

MEMORANDUM

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**Tax distortions, household production
and black-market work**

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**Tax distortions, household production
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December 2000

Abstract

We study an economy where a required amount of household production can be carried out either by each household itself, or purchased in the regular or "black" market. We assume that the economy has two types of individuals, one with low productivities, and another with high productivities and a comparative advantage in the industrial (non-service) sector. We show for a "moderate" rate of taxation, that low-productivity workers tend to produce their own home services, while high-productivity workers purchase these in the regular market, given that a black market does not exist. The introduction of a black market is then socially gainful whenever black-market services are demanded by low-productivity workers, and socially harmful whenever such services are demanded by high-productivity workers. We find that when the tax rate is either in an intermediate range, or is very high, the black market leads to allocation gains, while the black market leads to allocation losses when the tax rate is in between these ranges.

Introduction

Most of economic theory presumes a sharp distinction between productive and consumptive activity, and between producers and consumers. Even so, considerable attention has been devoted to the fact that much economic activity, in poor and rich countries alike, lies in a "grey zone" between these two pure categories of activity. For example, in my own country, Norway, the most recent available time use studies indicate that the amount of household work activity (as opposed to leisure activity) constitutes approximately 1.7 million man years (in a population of 4.5 million, and where a man year is defined as 1800 hours of work), while the amount of regular paid work constitutes approximately 2.1 million man years; thus the two types of work are of roughly the same magnitude in terms of hours. Undoubtedly, much household work must be done by the household itself; but much need obviously not. One regularly hears about the business executive or surgeon who leave their regular chores at 4 p.m. in order to tend their gardens, to repair or paint the house, or to make sure his illegally hired neighbor arrives in time to fix his car or electric installations. The latter comment hints to a third work category, "black market" work, which is performed for pay but not officially registered and thus not subject to (VAT, income or social security) taxation. This amount of work is smaller but still sizable, of the order 2-10 % of GDP in most OECD countries, or even higher by some estimates.¹ Most work done in this market is personal services that, in principle, could be produced in the household itself.

The presence of high tax wedges leads to two main, and interrelated, problems in this context. The first is that much of the work done within the household could be carried out more efficiently by persons outside the household, who have a comparative

¹ See e.g. Pedersen (1998) for an extensive survey.

advantage in such activities. As noted by Sandmo (1990), however, taxation of regular market work, together with the inability to tax household work, serves as a tariff on trade between individuals, and pushes activity out of the paid sector and into the household sector, in an inefficient way. The second problem is that taxation of regular work may create a basis for the "black" sector just described, giving demanders and suppliers in the household service market incentives to meet secretly and thereby avoiding to pay taxes.

The purpose of this paper is to study these two issues in combination, in a model where a household production can be carried out in the three ways just described: by household members themselves, by outside hired persons in the regular market, or by hired persons in the "black" market. We assume that labor productivity is always greatest in regular market work, and always lowest in home production, while productivity in black market work lies in between the other two. We assume that individuals are of two types which differ in their absolute productivity in "industrial" (non-service) production, such that the high-productivity group has both a comparative and absolute advantage in industrial production, when compared to household services production. Each household requires a given amount of household work to be carried out, either by the household itself or by others. Each household also consumes a given amount of leisure, and thus has a given amount of total time available, for home and market production.

An efficient allocation in this economy implies that all required household work be carried out in the regular market. In our model, such an allocation can be implemented when the tax rate is sufficiently low to rule out either home production or a black market. The focus of this paper is however on cases where the tax rate is higher. Most of the discussion will be centered around a plausible case where, in the absence of a

black market, low-productivity households do their own household work, and high-productivity workers purchase household services from in the regular (white) market. In sections 3-4 we introduce a black market for services into this economy. Here sellers and buyers incur costly search and form pairwise matches, with probabilities less than one, and enter the black market until private gains from entry are exactly eliminated. At equilibrium either low-productivity or high-productivity workers will demand services in the black market, but never both. The welfare implications of the existence of a black market then turn out to depend drastically on whether low-productivity or high productivity workers demand black services. In the context of our particular example, when black-market services are demanded by low-productivity workers, the existence of the black market is socially gainful. Black-market work in this case implies an efficiency-improving reallocation of labor, since low-productivity individuals would else do this household work themselves, less efficiently than in the black market. For the government this shows up in two forms, both through higher regular taxes (since labor is freed to perform more regular market work), and through net collected fines for tax fraud. By contrast, when black-market services are demanded by high-productivity workers, the black market is socially wasteful. The reason is that high-productivity workers would else demand these household services in the regular market, where labor productivity is higher. This must now be added to the rent-seeking loss due to the search costs in the black market. For the government, this loss is manifested through a reduction in taxes on regular taxable work, which must be greater than any possible net revenues collected as fines on tax fraud.

More generally we find that the black market is gainful when it "steals" work from activity which else would have been performed within the households. It is socially wasteful when it instead "steals" work from activity which else would have been

carried out in the "white" market. This conclusion consequently holds regardless of whether black services are demanded by low- or high-productivity workers. We find that, in addition to the cases just described above, it is very well possible for high-productivity workers to demand black services when these else would have produced their own home services; the black market is then socially gainful. It is also possible for low-productivity workers to demand black services when the demanded services else would have been demanded in the white market, in which case the black market is socially wasteful. These additional cases are discussed and summarized in the final section 5.

An important question is what ranges for the tax rate give rise to the different cases described in section 5. When the tax rate is "low", all households will generally demand all household services in the white market given that a black market does not exist. Thus when the tax rate is "low", a black market is always socially unfavorable. We also find that when the tax rate is in an "intermediate" range, low-productivity workers who would else demand their own home services buy black services, implying that the black market is socially wasteful. On the other hand, when the tax rate is in a higher range, high-productivity workers, and when in an even higher range, all workers, buy black services, and would else produce their own home services; then the black market is favorable.

Considerable attention has been attracted by economists to both the separate issues of the implications of household production activity (e.g. Corlett and Hague (1953), Becker (1965), Boskin (1975), Gronau (1977), and more recently Sandmo (1990), Juster and Stafford (1991), Kleven (2000), Kleven, Richter and Sørensen (2000), Pigott and Whalley (1998) and Anderberg and Balestrino (1999)), and the presence of black labor markets (cf. e.g. Allingham and Sandmo (1972), Feige (1989), Cowell

(1990), Jacobsen and Sørensen (1997), Pedersen (1998), Schneider (2000) and Schneider and Enste (2000)). So far, however, to my knowledge no work to date seeks to integrate these two in a coherent theoretical model.²

The implications of such an integration are strong and striking, and the implications are clear with respect to when a black market is beneficial, and when it is not. Overall, it tends to be beneficial when the predominant demanders in the black market are low-productivity (and low-income) households, and harmful when high-productivity households are the predominant demanders. In the former case, an implication is that the government should be relatively lenient in its policy to punish and thereby discourage black-market activity; in the latter case these policies should be much less lenient.

Our results indicate that the presence of (possibly sizable) a black market for labor is by itself no very clear indication of efficiency or inefficiency; one has to consider this in view both of the rates of taxation, what groups are using and producing black services, and how these services else would have been produced. This accords in principle with Schneider's (2000) conclusion, that there does not appear to be any strong and unambiguous relationship between the extent of black markets, and indicators of efficiency (such as level of per capital GDP or rates of growth). Clearly, more research is required for settling such issues.

Note also that our results below are derived without consideration for distributional concerns, which in case are likely to only strengthen many of the qualitative effects derived (in particular, a lenient policy toward tax evasion should to a large degree benefit low-income households when these are the predominant participants in the

² The paper closest to our approach is probably Jacobsen and Sørensen (1997), who build a computable equilibrium model where both household and underground production are present. Their model however does not address the main issues here, which are the implications and efficiency of a black services market, and how this market interacts with other markets.

black market; low-income households are likely to dominate at least on the supply side of the black market, where uncertainty of returns is the greatest).

2. The basic model with no trade in the black market

Consider an economy where individuals may engage in the following three types of activities: 1) regular paid work which is subject to income taxation at rate τ , 2) "black market" work which is not declared as taxable income, and 3) household work. The economy has two sectors, namely the household service sector (sector 1), and the marketable goods ("industrial") sector (sector 2), both with constant returns to scale in labor only. Regular paid work may be done in both sectors, while black market and household work may be done only in the household service sector. Individuals are of two types, which differ in their relative productivities when engaged in regular paid work in sector 2 ("industrial production"), versus producing household services. Assume that the industrial good is sold in the world market at a given unit price, and that the productivity in this sector for the two groups of individuals (per unit time of labor input provided) is q_1 or q_2 , where $q_1 < q_2$. We assume that type 1 individuals have the same productivity q_1 in regular paid work, in both sectors, and that they generally do some work in both sectors (provided that a positive amount of regular paid work is done in the household sector). Type 2 individuals however do no regular paid work in sector 1 (since their productivity in paid work is always greater in sector 2). Assume that the productivity in black work in sector 1 for type 1 individuals, q_B , is less than the productivity of regular paid work in this sector, q_1 , while the productivity in home production for this group, q_H , is even lower, implying the relationship $q_1 > q_B > q_H$. Intuitively, activities 1 and 2 may imply advantages of specialization over activity 3;

and activity 1 may imply e.g. economies of scale which cannot be reaped in activity 2 (since these are likely to be very small, one-man, firms). We will assume that all black-market work is done by type 1 individuals; this will be the case when the productivity of the two types in black-market work is the same, which we will assume in the following.³ Type 2 individuals may however be more efficient than type 1 individuals in home production. Assume that the productivity of the latter is q_{2H} , which may consequently exceed q_H . We will however throughout maintain an assumption of a comparative advantage of type 2 individuals, in industrial versus household production, i.e., $q_2/q_1 > q_{2H}/q_H$.

Throughout we assume that each individual's (or household's) total work time (to be split between regular, black market and household work) is given and normalized to one. We also assume that the required amount of household work to be done in each household is fixed and equal to W_H . If a type 1 household then itself performs all its required household work, this will occupy an amount of time $H \in (0,1)$ for the household, where $H = W_H/q_H$. The equivalent time for a type 2 household is $H_2 = W_H/q_{2H}$, where $H_2 \leq H$.

From the above description, the efficient allocation in such an economy implies, under certain assumptions to be clear below, that all type 2 individuals engage in production in sector 2 only.⁴ Type 1 individuals may in such an efficient allocation engage in production in both sectors. It is however never efficient for any worker to engage in black labor, nor to do any housework himself or herself.

We will at the outset assume that some regular paid work always is performed in sector 1, e.g. because the government will never tolerate that all paid work in sector 1

³ Alternatively, type 2 individuals could have an absolute advantage in black-market production, but not a comparative advantage when compared to the industrial sector.

⁴ The relevant assumptions are with respect to the sizes of the two groups of workers, in particular, there cannot be "too many" type 2 workers relative to type 1 workers.

is performed as black work. We will assume that type 1 individuals engage in some regular paid work in both sectors.⁵ This in case implies that the price of regular paid services in sector 1 must be equal to one, for type 1 individuals to be indifferent between working in the two sectors. Note that type 1 individuals must do all required paid work in sector 1, since type 2 individuals strictly prefer to do all their paid work in sector 2.⁶

We will in the rest of this section consider a benchmark case where there exists no black market; in sections 3-5 below we come back to different cases where a black market is present. We will initially concentrate on the main case treated below, in sections 3-4, namely where type 1 households produce their own household services, and type 2 household purchase these services in the "white" market (where taxes are paid), in the absence of a black market. Define now $H_1 = W_H/q_1$ as the necessary time for a type 1 worker to produce the required amount of home services for one household, when done as regular paid work. Full income of a type 1 worker is, given that the worker himself produces his own home services, and that total work time for all is given and normalized to one:

$$RF(1) = W_H + (1-H)(1-\tau)q_1 = Hq_H + (1-H)(1-\tau)q_1 \quad (1)$$

The condition under which a type 1 worker now does all own required household work is

⁵ This implies an assumption that group 1 is sufficiently large to be able to produce all required paid sector 1 services, for their own group and for all type 2 households.

⁶ Conditions that must be fulfilled for our analysis to be relevant are then that the total amount of household work required in this economy can be performed by type 1 individuals alone. The relative fraction of type 1 individuals must then be "sufficiently high" (or alternatively, the relative fraction of total work hours required for household work "sufficiently low").

$$q_H \geq (1-\tau)q_1 \quad (2)$$

For type 2 individuals who do none of their own housework (and thus spend all their work time in sector 2 production), full income is simply

$$RF(2) = (1-\tau)q_2. \quad (3)$$

The condition (equivalent to (2)) under which such households do no household work in their own home is⁷

$$(1-\tau)q_2 \geq q_{H2}. \quad (4)$$

Note that there can be no inefficiency associated with time allocation for type 2 individuals in this case; these individuals do all their work in their most efficient activity, namely sector 2 production. Such a solution however requires the tax wedge $1/(1-\tau)$ to be sufficiently small, such that market production (in sector 2) is always more advantageous than home production for type 2 workers.

For type 1 individuals there is by contrast an allocation loss, since these do all their own home production even though they are more productive in market production. The magnitude of this loss is their maximum production minus their actual production, as follows:

$$L(1) = q_1 - [Hq_H + (1-H)q_1] = H(q_1 - q_H). \quad (5)$$

⁷ Condition (4) can easily be generalized to the case with a purely comparative advantage in sector 2 production for type 2 workers, and not merely an absolute advantage as here. The relevant condition would have q_{H2} on the right-hand side of (4), where in general $q_{H2} \neq q_H$.

The allocation loss per type 1 individual equals the magnitude of the required household work, times the difference in productivity between regular paid work and own-produced services. The total loss to society, given a total number N of individuals, a fraction α of which are of type 1:

$$TL = H(q_1 - q_H)\alpha N. \quad (6)$$

This loss is caused by the tax wedge which makes individuals of type 1 prefer to do their own household work.

3. Black-market services demanded by low-productivity individuals who else would produce their own services

Consider now the existence of a black market for home services. We will maintain assumptions (2) and (4), implying that type 1 households do their own household production in the absence of a black market, while type 2 households buy these services in the white market. Assume that part or all of the required household work for type 1 households is done in the black market (by type 1 individuals). We thus assume that type 2 households still choose to have all their household services performed by the white market. We will later come back to the basis for such an assertion.

We will assume that a bilateral trade between a household which demands black services, and a supplier of such services, gives rise to a bargaining situation between the two. In the case studied here, both individuals are of type 1. $H_B = W_H/q_B$ is the

amount of time required to perform a black work task (assuming that the task covers the total amount of household work required by the demander). Defining the wage paid in black work by w_B , the total work income from a black work assignment is $H_B w_B$. A demander of black services has a net gain from this assignment equal to

$$G_D = H(1-\tau)q_1 - H_B w_B, \quad (7)$$

where the first term is the gain in net after-tax income (working in regular paid work) when time H is freed up for market work, as a consequence of others doing ones housework. The supplier has net gain equal to

$$G_S = H_B w_B - H_B(1-\tau)q_1 - \gamma F, \quad (8)$$

where the second term is the supplier's opportunity cost in terms of possible net work income from regular paid work. The last term is the expected fine when being caught doing black work. Assume that only the supplier and not the demander is subject to this type of penalty.⁸ The two parties bargain over the net match surplus, in an asymmetric Nash bargain with relative bargaining strengths β and $1-\beta$ to the demander and the supplier, respectively. The solution to this bargain yields the following expressions for G_D and G_S :

$$G_D = \beta[(1-\tau)(H-H_B)q_1 - \gamma F] \quad (9)$$

$$G_S = (1-\beta)[(1-\tau)(H-H_B)q_1 - \gamma F]. \quad (10)$$

The expression inside the bracketed term in (9) and (10) is total match surplus, i.e. the total expected gain to the demander and supplier of black services, over the alternative for the demander, of doing ones own household work, and for the supplier, of working in regular paid work in sector 1.

Consider next the technology for matching demanders and suppliers of black services. Assume that one unit of search in the market for black services takes a fixed amount H_S of time, for both a demander and a supplier. This unit of search gives the demander a probability ρ of finding a supplier of black services, and it gives a supplier a probability π of finding a demander for black services. These probabilities are given by

$$\rho = h(\theta) \tag{11}$$

$$\pi = h(1/\theta), \tag{12}$$

where $\theta = N_{SB}/N_{DB}$, and N_{SB} and N_{DB} denote the numbers of suppliers and demanders of black services who are active in the market. We assume $h' > 0$, $h'' < 0$. The h function is analogous to the matching function in a more standard labor market context, as e.g. in Pissarides (1990). It is assumed to exhibit constant returns to scale, such that a doubling of the numbers of both demanders and suppliers of black services renders the matching probability constant. Assume that each demander and supplier searches only once. Note that since $\rho N_{DB} = \pi N_{SB}$ (which equals the number of realized black market assignments), we may write (12) as follows:

⁸ In most countries it is in fact not illegal to buy black services. One could still of course picture the possibility of a psychological cost of this type also for the demander.

$$\pi = \frac{1}{\theta} f(\theta). \quad (12a)$$

Consider an equilibrium in this market. First, suppliers and demanders of black services are in equilibrium when the expected gain from engaging in black services (over the relevant alternatives, which are household work for the buyer and regular sector 1 work for the seller) equals the value of the search cost, in terms of net work income foregone when search takes an amount H_S of time. This requires the following two conditions to be fulfilled:

$$\rho G_D = H_S(1-\tau)q_1 \quad (13)$$

$$\pi G_S = H_S(1-\tau)q_1. \quad (14)$$

Using the expressions (9) and (10) (where we in particular note that $G_S = [(1-\beta)/\beta]G_D$) and that $\rho = \theta\pi$, we derive the following simple condition:

$$\theta = \frac{1-\beta}{\beta}. \quad (15)$$

From (11)-(12) and (15), ρ and π can now be considered as fixed parameters. A question arising is whether there will exist an equilibrium in our model, where (13)-(14) can be fulfilled with equality. When G_D and G_S are exogenous this is not possible, since (13)-(14) (and (15)) could then not both be fulfilled in general. Note also that when (13)-(14) hold with equality, our model only solves for θ and not the levels of N_{DB} and N_{SB} . We will solve these problems below, by introducing a government decision rule which endogenizes both the expected penalty for black-market tax fraud,

γF (i.e., sets γF equal to the level required for (13)-(14) to hold), and the volume of black-market trade, ρN_{DB} . For now we can just take both the required level of γF , and a particular solution value for ρN_{DB} , as exogenously given.

An obvious condition on parameters which must hold for an equilibrium to exist is, from (11), (13) and (15),

$$\frac{H_s q_1}{h\left(\frac{1-\beta}{\beta}\right)} \in [G_{D\min}, G_{D\max}] \quad (16)$$

where

$$G_{D\min} = \beta[(1-\tau)(H-H_B)q_1 - F] \quad (17)$$

$$G_{D\max} = \beta(1-\tau)(H-H_B)q_1. \quad (18)$$

$G_{D\min}$ and $G_{D\max}$ are found from (9)-(10) setting γ equal to 1 and 0 respectively. No relevant solution can here be found when the expression on the left-hand side of (16) is less than $G_{D\min}$ or greater than $G_{D\max}$.

The left-hand side of (16) will be less than $G_{D\min}$ when the fine F to be paid when caught for tax evasion is less than the gain from evading taxes, and at the same time the search cost in the black market in terms of time, H_s , is small. In such cases black market activity is always preferable to regular market activity for individuals of type 1, and no solution can here make such individuals indifferent between the two sectors, as assumed. In particular, type 1 individuals would then be at a corner solution and purchase all their required household work in the black market.

The left-hand side of (16) exceeds $G_{D\max}$ when H_s is sufficiently large, making entry into the black market unprofitable even when the probability of being caught for tax

fraud is zero. This case is uninteresting since it would preclude the existence of a black sector at the outset.

Note that the solution derived thus far only determines θ and not the absolute number of black-market offenses, $N_B = \pi N_{SB} (= \rho N_{DB})$. Moreover, we have determined what the expected penalty for tax fraud γF must be, but we have not specified a mechanism for actually determining this penalty. To these issues we now turn.

There are two basic ways in which γF can be determined. The first is that the government sets this penalty in an overall welfare-maximizing way, i.e. so as to maximize a social welfare function (assuming here in case that other instruments of the government are not affected). Alternatively, one may study the behavioral strategy of a government agency which is in charge of enforcing tax laws, but which does not necessarily take an overall welfare-maximizing perspective. We will here adopt the latter position, and in later parts of the paper come back to the former. Assume that a fine F is imposed on a supplier when caught, which is set administratively and not subject to optimization in the current context.⁹ The average probability γ of catching a black supplier is however now viewed as endogenous. Assume that a government agency in charge of tax enforcement has the objective of maximizing net revenue from catching black offenders, i.e. it maximizes gross fines minus enforcement costs. Assume also that the agency views N_B as exogenously given; essentially, this agency has no direct concern for the number of tax offenses as such. Assume that γ is given by the function $\gamma = \gamma_1(C/N_B) \gamma_2(N_B)$, where C is total enforcement cost, and thus C/N_B enforcement cost per committed offense. Assume that $\gamma_1' > 0$, $\gamma_1'' < 0$, and $\gamma_2' > 0$,

⁹ Having an upper limit to F may appear arbitrary in view of Becker's (1968) seminal analysis which implies that (risk-neutral) individuals here should face an "infinite" punishment with an "infinitesimal" probability. In most societies tax cheating is however not viewed as a very serious crime, with legal limitations on a socially acceptable punishment for caught offenders. E.g., in Norway the tax authorities may (for "moderate" tax evasion

where primes denote derivatives. A greater enforcement cost per offense thus, reasonably, raises the probability of catching a given offense. We also assume that when the overall number of offenses rises, the probability of catching any given offense rises for a given enforcement cost per offense. This may follow from certain returns to scale in the enforcement technology, e.g. the tasks of enforcement officers can become more specialized when the corps of such officers is larger. The objective function of the enforcement agency can then be written as¹⁰

$$R = \gamma F \pi N_{SB} - C = \gamma_1(C/N_B) \gamma_2(N_B) N_B F - C. \quad (19)$$

R is now maximized with respect to C , where as noted N_B is viewed as exogenous.

This yields the following first-order condition:

$$\gamma_1'(C_B) \gamma_2(N_B) F = 1, \quad (20)$$

where $C_B = C/N_B$ denotes average enforcement costs per offense. We may now study

how increases in F and N_B affect C_B and consequently γ . We find

$$\frac{dC_B}{dN_B} = -\frac{\gamma_1' \gamma_2'}{\gamma_1'' \gamma_2} > 0 \quad (21)$$

$$\frac{dC_B}{dF} = -\frac{\gamma_1'}{\gamma_1''} > 0 \quad (22)$$

offenses) impose only a 40 % addition to the original tax claim. We will here essentially take the limitations on F as given and not go deeply into the issue of whether higher levels are socially desirable.

¹⁰ See e.g. Andrioni, Erard and Feinstein (1998) for more thorough discussion of government objectives in this context. A main point in much of the related literature is that a government agency here cannot automatically be viewed as welfare-maximizing; it may be subject to constraints in terms of decision making and preferences that makes an analysis along our lines practically relevant.

Increases in the number of offenders and the fine when catching an offender both raise average enforcement costs per offense. More offenders makes enforcement more gainful due to the noted scale economies in enforcement. A higher fine also makes enforcement more lucrative for the enforcement agency. Note that the agency here does not take into consideration the effect of an increase F on N_B . We find:

$$\frac{d\gamma}{dN_B} = \left[1 - \frac{(\gamma_1')^2}{\gamma_1''} \right] \gamma_1 \gamma_2' > 0 \quad (23)$$

$$\frac{d\gamma}{dF} = -\frac{(\gamma_1')^2 \gamma_2}{\gamma_1''} + \frac{d\gamma}{dN_B} \frac{dN_B}{dF}. \quad (24)$$

N_B clearly affects γ positively. The effect of increased F on the level of γ chosen by the agency is however more difficult to sign in general. Still a higher F has a direct positive effect on enforcement costs, through (22). The last expression in (24) is however generally is negative (the number of offenders drops when the fine increases), and the overall effect on γ uncertain.

Assume now that (16) holds. We next need to demonstrate that there exists an equilibrium where the enforcement agency has an incentive to set γ at the level required for equality in (13) and (14). Consider then a level of γ slightly lower than the level required for equality in (13). This will make it strictly advantageous to enter the black market, and N_B will rise. But by (23), the enforcement agency will respond to this by raising γ . One easily realizes that this leads to a stable equilibrium value of γ , whereby (13)-(14) hold with equality.

The alternative way of determining γF would be that the government sets both γ and F directly, and in an optimal way. Note then that an arbitrarily large number of black-market trades N_B can be implemented by setting γF "slightly lower" than the level

which yields equality in (13)-(14). At the same time enforcement costs can be made arbitrarily low in principle, by setting F "large" and γ "low". The government would then in any case (in view of footnote 9) like to set F as large as it can (at least in the context of the present model, with risk-neutral individuals). Overall, the government would like to encourage black-market work (and effect a high volume of such work) when black-market work is seen as gainful, and it would discourage such work when it is seen as harmful, through appropriate values of γF . We will come back to this perspective below, in discussion of different possible cases.

So far we have taken for given that only type 1 individuals engage in black-market activity. To find the condition under which this is the case, we must consider a possible bargaining solution between a type 2 buyer and type 1 seller of black services. Provided, as assumed, that type 2 individuals would else buy the required services in the white market the bargaining surplus of a type 2 buyer can then be written as

$$G_{D2} = H_1 q_1 - H_B w_B = H q_H - H_B w_B, \quad (25)$$

where w_{B2} again is the negotiated black-sector wage, and as before $H_1 q_1 = H q_H = W_H$. Since the bargaining surplus of the seller has the same form as (8) (only replacing w_B by w_{B2}), the bargaining solution with relative bargaining strengths β and $1-\beta$ now implies

$$G_{D2} = \beta [H q_H - H_B (1-\tau) q_1 - \gamma F] \quad (26)$$

$$G_{S1} = (1-\beta) [H q_H - H_B (1-\tau) q_1 - \gamma F]. \quad (27)$$

Assume that a potential type 2 buyer, in the same way as a type 1 buyer, spends an average time H_S searching for a seller of black services. Since type 2 individuals' net return to labor is $(1-\tau)q_2$, his search cost is $H_S(1-\tau)q_2$, which is greater than that of type 1 individuals. Consider now a type 2 buyer who considers entering the market for black services, where otherwise only type 1 workers are active in the black market, and where the matching probability ρ is the same for buyers of types 1 and 2. The condition under which it is disadvantageous for a type 2 buyer to enter this market is then

$$\rho G_{D2} < H_S(1-\tau)q_2. \quad (28)$$

Combining (28) with (13), we derive parametric conditions under which our derived equilibrium holds, as follows:

$$\rho\beta H[q_H - (1-\tau)q_1] < H_S(1-\tau)(q_2 - q_1). \quad (29)$$

This condition yields interesting insights. The term in the bracketed expression on the left-hand side of (29) represents the gain from a type 2 individual demanding black services, over the gain from a type 1 individual demanding the same services, as viewed by the seller and buyer of black services together. When (2) holds (type 1 individuals never demand white services), this difference is always positive. The difference is however small when q_H is close to $(1-\tau)q_1$, i.e. when there is little to gain for type 1 households by producing their own services. The term on the right-hand side of (29) represents the differential cost of searching for black services, for type 2 versus type 1 households. This differential is always positive.

Assume a situation where (29) and (2) both hold initially, such that type 1 households are the only demanders of black services, and these never purchase any white services. Such a case holds whenever τ is "relatively small". (τ can however not be so small that (2) fails to hold, since then type 1 individuals would never produce their own home services in the first place, and instead demand "white" market services). When τ now increases, the right-hand side drops relative to the left-hand side, and beyond some minimum level of τ the inequality in (29) fails to hold.

The upshot of this is that a relatively low rate of taxation leads to black-market work being demanded by low-productivity (and –income) households, but not by high-productivity (and –income) households. There are two main factors which contribute to this effect. First, a higher tax makes the time freed up for a type 1 worker, when his household work is instead purchased in the black market, less valuable. Secondly, an increase in τ will reduce search costs proportionally for all individuals (since labor time becomes less valuable to all), but by more in absolute value for type 2 individuals, who have the higher time cost at the outset.

We also note that (29) is more likely to hold under the following circumstances:

- The smaller is the amount of time required to produce ones own home services, H , relative to the amount of time required to find a match in the black market, H_S .
- The greater is the difference in labor productivity between the two types of labor, in regular paid production.

Note also that neither the expected penalty for tax cheating, γF , nor the productivity of black-market work, q_B , affects condition (29).

We will now consider welfare effects of black market work, given that only type 1 individuals demand such work and these else would have produced their own home services. Note then that there is no actual such gain experienced by market participants

themselves in the model: all potential gains are completely eroded by the condition of free entry into the black market sector. As a result all loss or gain will show up in the form of changes in government revenue. Consider this gain (or loss) as a function of the number of black market transactions, $N_B = \rho N_{DB}$. From the definition of G_D this can be written as

$$NR(1) = N_B \left[(H - H_B) \tau q_1 + \gamma \left(\frac{C}{N_B}, N_B \right) F - \frac{C}{N_B} \right], \quad (30)$$

where we find it convenient to write the expression for the average surplus created per realized black-market trade inside the square bracket, and where we now have written γ as simply a function of C/N_B and N_B , from (19). Here, clearly, $NR(1) > 0$, since both the first term and the sum of the two last terms (which equals R in (19)) are positive. The first term represents the government's increase in regular tax revenue. This increase has two components. First, there is an increase in tax revenue when labor is freed from home production for type 1 individuals, and these instead perform regular (taxable) work, represented by the term $H \tau q_1$ in (30). Secondly, there is a drop in regular tax revenue as a result of black work being done, instead of regular taxable work, represented by the term $H_B \tau q_1$. The former term however exceeds the latter. The reason is that the work time freed in the home production sector is greater than the work time required to perform the black-market work tasks.

The two last terms inside the square bracket in (30) represent R/N_B which is positive by virtue of assumptions about the γ function made above, and since R is assumed to be maximized by the government enforcement agency, by (20). It is relevant to view the cost C as a social loss resulting from government rent-seeking in view of limited

ability to raise F (since for a given level of enforcement γF , it is possible to increase F and reduce γ sufficiently, so as to make C arbitrarily small in principle). Note also that since we have assumed that the partial effect of N_B on γ is positive, average enforcement cost C/N_B drops with N_B . Thus the greater the volume of black-market work, the greater the social surplus created by each such deal.

Thus overall, there is a positive welfare effect when fewer type 1 individuals perform their own household work, and these instead demand such work in the black market. The reason is the initial allocation failure when such individuals do their own household work, and the property of our model, that such work is done more efficiently as hired black-market work. For a given rate of taxation τ , such a favorable reallocation cannot be accomplished in other ways; thus the black market can be said to lead to an efficiency improvement. Part of the potential allocation gain is however "eaten up" by two different "rent-seeking" effects. The first of these takes the form of search costs when there is free entry of sellers and buyers in the black market. The second takes the form of excessive enforcement costs by the government agency in charge of punishing tax fraud, when this agency is subject to a fixed maximum fine level and takes the volume of tax fraud as given.

4. Black-market services demanded only by high-productivity individuals

We will now look closer at possible solutions where equilibrium implies that all demanders in the black market are of type 2, i.e. belong to the high-productivity group. In such cases the fundamental equations describing equilibrium in the market for black work are

$$\rho G_{D2} = H_S(1-\tau)q_2 \quad (31)$$

$$\pi G_{S1} = H_S(1-\tau)q_1, \quad (32)$$

where G_{D2} and G_{S1} are given by (26)-(27). We now derive the following equilibrium value of θ , using (11) and (12a):

$$\theta = \frac{1-\beta}{\beta} \frac{q_2}{q_1}. \quad (33)$$

The equilibrium value of θ is here greater than under the case of only type 1 demanders, given in (15). The reason is that demanders now have higher marginal search costs than suppliers, and this requires the equilibrium ratio of suppliers to demanders to be higher, in order for type 2 suppliers to be willing to enter.

We now easily realize that a condition of the form (29) still is relevant for deciding which of the two groups, type 1 or type 2 individuals, will demand black services.¹¹ Thus under the opposite inequality in (29), black work will be demanded by type 2 individuals only. This holds provided that τ is "sufficiently high". Note however that τ cannot be so high that type 2 individuals else would do their own housework, i.e., (4) must hold (alternatively formulated, q_2 must be sufficiently high for (4) to hold).

Also here a condition must hold for black market trade to be viable, similar to condition (16) with only type 2 demanders. This condition is now:

¹¹ We still assume that the volume of black trades never is allowed to grow without bound, and that not all individuals of either type engage in black trades. This will imply that only one of the types will ever demand black work at equilibrium.

$$\frac{H_S q_2}{h \left(\frac{1-\beta}{\beta} \frac{q_2}{q_1} \right)} \in [G_{D2\min}, G_{D2\max}] \quad (34)$$

where now

$$G_{D2\min} = \beta [Hq_H - H_B(1-\tau)q_1 - F] \quad (35)$$

$$G_{D2\max} = \beta[Hq_H - H_B(1-\tau)q_1]. \quad (36)$$

$G_{D2\min} < H_S q_2/h$ can, in a similar way as in (16), be viewed as a minimum condition on F , the fine when caught for tax fraud. $G_{D2\max} > H_S q_2/h$ is, also as above, a basic "feasibility" condition whereby (for a given level of τ) the surplus from engaging in a black-market trade must be sufficiently high given no fines for tax evasion. Note then that G_{D2} can alternatively be written as

$$G_{D2} = \beta[-(H_B - H_1)q_1 + H_B \tau q_1 - \gamma F], \quad (30a)$$

where we have used that $Hq_H = H_1 q_1 = W_H$, and where $H_B - H_1 > 0$. It is thus clear that for a solution to exist, τ must be "sufficiently high" to make the second term in the square bracket in (30a) dominate "sufficiently" over the other terms.

Consider welfare implications of the existence of a black market which serves only type 2 demanders. Changes in welfare can in the same way as in section 3 above, be represented by changes in net total government tax revenue (i.e., gross tax revenue minus enforcement costs). In (30a), the term $-(H_B - H_1)q_1$ represents the "basic" efficiency change from the black market. Since $H_B > H_1$ (labor is more efficient in the production of regular taxable services, than in the production of black services), this

change is negative. The implication is an efficiency loss resulting from the black market when household production alternatively will be produced in the form of regular taxable services. The second term in the bracket, $H_B \tau q_1$, represents tax savings per trade for private-market participants, which are positive, while the third term, γF , represents expected fines for tax fraud, which enter negatively into agents' calculations. The net change in social value is, as in the previous case, the change in net government revenue. Letting N_{B2} denote the number of realized black-market trades in this case, this change can be written as

$$NR(2) = N_{B2} \left[-H_B \tau q_1 + \gamma \left(\frac{C}{N_{B2}}, N_{B2} \right) F - \frac{C}{N_{B2}} \right]. \quad (37)$$

Since $G_{D2} > 0$ in (30a), we here know that $H_B \tau q_1 - \gamma F > 0$, which implies $NR(2) < 0$. The lost tax revenue due to household work assignments done in the black market instead of the regular taxable market, must here be greater than the (gross) revenues raised from fines on black-market tax violations. Intuitively, such a greater loss of tax revenue is required for type 2 individuals to demand black services in the first place (or else G_{D2} would be negative).

We can still be assumed to have increasing returns in the enforcement of black-market penalties. This is however of less importance from a basic welfare point of view, since the welfare effect of increased volume of black-market trades is negative, even in cases where enforcement costs are zero.

Overall, thus, when black services are demanded by high-productivity individuals only, it is socially advantageous to limit the extent of the black market as much as possible in the context of our model. There are three main factors behind the real loss

involved in having a black market in this case: the allocation loss due to household services production being less efficient in the black than the white market; the search costs of households in the black market; and the enforcement costs of the government agency in charge of catching black-market tax cheaters.¹²

5. Other cases

There are two other important cases that need to be considered. The first of these is the case where both types of individuals buy only white services at the outset, before the establishment of a black market, implying that the opposite inequality holds in (2). This holds when the tax level is "low", but positive; the tax rate must exceed a minimum level making entry into the black market privately advantageous. This case is simple. We now find that only type 1 individuals can be demanders in the black market, since the only difference between the two types on the demand side is that search costs are lower for type 1. The equilibrium condition for these can now be derived as

$$\rho\beta[Hq_H - H_B(1-\tau)q_1 - \gamma F] = H_S(1-\tau)q_1. \quad (38)$$

This condition can be fulfilled given that τ is "not too small" (since $Hq_H - H_Bq_1 < 0$). Since black work is less efficient than white work (and there are search costs in the black market), there would be no basis for a black market when white work is (almost) not taxed. On the other hand, for higher values of τ but not so high that (2) holds, a

¹² See here assume no enforcement costs for tax compliance in the white sector. This is too simple, but one may argue that the latter costs are much lower than those in the black sector.

solution to (38) can be found for a nonnegative γ^F only provided that the average required black-market search time, H_S , is sufficiently small.

Like under the case in section 4 above, the existence of a black sector entails a social loss. The welfare loss per effected trade in the black market is however generally somewhat smaller in this case. The direct production loss is the same, but the search cost in the black market is smaller, since searching demanders have lower search cost.

The other additional case needing consideration implies that both types would produce their own household services in the absence of a black market. This requires that condition (4) no longer holds, but is instead fulfilled with an opposite inequality. For this case to hold the tax rate τ must be "relatively high". To study its implications we need to determine which group, 1 or 2, will be demanders of black services. If group 1 are black-service demanders, condition (13) will still hold for these, with G_D given from (9). If instead group 2 are black-service demanders, the relevant condition for these is (31) except that G_{D2} in this case is given by

$$G_{D2} = (1-\tau)(H_2q_2 - H_Bq_1) - \gamma^F . \quad (39)$$

We may now find the cases under which type 1 individuals purchase black services, while type 2 individuals purchase white services, by considering the condition under which a negative inequality is obtained in (31) with G_{D2} given from (39). This condition is

$$\rho\beta(H_2q_2 - H_1q_1) < H_S(q_2 - q_1) . \quad (40)$$

This condition depends only on fundamental parameters of the model and not e.g. on the tax rate τ . It is more likely to hold, the greater the productivity difference between the two types (since the difference in black-market search costs then are greater), and the smaller is H_2 (implying that type 2 individuals have less of a comparative advantage in market production). If (40) holds, the implication of the equilibrium is quite similar to that in section 3 above, where only type 1 individuals demand black services. In both cases, black market production is socially gainful in the sense of increasing the economy's total output for a given tax rate τ . The only main difference is that type 2 individuals now produce their home services themselves instead of buying these in the white market.

When the opposite inequality holds in (40), black-market services are demanded by type 2 individuals only, in the same way as in section 4. The main difference is that now these individuals would else produce these services themselves. The welfare implications of the existence of a black market are also much the same as in section 3. As in that case such a market is now socially gainful, in particular since net government tax receipts will increase as a result of its existence. It is also easy to see that it is more gainful than in section 3. The point is that the very reason why type 2 individuals engage in black-market trades and not type 1 individuals, is that the social surplus created by the former is greater than that created by the latter. On the other hand, the distributional implications may be less favorable, since the high-productivity (and –income) group now gets the added advantage of access to the black market (and are the predominant tax cheaters).

Table 1 gives a summary of some features of the different cases that may arise in our model. We here categorize cases according to home and market demand for household services in the absence of a black market, and according to the demand for

black services, for the two groups of households. We have 5 possible cases with black market demand, as indicated in the table; in each case black-market services is demanded by only one household type. The only "impossible" case in the table is the case where none do their own household work and type 2 households demand black work: this is excluded because type 2 households have higher search costs. Note also that cases where (in the absence of a black market) all do their own household work requires a higher tax than the case where only type 1 households do their own household work, which again requires a higher tax than the case where all demand market work. Note also that cases (1), (2) and (3) all imply that the black market is favorable, while cases (4) and (5) imply that the black market is unfavorable.

Table 1: Overview of main cases in the model

Black market demand by	Both types do their own home prod.	Low-prod only does own home prod.	None does own home prod.
Low-prod. workers	$\rho\beta(H_2q_2 - H_1q_1) < H_S(q_2 - q_1), \tau$ high (1)	$\rho\beta H[q_H - (1-\tau)q_1] < H_S(1-\tau)(q_2 - q_1)$ (3)	$\rho\beta[Hq_H - H_B(1-\tau)q_1 - \gamma F] = H_S(1-\tau)q_1$ (5)
High-productivity workers	$\rho\beta(H_2q_2 - H_1q_1) > H_S(q_2 - q_1), \tau$ high (2)	$\rho\beta H[q_H - (1-\tau)q_1] > H_S(1-\tau)(q_2 - q_1)$ (4)	Never (6)

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