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Characterization and Measurement of Duration Dependence in Hazard Rates Models

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Characterization and Measurement of Duration Dependence in Hazard Rate Models

by

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Abstract

As is known from the economic literature, the notion of negative/positive duration dependence defined in terms of a decreasing/increasing hazard function can solely be used as a basis for revealing whether negative/positive duration dependence is present or not. However, when concern is directed to comparison and measurement of the extent of duration dependence in hazard rate models alternative definitions and methods are called for. To this end we propose a stronger as well as a weaker version of the standard definition of duration dependence and demonstrate that these definitions form a useful basis for developing appropriate duration dependence orderings and summary measures of duration dependence.

Keywords: Hazard rate models, duration dependence orderings, summary measures of duration dependence, the Weibull distribution, PH and MPH models.

JEL classification: J64

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1. Introduction

Labor market and population economics are two fields where analysis of duration data plays a key role. Duration of unemployment, job durations, lifetimes of firms, spacing of births and duration of marriages constitute a selection of life history events that are paid much attention in the economic literature. Distributions of duration data are commonly specified in terms of the conditional probability of exit or the hazard function. Application of the hazard function is known to be particularly useful when concern is turned to situations where the probability of leaving the state in question depends on the time spent in the state. If the hazard function is increasing then the probability that a spell of an event will be completed is increasing with the duration of the event. In the economic literature increasing hazard function is normally referred to as a state of positive duration dependence. As demonstrated by Lippmann and McCall (1976) the job search models of unemployment predict positive duration dependence in cases where the "reservation wage" declines. By contrast, Flinn and Heckman (1983) and Van den Berg (1994) demonstrate that negative duration dependence, i.e. decreasing hazard rate, may arise from stigma or discouragement effects whilst Jovanovic (1979) shows that negative duration dependence plays a crucial role in job turnover models. These examples suggest that the notion of duration dependence plays a central role in analyses of the labor market. However, the existing framework for analyzing duration dependence can solely be used to reveal occurrence of positive/negative duration dependence, but does not provide appropriate methods for comparing and measuring the extent of duration dependence in distributions of duration spells. For example, ranking distributions in accordance with the criterion of non-intersecting hazard rates may be of limited value because the hazard rate does not account for the fact that a specific spell will have different occurrence probabilities when two or more distribution functions are compared. This fact suggests that we should use the quantile of the duration distribution function rather than the spell variable as the basic unit in a framework for comparison and measurement of duration dependence.

By defining strong duration dependence in terms of the sum of two comonotone random variables, where one of the variables is exponentially distributed, Section 2 demonstrates that the quantile-specific hazard rate may serve as a basis for ranking distributions with respect to the extent of strong duration dependence. As indicated in Section 2 the ordering defined by the quantile-specific hazard rate proves to be closely related to a well-known dispersion ordering in the statistical literature and a risk ordering in the economic literature. To deal with distributions where the quantile-specific hazard rates intersect weaker ranking principles than the criterion of non-intersecting quantile-specific hazard rate are called for. By aggregating the inverse quantile-specific hazard rate from above a weaker criterion defined by the quantile-specific expected remaining duration curve emerges. The relationship between this duration dependence ordering, denoted weak duration dependence, and second-degree stochastic dominance is explored in Section 3.

The strong as well as the weak duration dependence ordering depends on the mean durations of the distributions being compared. However, a mean independent duration dependence ordering is called for when it is considered important to distinguish between the mean duration and the extent of duration dependence as the origin of an attained ordering of duration distributions. To this end a mean independent version of the weak duration dependence ordering defined in terms of the scaled quantile-specific expected remaining duration curve is introduced in Section 3.

Many parametric distributions, such as the gamma and the Weibull distributions, have monotone hazard rates. However, economic as well as physical phenomena may exhibit hazard rates that are non-monotonic. The typical non-monotone hazard rate decreases initially, then becomes essentially constant, and ultimately increases. Such hazard rates are denoted bathtub shaped. By contrast, parametric distributions, such as the lognormal and the inverse Gaussian distribution, are found to have an upside down bathtub shape[1]. To summarize the duration dependence structure of non-monotone hazard rates summary measures of duration dependence are needed.

Section 4 uses the scaled quantile-specific expected remaining duration curve as a basis for defining summary measures of duration dependence. By introducing an appropriate ordering relation on the set of duration distributions an axiomatic based family of measures of duration dependence is obtained. One of these measures is shown to coincide with the inverse shape parameter of the Weibull distribution when the duration spell is Weibull distributed. As a result, a convenient statistical/economic interpretation of Weibull's shape parameter is obtained. Section 5 discusses the phenomena of duration dependence in proportional and mixed proportional hazard rates models and demonstrates that it is impossible to distinguish between observed/unobserved heterogeneity and relative duration dependence, when concern is directed to comparison and measurement of the extent of relative duration dependence. Section 6 provides concluding remarks.

2. Strong duration dependence

As is well known the exponential distribution acts as a reference distribution in duration analyses since it is characterized by a constant hazard and exhibits no duration dependence. A natural generalization is the Weibull distribution, which allows time dependence of the hazard function. The Weibull distribution, named after the Swedish physicist Waloddi Weibull, was originally introduced to describe the breaking strength of materials and appears to have become the most popular and widely

[1] See e.g. Lancaster (1990) for a discussion on non-monotone hazard rates.

used parametric family of failure distributions in statistics, biostatistics and economics and is defined by

(2.1) $$F_\alpha(y) = 1 - \exp\left(-(\lambda y)^\alpha\right), \quad y \geq 0 \quad \text{and} \quad \lambda, \alpha > 0$$

where α is called the shape parameter and λ is a scale parameter. The Weibull distribution has been considered to be rather flexible and appropriate for describing and analyzing duration of single spells and has been much applied in econometric analyses of the duration of unemployment spells, see e.g. Lancaster (1979), Nickell (1979) and Kiefer (1988). The hazard function of the Weibull distribution is shown to increase or decrease monotonically according to whether the shape parameter α is larger or smaller than 1. The case of an increasing hazard function is referred to in the economic literature as positive duration dependence. Similar, a decreasing hazard function is said to exhibit negative duration dependence.[2]

Since α is treated as a parameter that captures duration dependence one may ask whether F_{α_1} exhibits stronger negative duration dependence than F_{α_2} when $\alpha_1 < \alpha_2 < 1$? To answer this question a definition that clarifies what stronger negative (positive) duration dependence is supposed to mean is required.

The association of negative (positive) duration dependence with a decreasing (increasing) hazard rate suggests that we may use the criterion of non-intersecting hazard rates as a basis for ranking distribution functions according to the extent of duration dependence. However, as will be demonstrated below it is useful to introduce a slightly stronger ordering than what is provided by the criterion of non-intersecting hazard rates. To this end we introduce an alternative definition of duration dependence that associates duration dependence with the addition of comonotone random variables. As noted by Schmeidler (1989) and follows from Definition 2.1 below comonotone random variables are characterized by providing identical orderings of the state space from best to worst (or worst to best).

DEFINITION 2.1. The random variables $X_1, X_2, ..., X_n$ are comonotonic if and only if there exist non-decreasing functions $g_i : \mathbb{R} \to \mathbb{R},\ i = 1, 2, ..., n$ and a random variable U such that $X_i = g_i(U),\ i = 1, 2, ..., n$.

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[2] As an alternative to this definition of (global) duration dependence one may use the following less restrictive definition of (local) duration dependence, see e.g. Heckman and Singer (1986): If $\left(dh(y)/dy\right) > (<) 0$, at $y = y_0$, there is said to be positive (negative) duration dependence at y_0.

The following definition of duration dependence says that strong negative (positive) duration dependence occurs when a random variable is equal to the sum of two comonotone random variables, where one of the variables is exponentially distributed and the other has a c.d.f. with decreasing (increasing) hazard rate.

DEFINITION 2.2. A cumulative distribution function *G* defined on $\left[0,\infty\right\rangle$ exhibits strong negative (positive) duration dependence if there exists a pair of comonotone random variables *X* and *Z* such that $X+Z\sim G$, *X* is exponentially distributed on $\left[0,\infty\right\rangle$ and *Z* has a c.d.f. defined on $\left[0,\infty\right\rangle$ with decreasing (increasing) hazard rate.

Note that the property of strong negative or positive duration dependence is preserved under scale-transformations of the duration variable.

To illustrate the definition of strong negative duration dependence we consider the following example: Let (7, 15, 27, 48) and (10, 23, 43, 80) be independent outcomes from two different duration distributions, where the former set of outcomes is assumed to be from the exponential distribution function. Since the latter set of outcomes can be expressed by the following sum

$$(10,23,43,80)=(7,15,27,48)+(3,8,16,32)$$

and (3, 8, 16, 32) proves to be outcomes from a distribution with decreasing hazard rate, we can conclude that the distribution with outcomes (10, 23, 43, 80) exhibits strong negative duration dependence.

The following proposition demonstrates that strong negative (positive) duration dependence may be given various alternative interpretations.

PROPOSITION 2.1. Let *F* and *G* be cumulative distribution functions defined on $\left[0,\infty\right\rangle$, where *F* is the exponential distribution function with hazard rate λ. Assume that *G* have hazard rate *h* and differentiable inverse G^{-1}. Then the following statements are equivalent,

(i) There exists a pair of comonotone random variables *X* and *Z*, where *Z* has a cumulative distribution function defined on $\left[0,\infty\right\rangle$ with differentiable inverse and decreasing (increasing) hazard rate, such that $X\sim F$ and $X+Z\sim G$

(ii) $$G^{-1}(v) - G^{-1}(u) \geq (\leq) F^{-1}(v) - F^{-1}(u) = \frac{1}{\lambda}\log\left(\frac{1-u}{1-v}\right), \quad \text{for all} \quad 0 < u < v < 1$$

and $h(t)$ is decreasing (increasing)

(iii) $h(t) \leq (\geq) \lambda$ for all $t \in \left[0, \infty\right\rangle$

and $h(t)$ is decreasing (increasing).

PROOF. It is convenient to divide the proof of the equivalence between (i) and (ii) into two parts. First, we will prove the equivalence between (i) and (ii) when the conditions of decreasing (increasing) hazard rates of G and (the c.d.f. of) Z are abandoned. Then, the equivalence between (i) and (ii) follows from Proposition 2 of Landsberger and Meilijson (1994). Next, we will prove the equivalence between the decreasing (increasing) hazard rate of the c.d.f. of the comonotone variable Z in (i) and the decreasing (increasing) hazard rate h of G in (ii).

Assume that Z has c.d.f. K with inverse K^{-1}, density k and hazard rate m, and G and F have densities g and f and assume that (i) is valid. Since X and Z are comonotone random variables we then have that

$$G^{-1}(u) = F^{-1}(u) + K^{-1}(u) .$$

Differentiating this expression yields

$$\frac{1}{g\left(G^{-1}(u)\right)} = \frac{1}{f\left(F^{-1}(u)\right)} + \frac{1}{k\left(K^{-1}(u)\right)}$$

which is equivalent to

$$\frac{1}{h\left(G^{-1}(u)\right)} = \frac{1}{\lambda} + \frac{1}{m\left(K^{-1}(u)\right)} .$$

Thus, $h(t)$ is decreasing (increasing) if and only if $m(t)$ is decreasing (increasing).

What remains to be proved is the equivalence between the inequality conditions of (ii) and (iii). By dividing both sides of (ii) by $v-u$ and by letting $v \to u$ it follows that

(ii)

$\Leftrightarrow$

$$g\left(G^{-1}(u)\right) \le (\ge) \lambda(1-u) \quad \text{for all} \quad u \in [0,1]$$

$$\Leftrightarrow$$

$$h(t) \le (\ge) \lambda \quad \text{for all} \quad t \in [0, \infty\rangle .$$

Proposition 2.1 demonstrates that strong negative (positive) duration dependence differs from the standard definition of duration dependence by requiring that the decreasing hazard rates are finite and that the increasing hazard rates are strictly positive. This additional requirement corresponds to impose the dispersion condition in (ii) on the duration distributions.[3] Thus, for distributions with finite decreasing (positive increasing) hazard rates strong negative (positive) duration dependence may be associated with larger (smaller) dispersion than what is exhibited by an exponential distribution function with higher (lower) hazard rate.

The alternative interpretations of strong duration dependence provided by Proposition 2.1 form a useful basis for developing an ordering of distributions with respect to duration dependence.

DEFINITION 2.3. Let G_1 and G_2 be cumulative distribution functions defined on $[0, \infty\rangle$ with hazard rates that exhibit strong negative (positive) duration dependence. Then G_2 is said to exhibit stronger strong negative (positive) duration dependence than G_1 if and only if

(i) $$G_2^{-1}(v) - G_2^{-1}(u) \ge (\le) G_1^{-1}(v) - G_1^{-1}(u) \quad \text{for all} \quad 0 < u < v < 1$$

and the inequality holds strictly for some (u,v).

By applying Definition 2.3 we may claim that the distribution with the largest (smallest) dispersion exhibits the strongest strong negative (positive) duration dependence when distributions that exhibit strong negative (positive) duration dependence are compared. Note that Quiggin (1991, 1993) and Landsberger and Meiligson (1994) proposed to use the condition (i) in Definition 2.3 as a risk ordering in the area of choice under uncertainty.

It follows from Proposition 2.1 that (i) in Definition 2.3 is equivalent to the condition

(2.2) $$h_2\left(G_2^{-1}(u)\right) \le (\ge) h_1\left(G_1^{-1}(u)\right) \quad \text{for all} \quad u \in [0,1]$$

where h_1 and h_2 are the hazard rates of G_1 and G_2, respectively.

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[3] Bickel and Lehmann (1979) proposed to use the dispersion condition in (ii) in Proposition 2.1 as a dispersion ordering for arbitrary c.d.f.'s whereas Doksum (1969) originally introduced this ordering condition as a tail ordering.

Thus, the quantile-specific hazard rate, $h\left(G^{-1}(u)\right)$, captures the essential features of the notion of strong duration dependence and can be used as a device for ordering distributions with respect to the extent of strong duration dependence. Note that condition (ii) is invariant with respect to identical scale-transformations of the G_1 and G_2 distributed random variables.

By employing the quantile-specific hazard rate we may examine the question of whether the shape parameter of the Weibull distribution can be used to rank distributions with respect to duration dependence. Inserting for (2.1) in $h\left(F_\alpha^{-1}(u)\right)$ we get

(2.3) $$h\left(F_\alpha^{-1}(u)\right)=\alpha\lambda\left(-\log(1-u)\right)^{1-\frac{1}{\alpha}}, \quad 0\le u\le 1.$$

By closer examination of (2.3) we find that $h_{\alpha_1}\left(F_{\alpha_1}^{-1}(u)\right)$ and $h_{\alpha_2}\left(F_{\alpha_2}^{-1}(u)\right)$ may intersect when $\alpha_1>\alpha_2>1$ or $\alpha_2<\alpha_1<1$ and $\lambda_1<\lambda_2$. Moreover, the Weibull hazard rates do neither fulfill the conditions of being finite decreasing or positive increasing. Thus, the strong duration dependence orderings do not provide a ranking of members of the Weibull family of distribution functions. This result suggests that the notion of strong duration dependence is very restrictive and of minor interest in applied work. To achieve ranking of Weibull distributions and other distributions that do not obey the dispersion criterion (i) in Definition 2.3 a weaker ordering than strong duration dependence is required.

3. Weak duration dependence

In order to introduce a weaker ordering criterion than strong duration dependence we may draw on the parallel to stochastic dominance and define an ordering condition that is similar to second-degree stochastic dominance. However, since duration dependence turns the focus to long duration spells the aggregation should start from above rather than from below. This fact suggests that the following function

(3.1) $$R^*(u)=\frac{1}{(1-u)}\int_u^1\left(G^{-1}(v)-G^{-1}(u)\right)dv=E\left(Y-G^{-1}(u)\middle|Y\ge G^{-1}(u)\right),\quad 0\le u\le 1,$$

may form a helpful basis for ranking distributions that do not exhibit strong duration dependence.[4] Note that $R^*\left(G(y)\right)$ coincides with the well-known mean (expected) remaining duration function, which plays a key role in the statistical theory of reliability, see e.g. Barlow and Proschan (1975).

By using integration by parts we get that the R^*-curve may be given the following alternative expression in terms of the hazard rate,

(3.2)
$$R^*(u) = \frac{1}{(1-u)} \int_u^1 \frac{1}{h\left(G^{-1}(t)\right)} dt, \quad 0 \le u \le 1 .$$

Now, by replacing the requirement of decreasing (increasing) hazard rate in the definition of strong duration dependence by the weaker condition of increasing (decreasing) R^*-curve we arrive at the following alternative definition of duration dependence.

DEFINITION 3.1. A cumulative distribution function G defined on $\left[0,\infty\right\rangle$ exhibits weak negative (positive) duration dependence if the R^*-curve of G is increasing (decreasing).

Note that the R^*-curve increases (decreases) and is lying above (below) the mean if the duration distribution exhibits weak negative (positive) duration dependence.

Differentiation of R^* yields

(3.3)
$$\frac{dR^*(u)}{du} = \frac{1}{\left(1-u\right)^2} \int_u^1 \left(1-t\right) d'(u) du ,$$

where $d'(u)$ is the derivative of $d(u)$ (provided that it exists) and $d(u)$ is defined by

(3.4)
$$d(u) = \frac{1}{h\left(G^{-1}(u)\right)}, \quad 0 \le u \le 1 .$$

Then it follows straightforward from (3.3) and (3.4) that $R^*(u)$ is monotonically increasing (decreasing) when the hazard rate is monotonically decreasing (increasing). Accordingly, duration dependence defined in terms of the R^*-curve is weaker than strong duration dependence.

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[4] Note that Fernández-Ponce et al. (1998) and Shaked and Shanthikumar (1998) proposed to use $\left(1-u\right)R^*(u)$ as a device for comparing the right-spread variability of distribution functions, whereas Jewitt (1989) used $R^*(u)$ to characterize behavior under risk.

Assume that G_1 and G_2 have R^*-curves R_1^* and R_2^*. As can easily be observed from (3.1), $R_2^*(u) \geq R_1^*(u)$ for all $u \in [0,1]$ only if the mean of G_2 is greater or equal to the mean of G_1. Note that the strong negative duration ordering (2.2) is satisfied only if the mean of G_2 is greater than the mean of G_1. However, when comparing distributions with respect to the extent of duration dependence a mean independent ordering relation may be called for. Otherwise, we would not be able to distinguish between the general level of duration and the extent of duration dependence as the origin of an attained ordering of distributions. The scale invariance property appears particular attractive when comparison are made between distributions that are formed under highly different conditions. For example, the stigma effects that follows from longer unemployment spells may be considered to be relative in the sense that both the unemployed and the employer account for the general level of unemployment spells in their behavior. Thus, while six months duration spells may create stigma effects in the top of a business cycle it may be considered to be a normal spell length during a recession.

A mean independent ordering of weak duration dependence is provided by the following function

(3.5) $$R(u) = \frac{R^*(u)}{\mu}, \quad 0 \leq u \leq 1 .$$

Thus $R(u)$ can be interpreted as the relative or scaled mean remaining duration for an individual with longer duration spell than the u-quantile.

DEFINITION 3.2. Let G_1 and G_2 be cumulative distribution functions defined on $\left[0,\infty\right\rangle$ with R-curves R_1 and R_2, respectively. Then G_2 is said to exhibit stronger weak negative (positive) relative duration dependence than G_1 if and only if

(i) $R_1(u)$ and $R_2(u)$ are increasing (decreasing)

and

(ii) $R_2(u) \geq (\leq) R_1(u)$ for all $u \in [0,1]$ and the inequality holds strictly for some u.

Replacing R by R^* in Definition 3.2 yields an absolute version of the weak duration dependence ordering. Note that a constant R-curve (equal to 1) and a constant hazard rate are equivalent conditions of a c.d.f. G, which are fulfilled if and only if G is the exponential distribution function. Thus, the *R*-curve increases (decreases) and is lying above (below) the horizontal line 1 if the duration distribution exhibits weak negative (positive) duration dependence.[5]

Note that every duration distribution G with strictly log-convex survivor function, i.e. $\log\left(1-G(y)\right)$ is strictly convex, has an increasing *R*-curve and thus exhibits weak negative duration dependence. By contrast, every duration distribution with strictly log-concave survivor function has a decreasing *R*-curve and exhibits weak positive duration dependence.

To illustrate the relevance of the weak relative duration dependence ordering consider the Pareto family of duration distributions defined by

(3.6) $$G(y) = 1-\left(1+\gamma y\right)^{-\beta}, \quad y \geq 0$$

where $\gamma > 0$ and $\beta > 1$.

By straightforward calculation we find that the mean μ, the quantile-specific hazard rate $h\left(G^{-1}(u)\right)$ and the *R*-curve is given by

(3.7) $$\mu = \frac{1}{\gamma\left(\beta-1\right)},$$

(3.8) $$h\left(G^{-1}(u)\right) = \beta\gamma\left(1-u\right)^{\frac{1}{\beta}}$$

and

(3.9) $$R(u) = \left(1-u\right)^{-\frac{1}{\beta}}.$$

It follows from (3.8) that $h(t)$ is decreasing and that $h(t) \leq \beta\gamma$ for all $t\left[0,\infty\right\rangle$. Thus, we get from Proposition 2.1 that the Pareto distributions exhibit strong negative duration dependence. However, as indicated above an ordering of distributions based on strong duration dependence cannot be achieved when we restrict to distributions with equal means. As an example assume that G_1 and G_2 are defined by (3.6) where $\beta_1 = 11$ and $\gamma_1 = 0.1$ and $\beta_2 = 3$ and $\gamma_2 = 0.5$, respectively. Thus, G_1 and G_2 have equal means. Moreover, as displayed in Figure 1 the quantile-specific hazard rate h_2 of G_2 lies above the quantile-specific hazard rate h_1 of G_1 for small and medium sized quantiles $(u < 0.72)$ and below the quantile-specific hazard rate of G_1 for large quantiles $(u > 0.72)$. Although the quantile-specific

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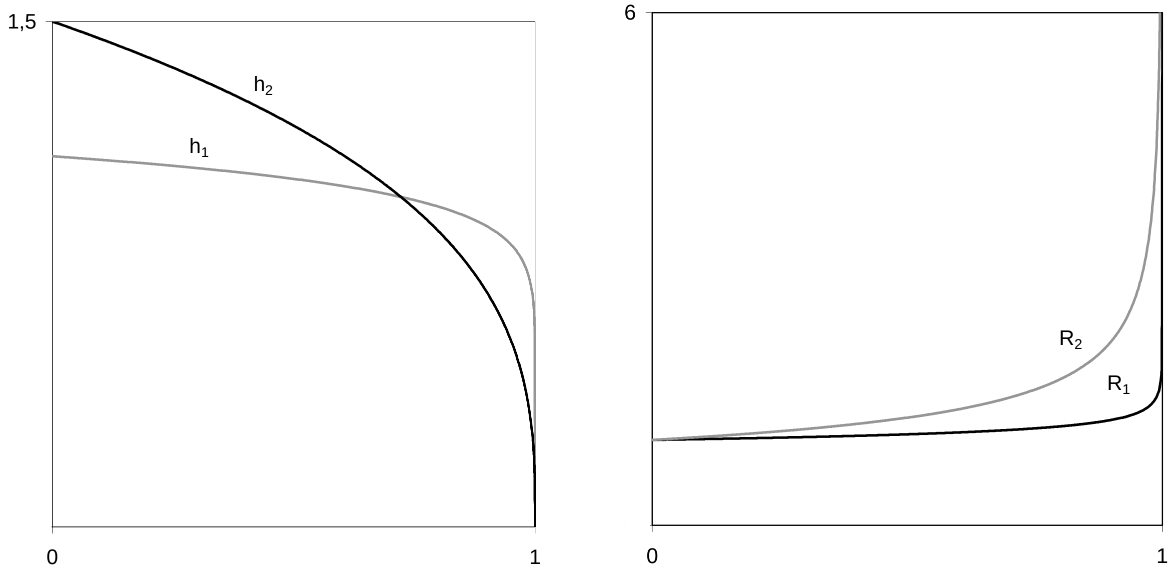
[5] Note that $R(u) \leq (\geq) 1$ for all u is equivalent to the criterion “new better (worse) than used in expectation” used in the statistical theory of reliability.

hazard rates intersect it appears intuitive plausible to claim that G_2 exhibits stronger negative duration dependence than G_1 since longer duration spills may occur more frequently from G_2 than from G_1. Actually, this intuition is captured by the notion of weak negative (relative) duration dependence. Since $R_1(u)$ and $R_2(u)$ are increasing and $R_2(u) > R_1(u)$ for all $u \in \langle 0,1 \rangle$ we get from Definition 3.2 that G_2 exhibits stronger weak negative (relative) duration dependence than G_1. Since the means of G_1 and G_2 are equal to 1 note that $R_1^*(u) = R_1(u)$ and $R_2^*(u) = R_2(u)$ in this example.

Figure 1. Plots of the quantile-specific hazard rates and the scaled quantile-specific mean remaining duration curves of two different Pareto distributions



An interesting question is whether there is any relationship between weak duration dependence dominance and second-degree stochastic dominance. To discuss this question it is convenient to introduce the following notation,

(3.10)
$$\tilde{R}(u) = \frac{E\left(Y \middle| Y \geq F^{-1}(u)\right)}{\mu} = \begin{cases} \dfrac{1}{\mu(1-u)} \displaystyle\int_u^1 F^{-1}(t)dt, & 0 \leq u < 1 \\ \dfrac{F^{-1}(1)}{\mu}, & u = 1. \end{cases}$$

Thus, $\tilde{R}(\cdot)$ is a scaled conditional mean function. The following result gives an alternative condition for the weak relative duration dependence ordering.

PROPOSITION 3.1. Let F_1 and F_2 be cumulative distribution functions with R-curves R_1 and R_2 and scaled conditional mean functions $\tilde{R}_1$ and $\tilde{R}_2$. Then the following statements are equivalent,

(i) $$R_1(u) \geq R_2(u) \text{ for all } u \in [0,1]$$

(ii) $$\tilde{R}_1(v) - \tilde{R}_1(u) \geq \tilde{R}_2(v) - \tilde{R}_2(u) \text{ for all } 0 \leq u \leq v \leq 1$$

PROOF. By dividing both sides of (ii) by $v-u$ and by letting $v \to u$ it follows that

$$\tilde{R}_1(v) - \tilde{R}_1(u) \geq \tilde{R}_2(v) - \tilde{R}_2(u) \text{ for all } 0 \leq u \leq v \leq 1$$

$$\Leftrightarrow$$

$$\frac{1}{\mu_1(1-u)^2}\int_u^1 \left(G_1^{-1}(t) - G_1^{-1}(u)\right)dt \geq \frac{1}{\mu_2(1-u)^2}\int_u^1 \left(G_2^{-1}(t) - G_2^{-1}(u)\right)dt \text{ for all } 0 \leq u \leq 1$$

$$\Leftrightarrow$$

$$R_1(u) \geq R_2(u) \text{ for all } 0 \leq u \leq 1 .$$

The latter equivalent statement follows directly from the definitions (3.5) and (3.1) of R and R^*.

Since $\tilde{R}_1(0) = \tilde{R}_2(0)(=1)$, the next result follows directly from Proposition 3.1 and demonstrates that the $\tilde{R}$-curve ordering is weaker than the R-curve ordering.

PROPOSITION 3.2. Let F_1 and F_2 be cumulative distribution functions with R-curves R_1 and R_2 and conditional mean functions $\tilde{R}_1$ and $\tilde{R}_2$. Then

(i) $$R_1(u) \geq R_2(u) \text{ for all } u \in [0,1]$$

implies

(ii) $$\tilde{R}_1(u) \geq \tilde{R}_2(u) \text{ for all } u \in [0,1]$$

When the comparisons of *R*-curves are restricted to distributions with equal means it follows from Proposition 3.2 that weak negative (positive) duration dependence dominance implies second-degree downward (upward) stochastic dominance.

Now, we will examine whether Weibull distributions with different shape parameters can be ordered by R or $\tilde{R}$. By inserting for (2.1) in (3.1) and (3.10) we get

(3.11) $$R(u) = \frac{\Gamma\left(\frac{1}{\alpha}, -\log(1-u)\right)}{\alpha\Gamma\left(1+\frac{1}{\alpha}\right)(1-u)}, \quad 0 \le u \le 1$$

and

(3.12) $$\tilde{R}(u) = \frac{\Gamma\left(1+\frac{1}{\alpha}, -\log(1-u)\right)}{\Gamma\left(1+\frac{1}{\alpha}\right)(1-u)}, \quad 0 \le u \le 1,$$

where $\Gamma(v,z)$ is the incomplete gamma function.

As for the strong duration dependence criterion we find that *R*-curves as well as $\tilde{R}$-curves formed by different α-parameters may intersect and thus do not offer a method for ranking Weibull distributions according to the extent of duration dependence. To obtain a complete ranking of Weibull distributions summary measures of duration dependence are called for.

4. Measures of duration dependence

To deal with situations where the *R*-curves intersect a weaker ranking principle than dominance of *R*-curves is called for. In order to reach unambiguous conclusions in these cases summary measures of duration dependence are needed. Moreover, summary measures provide a quantification of the extent of duration dependence. The area below the *R*-curve emerges as immediate candidate for summarizing the information content of the *R*-curve. The formal definition of the area below the *R*-curve is given by

(4.1) $$D(G) = \int_0^1 R(u)\,du,$$

which can be interpreted as the ratio between the average of the mean remaining duration and the overall mean duration. Note that the area below the $\tilde{R}$-curve is equal to $D+1$. Thus, *D* preserves the ordering of non-intersecting *R*-curves as well as the ordering of non-intersecting $\tilde{R}$-curves. The *D*-

coefficient has range $[0,\infty\rangle$ and takes the value 1 when *G* is the exponential distribution function. Recalling the discussion in Sections 2 and 3 we may claim that *G* on average exhibits positive relative duration dependence when *D*(*G*) takes lower values than 1; the lower value the stronger positive relative duration dependence. By contrast, when *D*(*G*) takes values larger than 1 the duration distribution *G* on average exhibits a tendency of negative duration dependence; the higher value the stronger is the tendency of negative duration dependence. Now, by inserting for (3.5) and (3.2) in (4.1) and using integration by parts the following alternative expression for *D* is obtained,

(4.2)
$$D(G) = -\frac{1}{\mu}\int_0^\infty (1-G(y))\log(1-G(y))\,dy.$$

Specifically, let *G* be the Weibull distribution. By inserting for (2.1) in (4.2) we get

(4.3)
$$D(F_\alpha) = \frac{1}{\alpha}.$$

Thus, we have justified that Weibull's shape parameter can be used as a summary measure of duration dependence and, moreover, has a convenient geometric interpretation in terms of the area below the scaled quantile-specific mean remaining duration curve. Consequently, the question posed in the introduction of Section 2 has got an adequate answer.

Since no single summary measure can reflect all aspects of duration dependence exhibited by the *R*-curve it is important to introduce alternative measures that may complement the information provided by the *D*-coefficient. To this end we will use an axiomatic approach similar to Kolmogorov's (1930) and Nagumo's (1930) characterization of quasi-linear means and von Neumann and Morgenstern's (1944) theory for choice under uncertainty.[6]

Let $\Im$ be the family of distribution functions defined on $[0,\infty\rangle$ and let $\precsim_n$ be a negative relative duration dependence ordering that is assumed to satisfy the following conditions,

Condition 1A (Weak negative relative duration dependence dominance). Let $G_1, G_2 \in \Im$ have *R*-curves R_1 and R_2 and weak negative duration dependent hazard rates. If

$$R_1(u) \le R_2(u) \quad \text{for all} \quad u \in [0,1]$$

and the inequality holds strictly for some *u*
then $G_1 \prec_n G_2$.

Condition 2 (Order). $\precsim_n$ is a transitive and complete ordering on $\Im$.

Condition 3 (Continuity). For each $G \in \Im$, the sets $\left\{G^* \in \Im : G \precsim_n G^*\right\}$ and $\left\{G^* \in \Im : G^* \precsim_n G\right\}$ are closed (with respect to the topology of convergence in distribution).

Condition 4 (Comonotonic independence). Let G_1, G_2, $G_3 \in \Im$ have means μ_1, μ_2 and μ_3, and let $\alpha \in [0,1]$. Then $G_1 \precsim_n G_2$ implies $\left(\alpha \frac{G_1^{-1}}{\mu_1} + (1-\alpha)\frac{G_3^{-1}}{\mu_3}\right)^{-1} \precsim_n \left(\alpha \frac{G_2^{-1}}{\mu_2} + (1-\alpha)\frac{G_3^{-1}}{\mu_3}\right)^{-1}$.

Condition 1A ensures that the ordering $\precsim_n$ preserves weak negative relative duration orderings and may thus be considered as an essential assumption for $\precsim_n$. Conditions 2 and 3 are standard and well-known assumptions for most ordering relations. Condition 4 was originally introduced by Yaari (1987) as an alternative to the standard independence axiom in the theory for choice under uncertainty, and formed the basis of the so-called rank-dependent (linear) utility theory for choice under uncertainty. Condition 4 requires the ordering relation $\precsim_n$ to be invariant with respect to the addition of comonotone random variables, i.e. variables with rank-correlation equal to 1. As an illustration consider the following example where (4, 10, 26, 60) and (3, 8, 24, 64) are two sets of independent outcomes from two distributions with equal means and increasing *R*-curves. Moreover, since the *R*-curve of (3, 8, 24, 64) is lying above the *R*-curve of (4, 10, 26, 60) it follows from Condition 1A that $(4, 10, 26, 60) \precsim_n (3, 8, 24, 64)$. Now, assume that these two sets of outcomes are mixed with the following set of independent outcomes (3, 4, 5, 6) from a third distribution. Then Condition 4 implies that $\alpha(4, 10, 26, 60) + (1-\alpha)(3, 4, 5, 6) \precsim_n \alpha(3, 8, 24, 64) + (1-\alpha)(3, 4, 5, 6)$ for $\alpha \in [0,1]$, which after a rearrangement is found to be equivalent to $\alpha(1, 6, 21, 54) + (3, 4, 5, 6) \precsim_n \alpha(0, 4, 19, 58) + (3, 4, 5, 6)$. Note that Condition 4 is closely related to Definitions 2.2 and 2.3 since G_1 and G_2 in condition (i) of Definition 2.3 are distribution functions of random variables that can be expressed as sums of comonotone random variables, where one of the variables in each of the sums is exponentially distributed. The essential difference between Definition 2.3 and Condition 4 is that Definition 2.3 concerns duration distributions that exhibit strong negative (or positive) duration dependence, whereas Condition 4 is valid for all duration distributions.

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[6] For a review of the early axiomatic treatment of quasi-linear means see Muliere and Parmigiani (1993).

THEOREM 4.1A. A negative duration dependence ordering $\precsim_n$ defined on $\Im$ satisfies Condition 1A and 2-4 if and only if there exists a continuous and increasing function $q(\cdot)$ defined on the unit interval, such that for all $G_1, G_2 \in \Im$

(4.4) $$G_1 \precsim_n G_2 \Leftrightarrow \int_0^1 q(t)\,dt \le \frac{1}{\mu_1}\int_0^1 (1-t)\,q(t)\,d\,G_1^{-1}(t) \le \frac{1}{\mu_2}\int_0^1 (1-t)\,q(t)\,d\,G_2^{-1}(t)\,.$$

Moreover, q is unique up to a positive affine transformation.

PROOF. Note that there is a one-to-one correspondence between G and its inverse G^{-1}. Hence, the ordering relation $\precsim_n$ defined on the set of distribution functions $\Im$ is equivalent to the ordering relation defined on the family of the inverses of the members of $\Im$. Then it follows from Conditions 2-4 that the basic conditions of Theorem 3 (Chapter 3) of Fishburn (1982) are satisfied and thus that there exists a continuous bounded function $\tilde{q}(\cdot)$ satisfying

$$G_1 \precsim_n G_2 \Leftrightarrow 0 \le \frac{1}{\mu_2}\int_0^1 \tilde{q}(t)\,d\,G_2^{-1}(t) - \frac{1}{\mu_1}\int_0^1 \tilde{q}(t)\,d\,G_1^{-1}(t) = \int_0^1 \frac{\tilde{q}(t)}{1-t}\left(\frac{1}{\mu_2 h_2\left(G_2^{-1}(t)\right)} - \frac{1}{\mu_1 h_1\left(G_1^{-1}(t)\right)}\right)dt$$

where $\tilde{q}(\cdot)$ is unique up to a positive affine transformation.

Let $\tilde{q}(t) = (1-t)\,q(t)$. Then by observing that

$$\int_0^1 \left(\frac{1}{\mu_2 h_2\left(G_2^{-1}(t)\right)} - \frac{1}{\mu_1 h_1\left(G_1^{-1}(t)\right)}\right)dt = \frac{1}{\mu_2}\int_0^1 (1-t)\,d\,G_2^{-1}(t) - \frac{1}{\mu_1}\int_0^1 (1-t)\,d\,G_1^{-1}(t)$$
$$= \frac{1}{\mu_2}\int_0^1 \left(1-G_2(y)\right)dy - \frac{1}{\mu_1}\int_0^1 \left(1-G_1(y)\right)dy = 1-1 = 0$$

and by using integration by parts we get that

$$\int_0^1 q(t)\left(\frac{1}{\mu_2 h_2\left(G_2^{-1}(t)\right)} - \frac{1}{\mu_1 h_1\left(G_1^{-1}(t)\right)}\right)dt = \int_0^1 (1-u)\,q'(u)\left(R_2(u) - R_1(u)\right)du\,.$$

By using Lemma 1 in the Appendix it follows that

$$\int_0^1 (1-u)\,q'(u)\left(R_2(u) - R_1(u)\right)du > 0$$

only if $q'(u) > 0$ for all $0 < u < 1$.

Since $R(t) = 1$ for all $t \in [0,1]$ when F is an exponential distribution function, it follows from Condition 1A that

$$\int_0^1 q(t)dt \le \int_0^1 (1-t)q(t)\,d\,G_1^{-1}(t)$$

for any distribution G_1 with strong negative duration dependent hazard rate.

The necessary part of Theorem 1A follows by straightforward verification.

Next, let $\precsim_p$ be a positive relative duration ordering that is assumed to satisfy Conditions 2-4 when $\precsim_n$ is replaced by $\precsim_p$. Moreover, $\precsim_p$ is assumed to satisfy

Condition 1B (Weak positive relative duration dependence dominance). Let $G_1, G_2 \in \Im$ have R-curves R_1 and R_2 and weak positive duration dependent hazard rates. If

$$R_1(u) \ge R_2(u) \quad \text{for all} \quad u \in [0,1]$$

and the inequality holds strictly for some u

then $G_1 \prec_p G_2$.

Then, by replacing Condition 1A with Condition 1B, Theorem 1B follows directly from the proof of Theorem 1A.

THEOREM 4.1B. A positive relative duration dependence ordering $\precsim_p$ defined on $\Im$ satisfies Conditions 1B and 2-4 if and only if there exists a continuous and increasing function $q(\cdot)$ defined on the unit interval, such that for all $G_1, G_2 \in \Im$

(4.5) $$G_1 \precsim_p G_2 \Leftrightarrow \frac{1}{\mu_2}\int_0^1 (1-t)q(t)\,d\,G_2^{-1}(t) \le \frac{1}{\mu_1}\int_0^1 (1-t)q(t)\,d\,G_1^{-1}(t) \le \int_0^1 q(t)\,dt\,.$$

Moreover, q is unique up to a positive affine transformation.

Now, let D_P be functional, $D_P : R \to [0,\infty\rangle$, defined by

(4.6) $$D_P(G) = \int_0^1 R(u)\, d\, P(u)$$

where R is the R-curve of G and P is a bounded weight-function. For convenience and with no loss of generality we assume $P(0) = 0$ and $P(1) = 1$. This is a normalization condition which ensures that $D_P(G)$ takes the value 1 when G is the exponential distribution function. Moreover, G is said to exhibit average negative relative duration dependence when $D_P(G) > 1$ and average positive relative duration dependence when $D_P(G) < 1$. Note that D_P is equal to D when $P(t) = t$ for all t. Thus, the D-coefficient is rationalizable under Conditions 1A, 1B and 2-4.

By inserting for R defined by (3.2) and (3.5) in (4.6) and changing the order of integration we get

(4.7) $$D_P(G) = \frac{1}{\mu}\int_0^1 (1-u)\int_0^u \frac{1}{1-t}\, d\, P(t)\, d\, G^{-1}(u)\,.$$

Then it follows from Theorems 4.1A and 4.1B that D_P is a measure of duration dependence that is rationalizable under Conditions 1A, 1B and 2-4 provided that

(4.8) $$P'(t) > 0 \quad \text{for all} \quad 0 < t < 1\,.$$

This means that the duration dependence measure D_P is more sensitive to changes in the upper part than in the lower part of the quantile-specific expected remaining duration curve. Moreover, condition (4.8) proves to play a key role in the following dominance result.

THEOREM 4.2. Let G_1 and G_2 be cumulative distribution functions with R-curves R_1 and R_2. Then the following statements are equivalent,

(i) $$R_1(u) \geq R_2(u) \;\text{ for all }\; u \in [0,1]$$

and the inequality holds strictly for some u

(ii) $D_P(G_1) \geq D_P(G_2)$ for all continuous and differentiable P with $P'(u) > 0$ for all $u \in \langle 0,1 \rangle$.

PROOF. From the definition (4.6) of $D_P(G)$ it follows that

$$D_P(G_1) - D_P(G_2) = \int \left(R_1(u) - R_2(u)\right) P'(u)\, du\,.$$

Thus, if (i) holds then $D_P(G_1) > D_P(G_2)$ for all continuous and differentiable *P* with $P'(u) > 0$ for all $u \in \langle 0,1 \rangle$.

Conversely, by assuming that (ii) is true, application of Lemma 1 in the Appendix gives (i). Hence, the equivalent of (i) and (ii) is proved.

The characterization of increasing (decreasing) weak relative duration dependence provided by Theorem 4.2 shows that non-intersecting *R*-curves can be ordered without specifying further the functional form of the weight-function *P* other than *P* being increasing.

As will be demonstrated below the following family of *P*-functions

$$P_k(u) = \frac{1}{\sum_{i=2}^{k+1} \frac{1}{i}} \left(\sum_{i=1}^{k+1} \frac{u^i}{i} - u^{k+1} \right), \quad 0 \le u \le 1, \quad k = 1, 2, ... \tag{4.9}$$

with $P_k'(u) > 0$ for all $u \in \langle 0,1 \rangle$ proves to form the basis of the following convenient family of duration dependence measures

$$D_k(G) = \frac{1}{\sum_{i=2}^{k+1} \frac{1}{i}} \left(\frac{E \max_{i \le k+1} X_i}{\mu} - 1 \right), \quad k = 1, 2, ... \tag{4.10}$$

where $X_1, X_2, ..., X_{k+1}$ is a random sample of size $k+1$ drawn from *G*.

The expression (4.10) for D_k is obtained by inserting for the derivative of P_k defined by (4.9) in (4.7), which yields

$$\begin{aligned} D_k(G) &= \frac{1}{\mu \left(\sum_{i=2}^{k+1} \frac{1}{i} \right)} \int_0^1 (1-u) \int_o^u \left(\sum_{i=1}^{k} i\, t^{i-1} \right) dt\, d\, G^{-1}(u) \\ &= \frac{1}{\mu \left(\sum_{i=2}^{k+1} \frac{1}{i} \right)} \int_0^1 (1-u) \left(\sum_{i=1}^{k} u^i \right) d\, G^{-1}(u) = \frac{1}{\mu \left(\sum_{i=2}^{k+1} \frac{1}{i} \right)} \int_0^1 G(y) \left(1 - G^k(y) \right) dy. \end{aligned} \tag{4.11}$$

From expression (4.10) we get that D_k claims that there is average negative relative duration dependence if and only if

$$E \max_{i \le k+1} X_i \ge \mu \sum_{i=1}^{k+1} \frac{1}{i}. \tag{4.12}$$

Thus, D_1 claims that there is average negative relative duration dependence when the expected maximum of two randomly drawn observations from the duration distribution G is larger than 1.5 times the mean.[7]

Note that D_k increases its focus on large duration spells when k increases. At the extreme as $k \to \infty$, $\left(\sum_{i=2}^{k+1} \frac{1}{i}\right) D_k$ approaches $\left(G^{-1}(1) - \mu\right) / \mu$ where $G^{-1}(1)$ denotes the largest duration spell.

5. Duration dependence in PH and MPH models

The duration distributions considered in Sections 2-4 may be considered as individual-specific. However, in practical situations lack of data normally makes it impossible to obtain separate estimates of individual-specific duration distributions. Thus, in these situations a parametric or semi-parametric modeling framework is needed. The most popular and extensively used econometric models for duration data are the proportional hazard rate (PH) and the mixed proportional hazard rate (MPH) models. A convenient feature of the PH and the MPH models is that they allow for a decomposition of the hazard rate into a duration dependent term and an individual-specific component. The PH and MPH models are defined by

$$h(t) = aq(t) , \tag{5.1}$$

where $q(t)$ is the duration dependence term and a is a term that capture observed heterogeneity in the PH model case whereas it is supposed to account for observed as well as unobserved heterogeneity in the MPH model case.

When analyzing economic data the importance of distinguishing between the effects of genuine duration dependence and unobserved heterogeneity on exit rates has long been acknowledged simply because the policy response to the effect of these two factors may differ. Although the specification (5.1) suggests that genuine duration dependence and unobserved (and/or observed) heterogeneity are separate phenomena within the MPH models, we can in general not claim that observed and unobserved heterogeneity does not affect the extent of duration dependence. One notable exception is the Weibull duration distribution where expressions (2.3), (3.11) and (4.3) demonstrate that the measurement of relative duration dependence is independent of the scale parameter λ, i.e. the term a in (5.1) does not have any influence on the extent of relative duration dependence. To further

[7] Note that $D_1/2$ is equal to the Gini coefficient, which is the most widely used device for measuring inequality in distributions of income.

explore the relationship between the term *a* and the extent of relative duration dependence it will be convenient to introduce the following notation,

$$Q(t)=\int_0^t q(v)dv\,. \tag{5.2}$$

Provided that (5.1) is valid we may use the following expression for the duration distribution *G*

$$G(t)=1-\exp\left(-a\,Q(t)\right). \tag{5.3}$$

It follows from the definitions of *R*, *D* and D_P that *D* and D_P depend on *a* if and only if *R* depend on *a*. Thus, without loss of essential information we may restrict to employ *D* as basis for examining the impact of *a* on the extent of relative duration dependence for various parametric choices of *q*.

By inserting for (5.3) in (4.1) we get

$$D(G)=\frac{a\int_0^\infty Q(t)e^{-aQ(t)}dt}{\int_0^\infty e^{-aQ(t)}dt}\,. \tag{5.4}$$

Now, let us consider the following family of distribution functions defined by

$$G(y)=1-\left(1-\theta t\right)^{\frac{\alpha a}{\theta}} \tag{5.5}$$

where $t\geq 0$ and $\theta<0$, or $0\leq t\leq 1/\theta$ and $\theta>0$. The parameters *a* and α are assumed to be positive. Note that *G* is a Pareto distribution when $\theta<0$ and a power-function distribution when $\theta>0$. The hazard rate of *G* is given by

$$h(y)=\frac{\alpha a}{1-\theta y}\,. \tag{5.6}$$

It follows from (5.6) that *h* is decreasing when $\theta<0$ and increasing when $\theta>0$. Inserting for $Q(t)=\int_0^t h(y)\,dy/a$ in (5.4) yields

$$D(G)=\frac{\alpha a}{\alpha a+\theta}\,. \tag{5.7}$$

Thus, as opposed to Weibull distributed duration spells the extent of negative relative duration dependence decreases with increasing *a* in the case of Pareto distributed spells and increases with increasing a when G is a power-function distribution. This means that we cannot distinguish between observed/unobserved heterogeneity and relative duration dependence when concern is directed to comparison and measurement of the extent of relative duration dependence, even in cases when non-parametric identification of the components of the MPH model is obtained. Note that identification of the shape parameter (α) of the MPH model with a Weibull base-line hazard requires that the additional condition of a finite mean of the mixing distribution is imposed. For a further discussion on this result and more general identification results for generalized accelerated failure-time models we refer to Ridder (1990). Note, however, that the results of Ridder (1990) is closely linked to the above results since non-parametric identification of the MPH models can solely be achieved up to a positive affine transformation. As demonstrated by Brinch (2001) a similar identification result can be obtained for a general family of mixed hazard rate models, provided that data on time-varying covariates is available.

6. Concluding remarks

Most analyses of unemployment duration data are primarily concerned with the occurrence of negative duration dependence that may arise from stigma or discouragement effects, see e.g. Lancaster (1979), Flinn and Heckman (1983), Heckman and Singer (1986) and Van den Berg (1994). These studies discuss and employ alternative methods for distinguishing duration dependence from unobserved heterogeneity. A situation dominated by negative duration dependence may require a different policy from what is required in a situation dominated by unobserved heterogeneity. However, since policy instruments may affect the strength of the duration dependence as well as the overall mean duration a conflict between diminished negative duration dependence and reduced mean duration may arise. To deal with this problem a method that separates the effects from changes in the mean duration and the duration dependence is called for. To this end this paper introduces appropriate definitions of duration dependence and methods for comparison and measurement of duration dependence in hazard rate models. However, since the proposed methods are general in nature they are applicable for other purposes than analyses of unemployment duration spells.

Although a social decision-maker primarily may be concerned with the presence and structure of duration dependence in individual-specific hazard rates the aggregate effects that emerge in the distribution of duration spells across individuals are normally paid equally much attention. The focus is then turned to concentration and questions of how the burden of unemployment is distributed among the unemployed. Is the distribution of duration spells for the unemployed characterized by a large group of people with short spells and a small group of people that suffers from long spells? To examine this type of questions the framework proposed in Sections 3 and 4 can be applied. However, a reinterpretation that is consistent with the problem under study is required.

Appendix

LEMMA 1. Let H be the family of bounded, continuous and non-negative functions on [0,1] which are positive on $\langle 0,1 \rangle$ and let g be an arbitrary bounded and continuous function on [0,1]. Then

$$\int g(t)\,h(t)\,dt > 0 \text{ for all } h \in H$$

implies

$$g(t) \geq 0 \text{ for all } t \in [0,1]$$

and the inequality holds strictly for at least one $t \in \langle 0,1 \rangle$.

The proof of Lemma 1 is known from mathematical textbooks.

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