=1	H=4 W=2	H=4 W=3	Total
The initial period (Y1)	H=1 W=1	16	8	3	2	4	1	7	6	1	9	8	0	65
	H=1 W=2	2	0	1	0	0	3	0	0	5	0	1	3	15
	H=1 W=3	1	0	108	0	0	34	1	0	81	0	0	79	304
	H=2 W=1	0	3	0	0	1	0	0	0	0	0	0	0	4
	H=2 W=2	0	0	0	0	0	0	0	0	1	0	0	0	1
	H=2 W=3	0	0	3	0	0	2	0	0	10	0	0	3	18
	Total	19	11	115	2	5	40	8	6	98	9	9	85	407

Table 26. Transfer matrix for GroupB=7 (zero columns and rows not shown, H=*i* indicates husband's state, W=*j* indicates wife's state).

		The following period (T1,T2)												
		H=1 W=1	H=1 W=2	H=1 W=3	H=2 W=1	H=2 W=2	H=2 W=3	H=3 W=1	H=3 W=2	H=3 W=3	H=4 W=1	H=4 W=2	H=4 W=3	Total
The initial period (Y1)	H=1 W=1	24	12	3	9	3	1	13	8	4	21	20	3	121
	H=1 W=2	2	1	4	0	1	0	0	0	3	0	2	3	16
	H=1 W=3	1	1	136	0	0	67	0	1	87	1	1	135	430
	H=2 W=1	0	0	0	0	0	0	1	1	2	0	0	0	4
	H=2 W=2	1	0	0	0	0	0	0	0	0	0	1	0	2
	H=2 W=3	0	0	6	0	0	6	0	0	11	0	0	2	25
	Total	28	14	149	9	4	74	14	10	107	22	24	143	598

Table 27. Transfer matrix for GroupB=8 (zero columns and rows not shown, H=*i* indicates husband's state, W=*j* indicates wife's state).

State at Y1	N obs.	Variable	N obs.	Mean	Std. Dev.	Min	Max
Full-time work	18 288	Salary	18 288	229.570	81.683	36.167	1 240.000
		Days worked	18 288	364.99	0.26	351	365
		Potential OP pension	4 770	102.809	47.185	37.073	482.796
Part-time work	739	Salary	739	161.066	84.456	36.167	507.686
		Days worked	739	157.67	116.16	0	348
		Potential OP pension	150	90.595	45.123	39.027	260.392

Table 28. Initial earnings for husbands in different initial states (1000 NOK).

State at Y1	N obs.	Variable	N obs.	Mean	Std. Dev.	Min	Max
Full-time work	10 181	Salary	10 181	137.555	65.004	36.167	606.427
		Days worked	10 181	364.99	0.27	352	365
		Potential OP pension	2 477	69.698	21.264	37.214	199.770
Part-time work	749	Salary	749	88.024	61.015	36.167	424.376
		Days worked	749	131.54	110.94	0	349
		Disability pension	236	61.097	19.564	0	124.853
		Potential OP pension	130	64.289	21.210	38.482	142.842
Out of labour force	7 071	Potential salary	7 071	83.163	47.904	37.033	346.326
		Disability pension	3 591	60.952	20.462	0	165.756
		OP pension	0	.	.	.	.
Retirement	1 026	NIS pension	1 025	68.628	15.633	0	137.351
		AFP pension	304	21.188	35.748	0	124.508
		Disability pension	205	63.217	20969	29.469	121.474
		OP pension	79	59.926	16.627	39.355	115.033

Table 29. Initial earnings for wives in different initial states (1000 NOK).