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Competitive effort and employment determination

with team production

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Competitive effort and employment determination with team production

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Abstract

We study a labor market where workers' disutilities of effort differ, firms' outputs depend on the joint efforts of many workers, and individual worker characteristics cannot be observed or inferred by firms. Under assumptions similar to those in Holmström (1982), we demonstrate that an efficient effort allocation then in principle can be implemented by firms paying the same wage to all workers, and more productive workers putting up greater efforts than less productive ones. When the labor market is competitive, however, a first best cannot be implemented due to worker adverse selection. When there are two types of workers and the fraction of the ”bad” type is not too high, a competitive equilibrium implies that all workers' productivities are below first best, and that there may or may not be full employment, with possible unemployment evenly distributed among the two types. When the share of ” bad” workers is greater, some (or even all) firms attract bad workers only, and unemployment disproportionately borne by the high-productivity type.

1. Introduction

An important and prevalent, and at the same time puzzling, aspect of labor markets and their organization is the common observation that worker compensation at a given workplace is often not very closely related to individual productivity. Several explanations have been offered for this phenomenon.[1] Some of these stress the difficulty of observing individual productivities, and thereby the inability of firms to actually differentiate wages. Others are of a more sociological or organizational nature, and emphasize e.g. potential negative effects on workers' morale of high wage differentiation among workers who jointly perform similar tasks, even though their actual contributions to output may differ widely. A problem with many of the former types of explanations is that they often do not account well for exactly why workers' productivities should differ by much more than the (possibly small) differences in wages. In particular, a common notion is that when the wage is the same for all (and there are no possibilities for promotions), there should be no reason for some workers to voluntarily work harder than others. A problem with the second type (where it must be presumed that workers' individual productivities can be observed) is that workers with high productivities should tend to move on to other firms who are willing to reward them more in line with actual outputs, as should occur in any efficiently functioning market economy.

Our purpose here is to study a model which can explain such observations, and which differs from other standard models in the groups referred to above. Departing from Holmström (1982), we assume that workers in a given firm differ according to productivity, but that only total output, and not individual workers' contributions to this, can be measured by each firm. We moreover make the rather extreme assumption that firms have no ways of screening workers according to productivity, nor distinguishing more from less

1 See e.g. Agell (1999) and references quoted there. Much of Agell's argument toward defending a "flat" wage structure is largely based on other arguments than those exposed here, such as a common acceptance of norms and the value of the wage scheme in the context of social insurance. See however also Hibbs and Locking (2000), who argue that e.g. the extreme wage equality experienced in Sweden has been good for reallocation of labor across sectors, but not generally for inducing efficient effort.

productive individuals.

In section 2 of the paper, we first demonstrate that a socially efficient allocation of worker effort implies that the marginal disutility of effort equals the marginal productivity contribution to total output for each worker. We also show that such a solution (under certain assumptions which coincide with those made by Holmström) in principle can be implemented through a bonus scheme, whereby all workers receive the same positive bonus given that aggregate output is observed to be greater than a given minimum level. At this solution individual workers' contributions to output differ according to ability, in an optimal way, but all workers' wages are the same. Under our assumptions, both marginal and average disutility of effort is greater at the optimal solution for those workers who contribute more to total output, than for those who contribute less. With equal wages to all this implies that expected utilities differ, and are generally lower for workers with higher productivities.

The results in section 2 follow more or less directly from Holmström's analysis, when explicitly extended to account for worker heterogeneity. Sections 3-4 however extend Holmström's analysis in less trivial ways, by embedding firms in a competitive labor market. We now simplify the analysis by assuming that there exist only two types of workers, those with ”low” and those with ”high” productivities (type 1 and 2) respectively. We assume that firms are identical and behave competitively and identically, the number of firms in the economy is given, there is perfect interfirm mobility of labor, and workers have given and identical options outside of the labor market (e.g. in the form of home production or leisure). This is a problem reminescent of the Rothschild-Stigitz (1976) insurance market problem, where an equilibrium is shown to frequently not exist. We resolve the existence problem by assuming a particular type of behavior for low-productivity workers: such workers always go to firms in fixed proportions to high-productivity workers (for fear of else having their types singled out).

In section 3 we study competitive equilibria with full employment under these assumptions. At such equilibria firms can be viewed as competing for high-productivity (type 2) workers only. The participation constraints for these workers must then bind on any given firm, in the sense that their utilities must be equal in all firms and generally exceed their outside-option value. We demonstrate that the implication of such competition for ”good” workers only is to reduce the level of effort that will be implemented by firms at equilibrium, below the first-best level, and more so the greater the overall fraction of ”bad” (type 1) workers in the economy is. This is due to an adverse selection problem at the competitive equilbrium, when firms attract a mix of the two types of workers, and not just the preferred, high-productivity, type. This problem reduces the effort which the firms desires to implement for the high-productivity type, and it results in too low effort also for the low-productivity type.

In section 4 we consider cases where not all workers are employed at equilibrium. In such cases the utility of employed type 2 workers must equal their utility in the unemployed state. We here demonstrate that three different cases may arise. First, when the adverse selection problem is ”not too serious”, in the sense that the fraction of low-productivity workers is ”low”, such an equilibrium implies that a random fraction of all workers are employed, and all active firms employ a mix of the two worker types. Secondly, when the fraction of low-productivity workers is higher, two types of firms coexist in the market: one group of firms which attracts only type 1 workers, and another group which attracts a mix of the two types. Finally, when the adverse selection problem is sufficiently serious (there are ”very few” type 2 workers), all active firms may employ the low-productivity workers only. In all such equilibria, firms which employ a mix of the two types always select too low effort levels, as under the full employment solution. Firms which only employ low-productivity workers by contrast suffer no adverse selection problem, and select first-best efficient efforts for this group.

The paper is related to other work in the literature, notably McAfee and McMillan (1991), who also study a combined moral hazard-adverse selection problem with team production. In the same way as Holmström (1982) but as different from us, they focus on the internal allocation problem within the firm, and thus do not consider a competitive market for labor. Another difference is that the total disutility associated with working is lower for higher-productivity types in their formulation, but higher in our formulation. McAfee and McMillan then show that a direct-revelation mechanism can be constructed that permits truthful revelation of individual types, and that this in turn permits an efficient solution to be implemeneted. Here this is not feasible at a competitive equilibrium, due to the fact that under the informational constraints given, it is never possible to design a contract which low-productivity types prefer over that offered to high-productivity types. Separation of the two types must here rely on some firms employing low-productivity workers only. One should then also note that the possible generalization of our analysis, from two to more types (or a continuum of types as in McAfee and McMillan's formuation) is not straightforward and must in case await further research.

The paper is as noted also rather closely related to the seminal Rothschild-Stiglitz (1976) paper on equilibrium in competitive insurance markets with asymmetric information, with two types of insurance customers, corresponding to our two types of workers here. There are close parallels between the two papers, e.g. in terms of potential problems of existence of a competitive equilibrium when the fraction of "good" individuals is high, and the existence of an equilibrium where only low-quality individuals participate in the market whenever the fraction of such individuals is high. There are however also differences, which serve to ensure existence of equilibrium in more cases here than in Rothschild-Stiglitz, as noted in the final section below.

We will stress that our assumptions concerning the impossibility of measuring or inferring individual workers' contributions to a firm's output, are rather extreme and likely to be unrealistic in most cases. The analysis of such a pure case may still be of interest both from theoretical point of view, and as a starting point for analyses of how many actual labor markets function; in particular, why many talented individuals, who would make a larger contribution to aggregate social output by working as part of a large organization, instead opt to start their own enterprises, often at considerable risk.2

2. The effort allocation problem of a single firm

We initially consider the basic problem studied by Homström (1982), section 2, extended to explicitly embody the issue of worker heterogeneity. Here a single firm implements the allocation of effort for a given worker stock, consisting of n workers (where n may be large). Worker i puts up subjective effort $e_i \geq 0$ on the job, and $v(e_i)$ is subjective

2 Undeniably, often new startups by talented individuals are socially efficient as well. Our point here is that many startups may be initiated also in the opposite case.

disutility associated with this effort. The v function is assumed to be the same for all workers, and v', v'' > 0, for all $e_i \geq 0$. Assume that workers differ in their productivities, in the sense that for a given subjective effort, some workers contribute more to the firm's output than others. Denote the productive efficiency of worker i putting up effort e_i by $\alpha_i e_i$. The average measure of productivity for all workers in the firm, a, can then be written as $a = \Sigma\alpha_i e_i/n$, where the sum is taken over all n workers in the firm. In line with Holmström we assume that the firm's output can be expressed as $x = f(\Sigma\alpha_i e_i) + \varepsilon$, where ε is a stochastic variable with zero mean and continuous distribution $G(\varepsilon)$ on the domain $[-q, \infty)$, where $-q$ is some lower bound on ε (in particular, total output cannot be negative). The presence of the stochastic component ε is assumed to imply an imperfect correspondence between the sum of individual productivities and total output. The price of the firm's output is constant and equal to unity. x is then the amount of output that can be sold by the firm at this price. Workers are assumed to be risk neutral and their utilities given by $u_i = w_i - v(e_i)$.

The efficient allocation of workers' efforts is given by maximizing the expression

(1) $$W = Ex - \Sigma v(e_i) = f(\Sigma\alpha_i e_i) - \Sigma v(e_i).$$

W > 0 at the optimal solutions for the e_i is here a necessary condition for this problem to be meaningful. We find the following first-order conditions

(2) $$\alpha_i f'(na) - v'(e_i) = 0,$$

which must hold for all i.

The firm is assumed not to observe individual workers' types, efforts, nor their individual contributions to total output. Workers themselves however know their types. Individual workers' contributions to the total labor input in the firm are assumed to be fully independent. Assume also that workers do not observe each others' efforts not seek to cooperate in the determination of effort. We consider a one-period model, such that reputation or learning about workers' abilities are of no concern.

Since the firm can only observe total output, it has no way of differentiating workers' wages except by randomization. It can be shown that randomizing wages cannot dominate; we accordingly assume that all workers are paid the same wage. Consider now a payment scheme whereby each worker in the firm is paid w_1 if total output as observed by the firm at least equals some minimum level, call it x^*, and is otherwise paid w_0. Assume $w_0 \geq 0$, i.e., workers are never required to make net payments to the firm. The utility of worker i can then be written as

$$u_i = w_1 - v(e_i), \qquad x \geq x^*, \tag{3}$$
$$ = w_0 - v(e_i), \qquad x < x^*.$$

From the definition of x we find, for given e_i , that x^* and ε^* are related in the following way:

$$x^* = f(\Sigma\alpha_i e_i) + \varepsilon^*. \tag{4}$$

Define now P as the probability that x will exceed x^*, in which case worker i will receive the wage w_1 instead of the minimum wage w_0. We then have

$$P = \text{prob}(x \geq x^*) = \text{prob}(\varepsilon \geq \varepsilon^*) = 1\text{-}G(\varepsilon^*) = 1\text{-}G(x^*\text{-}f(\Sigma\alpha_i e_i)). \tag{5}$$

The effect of an increase in effort for worker i on this probability is given by

$$dP/de_i = g(\varepsilon^*)\alpha_i f'(na). \tag{6}$$

The expected utility of worker i is now given by

$$Eu_i = P(w_1 - v(e_i)) + (1\text{-}P)(w_0 - v(e_i)) = w_0 + P(w_1 - w_0) - v(e_i). \tag{7}$$

Using (6), worker i's first-order condition for optimal effort e_i is given by

$$g(\varepsilon^*)\alpha_i f'(na)(w_1 - w_0) = v'(e_i). \tag{8}$$

Comparing the efficient solution, (2), to the solution implementing individual worker effort, (8), we derive the following result:

Proposition 1*: Assume that the firm observes only aggregate output, and workers have individual productivity parameters α_i . Then the efficient allocation is implemented by the wage scheme*

$$(9) \qquad w_1 = w_0 + 1/g(\varepsilon^*),$$

which implies that all workers are paid the same wage, independent of individual productivities.

The proof is straightforward, comparing the expressions (2) and (8). The efficient allocation is here implemented by giving all workers the same wage, even though their productivities may differ widely. This result is striking, and perhaps surprising. Note in particular that at an optimal solution just described, the expected utility of worker i is given by

$$(10) \qquad Eu(i) = w_0 + P(w_1\text{-}w_0) - v(e_i) = w_0 + [1\text{-}G(\varepsilon^*)]/g(\varepsilon^*) - v(e_i),$$

where we have inserted for P from (5). (10), together with the optimality condition (2), imply that workers' expected utilities differ, and are lower for higher-productivity workers, since the optimal efforts are greater for these, while the wage is the same for all. Intuitively, a high-productivity worker knows that he or she has a relatively great impact on total output, and consequently a great impact on the probability that group output will exceed the target x*, for any given effort. This makes it optimal for a high-productivity worker to put up a higher marginal (and here thus total) effort, for a given bonus (w_1–w_0). The share of output ascribed to each individual worker will then also differ and be proportional to the individual efficiency coefficients α_i.

So far we have shown simply that an optimal solution, if it exists, must have the properties described by Proposition 1. We have said less about whether such a solution can and will be implemented. We will here consider the implementation problem as one of intrafirm allocation, where our main concern is studying whether an efficient solution exists. For this purpose we assume, following Holmström (1982, page 328) and writing $G(\varepsilon) = G(x\text{-}f(na))$:

Assumption 1: G is convex in na.

Assumption 2: $g(x\text{-}f(na))/[1\text{-}G(x\text{-}f(na))] \rightarrow \infty$ as $x \rightarrow \infty$..

Assumption 1 here assures global optimality of agents' actions. Assumption 2 can be shown to have the natural interpretation that the signal provided to the firm about workers' efforts, inferred from observed output, becomes very precise when observed output becomes very large.3

Consider now a labor market where all firms are identical, implying that they have identical production functions and that only group (and not individual) productivities can be measured in any firm.4 Assume that all workers have opportunities outside of the labor market with a common value B, which may represent the value of home production, unemployment compensation or leisure. Assume also that workers' productivity differences are purely general, i.e., a given worker has the same productivity in all firms. One could then in principle visualize a possible equilibrium where the workers employed by any given firm is a random sample of all workers in the economy. With a large number of workers in all firms, average worker productivity will then be (approximately) equal for all firms.

Consider such a possible solution. Since firms have no way of distinguishing between workers, they must pay the same wage to all. Denoting firm j's profits by R(j), the firm's expected profits are given by

$$ER(j) = f(n(j)a(j)) - n(j)w_0 - n(j)[1\text{-}G(\varepsilon^*(j))]/g(\varepsilon^*(j)), \tag{11}$$

where n(j) is the number of workers in firm j, n(j)a(j) is the value of the aggregate (effort-augmented) labor input for firm j, and $\varepsilon^*(j)$ is the cutoff level for ε selected by firm j. Feasibility of the optimal solution now requires $ER(j) \geq 0$ in (11), and $Eu(i) \geq B$ for all i.

3 See also Holmström (1979) and Milgrom (1981) for further discussions of these expressions and their interpretations.

4 We are thus here implicitly assuming that no firm can profitably reduce its scale of production to a size so small that individual workers' contributions to output can be identified.

Let α_m denote the upper support of the productivity distribution over workers. We can then show the following:

Proposition 2: *Assume a given number of firms m, and a given total number of workers N, no firm can observe individual workers' productivities, workers have outside opportunities with a common value b, Assumptions 1-2 hold, and firms have unbounded wealth. Then a first-best allocation implies that all workers are employed in firms whenever the following condition holds:*

$$(12) \qquad af'(na) \geq v(e_m) + B,$$

where $n = N/m$, $e_m = e(\alpha_m)$, and a and all e_i correspond to the optimality conditions (2). Moreover, such an allocation can in principle be implemented by firms using the wage scheme (9).

Proof: Consider setting $w_0 = v(e_m)+B$. Then the labor market participation constraint Eu(i) ≥ B must hold for all workers, in particular for workers with α_m. From Assumption 2, for a sufficiently high level of ε, [1-G(ε)]/g(ε) tends to zero, implying that Ew = $v(e_m)$+B is sufficient in the limit, to fulfill the participation constraint for all workers. Consider next firms. (12) is the condition that firms' marginal profits with respect to n are positive given that employment in all firms is n and all attract a random sample of workers. This implies that all workers should optimally be employed in firms. Consider finally possible states where $\varepsilon>\varepsilon^*$. For such states, from (9), $w= w_1 = v(e_m) + 1/g(\varepsilon^*)$. When firms have unbounded wealth, such wages can be paid regardless of $g(\varepsilon^*)$. Q.E.D..

Proposition 2 is based on Theorem 4 in Holmström (1982), with the additional consideration of implementability through the requirement that workers' participation constraints be fulfilled. Under our assumptions (all workers have the same outside utility B and there is no intrafirm worker mobility) these constraints are fulfilled whenever they hold for the highest-productivity workers. Note the role (as in Holmström) of the unbounded wealth requirement for the case where (12) is "just barely" fulfilled. In this case implementation requires that w_1–w_0 tends to zero, and that ε tends to its upper support. $g(\varepsilon^*)$ could then in principle be small, implying that the wage premium is large (in the (rare) event that such a premium is paid). Without the unbounded wealth requirement firms would not in general be able to cover their liabilities in such events, and the scheme would break down.5

Proposition 2 goes somewhat further than Holmstrom's analysis by adding considerations about workers' participation given the implementation schemes offered by firms. It is here clear that high-productivity workers are both hardest to give incentives, and at the same time most attractive to firms. So far there are however no parameters by which firms can compete for workers. This weakness will be remedied in the following sections. Note that, under our assumptions, those workers with the highest productivities end up with the worst outcomes, in terms of their combination of wage and effort (the wage is the same to all, but the effort is greater for those with high productivities). We thus have the (rather perverse) situation that high-productivity workers would prefer to be lower-productivity ones. The fact that they know they are good leaves then with the incentive to put up high effort given the wage scheme offered by the firm.

Two further comments are here in order. First note that Proposition 2 gives a sufficient condition for full employment being first-best efficient in an economy of the type desrcibed. The main point is that workers at productivity levels below maximum enjoy strictly positive utilities in excess of their outside options, at an equilibrium with full employment. One problem to be addressed below arises when (12) is not fulfilled. In such cases the implementation of a first-best solution in general requires that workers with different productivities be paid different wages, which is here not feasible.

The second, related, point is that Proposition 2 deals with the feasibility of implementing a first-best solution, not with its actual implementation, .e.g competitive economy. This issue will be dealt with in more detail in the next section.

3. Full-employment equilibria under labor market competition

Consider now an economy which corresponds to that in section 2 above, except that

5 One might here perhaps argue that the firm could be able to sign a future financial contract whereby the firm is insured against the event of a "very high" wage payment, resulting when $\varepsilon>\varepsilon^*$. We will not go into this issue in detail, just mention that moral hazard considerations are likely to make this difficult.

firms now are assumed to compete for workers, in a perfectly competitive labor market, an issue that was not considered by Holmström (1982). We focus in this section on the case where market equilibrium implies full employment, i.e., all those workers who supply their labor to the market are employed by firms. We still assume that competition among workers themselves is irrelevant, since workers act independently, their individual contributions to the firm's effort-augmented composite labor input are independent, and individual efforts cannot be observed by others than each worker himself or herself. We will study what types of solutions are implementable in such a setting, given that the N workers are free to move between m identical firms, and where firms maximize profits. We focus on one particular firm, and assume that the number of workers in this firm, n, is a choice variable for the firm. We will assume perfect mobility of labor, and denote the net utility provided by the labor market (”other firms”) to a worker of type i by Q_i. From the the previous section we maintain the following assumptions:

a) The wage paid to all workers must be the same;
b) The distribution of workers according to productivity must be the same in all firms;
c) Any feasible set of worker effort levels is in general implementable, provided that the constraint on relative effort levels derived from (8), expressed as $v'(e_i)/\alpha_i = k$ must hold, where k is a positive constant that can be decided by the firm.

We also take as given that efforts can be enforced through a wage scheme (9), such that expected wage payments (in the limit) equal a base wage w_0, and firms have unbounded wealth. A symmetric equilibrium in the market, with identical firms, must here imply that the particular firm we consider provides net utility Q_i to workers of type i, for any i, and that these utilities are identical across firms. We make the following two additional, more specific, assumptions:

<u>Assumption 3</u>: There are two types of workers, namely low-productivity ones (type 1), with productivity coefficients $\alpha < 1$, and high-productivity ones, with productivity coefficients equal to unity, and the relative shares of the two types are γ and $1-\gamma$, respectively.

<u>Assumption 4</u>: $v(e) = he + be^2/2$, where h and b are positive constants. The first and second derivatives of the v function can then be written as $v'(e) = h + be$, and $v''(e) = b$.

Assumptions 3-4 will lead to relatively simple analytics and the derivation of easily interpretable results, without seriously reducing the generality of the discussion.

By c) above, the following constraint must be observed by a firm which employes workers of both types:

$$v'(e_1) = \alpha v'(e_2).$$

Note also that, at equilibrium, the following conditions must hold for the two types of workers:

$$(14) \qquad w_0 = v(e_i) + Q_i, \qquad i = 1,2,$$

where w_0 is the common wage offered by the firm to all its workers. Note here that an implication of (13) is $e_1 < e_2$, and consequently, $v(e_1) < v(e_2)$. From (14), this implies $Q_1 > Q_2$, i.e., lower-productivity (type 1) workers must have the higher utility levels at an equilibrium with full employment. At the same time type 1 workers provide less output than type 2 workers for the firm, and are paid the same wage as the latter. As before, assume also that all workers have an outside option with value B interpreted as the (exogenous) value of leisure of home production. At a competitive equilibrium with full employment, $B \leq Q_i$ for i=1,2.

It is here clear that, ideally, firms prefer to attract only type 2 workers, since these are more productive and all must be paid the same wage. This leads to the following preliminary result:

Lemma 1: *At a competitive equilibrium with full employment, under assumptions a-c above, the participation constraint (14) will be binding on firms only for the best workers, i.e., i=2.*

At a full-employment equilibrium there is consequently effective (positive) competition among firms only for workers with the highest productivities. For low-productivity workers the situation is the diametrically opposite, namely that firms would wish to discourage these from joining. This implies that the expected utility constraint (14) for "bad" workers (i=1) plays an opposite role in that firms would ideally prefer to make these utilities as low as possible at the competitive equilibrium. Analytically this turns out to create a potential problem for existence of a competitive equilibrium in such a setting. To overcome this existence problem we impose the following condition:

Assumption 5: Consider a possible competitive equilibrium where some firms attract workers of both types. Then the ratio of the two types attracted is the same in all firms. The fraction of type 1 workers attracted is independent of the exact utility level offered to these.

The role of this assumption is to ensure the existence of a competitive equilibrium with effective competition for type 2 workers only. This will become clear from the discussion below. The basic idea exploited is that ”bad” workers do not wish to risk being singled out by firms; the least risky strategy in this regard is to spread out evenly across firms which employ both worker types.

Note that there is no mechanism available to firms, for selectively attracting high-productivity workers. In particular, a higher wage would not help, since this would attract workers of all categories and not just the high-productivity ones. In this light assumption b above appears reasonable: firms have to take the distribution of employed workers as given, and this distribution is the same for all firms (and identical to the market distribution).6

The firm’s problem can now be formulated as maximizing the following lagrangian

$$L = f(n(\gamma\alpha e_1 + (1-\gamma)e_2)) - nw_0 - \lambda\left[v(e_2) + Q_2 - w_0\right] - \mu\left[v'(e_1) - \alpha v'(e_2)\right] \tag{15}$$

with respect to e_1, e_2, w_0 and n. The two first terms in (15) represent firm profits under the preferred scheme, given that the fractions of the two types of workers attracted to the firm in question correspond to their relative market shares. The participation constrant for type 2 (”good”) workers is represented by the third term, where each firm takes Q_2 as given by the labor market as a whole. When this is fulfilled with equality, the required number of ”good”workers is assumed to be attracted, accompanied by a number of ”bad” (type 1)

6 We are here thus focussing on symmetric equilibria where all firms’ profits must be the same. This is analytically

workers corresponding to relative market frequencies, in accordance with Assumption 5. The last term represents the constraint on relative efforts that are implementable using a fixed-wage scheme, given by (13). This maximization yields the following set of first-order conditions:

(16) $$\frac{\partial L}{\partial e_1} = n\gamma\alpha f'(.) - \mu v''(e_1) = 0$$

(17) $$\frac{\partial L}{\partial e_2} = n(1-\gamma)f'(.) - \lambda v'(e_2) + \mu\alpha v''(e_2) = 0$$

(18) $$\frac{\partial L}{\partial w_0} = -n + \lambda = 0$$

(19) $$\frac{\partial L}{\partial n} = [\gamma\alpha e_1 + (1-\gamma)e_2]f'(.) - w_0 = 0.$$

(16)-(19), together with the two side constraints, now together determine the 6 variables e_1, e_2, w_0, λ, μ and Q_2. A symmetric full-employment equilibrium moreover implies that n can be taken as given for the market as a whole, and equals N/m. (16) and (18) may be used to eliminate μ and λ respectively. Using the assumption that v(e) is quadratic, we now find from (17):

$$[1-\gamma+\alpha^2\gamma]f'(.) = v'(e_2).$$

We can now derive the following conclusion:

Proposition 3: *Consider a symmetric competitive labor market equilibrium in an economy otherwise described in Proposition 2, Assumptions 1-5 hold, and equilibrium implies full employment. At such an equilibrium workers' efforts are below first-best optimal. The deviation from first-best optimality is greater the larger the fraction of the low-productivity type, and the greater the difference in productivity between types.*

Proof: One may here compare (20) to the first-best optimality condition (2), which here takes the form f'(.) = v'(e_2). One then finds that v'(e_2) is lower in (20) than in the first-best

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convenient, but does not deny the principal possibility of existence of asymmetric equilibria.

case, using concavity of the firm's profit function. As a consequence e_2 is lower than first best in (20). From (13), e_1 is then also lower than first best. From (20), the discrepancy from first best increases in α and γ. Q.E.D.

Proposition 3 implies a tendency for firms to implement below-first-best efforts in a competitive market solution. Intuitively, a lower than optimal effort is a cost-effective mechanism for firms of attracting the (attractive) high-productivity workers, since it permits the wage to be reduced not only for these workers, but also for the (unattractive) low-productivity workers. Low-productivity workers' efforts are then also lowered, but this counts for less since the latter contribute less to output, and the output loss resulting for these is more than made up through lower wages.

Our derived solution depends on Assumption 5, which may appear arbitrary and somewhat artificial. When this assumption does not hold, a problem arises since lowering the effort requirement of all workers, in such a way that (13) is fulfilled, and holding Q_2 fixed in (14), implies that the utility level Q_1 of low-productivity workers is reduced.7 Technically this implies that starting from a symmetric "equilibrium" just derived, any firm could reduce the effort requriement "a little", still holding Q_2 fixed, and thereby reducing Q_1 below the level of utility enjoyed by type 1 workers in other firms. This is exactly the existence problem pointed out by Rothschild and Stigliz (1976), in their seminal work on competitive insurance markets.8 We will however argue that there are some fundamental differences between insurance and labor markets, which makes the possibilities for existence greater in the present case. Assumption 5 here "saves" the model by "forcing" workers to remain with their initial firm under such circumstances. Intuitively, starting from an initial symmetric equilibrium (of the type derived), it could appear as a risky strategy for a low-type worker to quit his or her initial firm in pursuit of a (marginally) higher utility elsewhere, since this could permit public identification of the worker's type.9

We will point out that derived solution in principle can be "saved" also in the absence

7 This is easily seen differentiating the expressions (14) for i = 1,2, inserting from (13) and holding Q_2 fixed.

8 See also Hirschleifer and Riley (1992), chapter 11, for a more general exposition of this problem. Our resolution of this problem here is essentially to remove the client anonymity assumption, on grounds that the act of changing employer is a much more conspicuous event than the act of changing insurer.

9Alternatively, firms could be viewed as improving the contracts to type 2 workers slightly, such as to attract only these. Assumption 5 then correspondingly implies that a required number of type 1 workers at the same time would move.

of Assumption 5, by invoking the concept of a socalled ”reactive” (or ”anticiaptory”) equilibrium, introduced by Wilson (1977) and Riley (1979) and further developed by Kohlberg and Mertens (1986) and Engers and Fernandez (1987). The basic idea here is that given an initial set of contracts (corresponding to the derived equilibrium), a change in the contract offered by one particular firm would invoke a reactive response on behalf of the other firms (who would change their contracts accordingly), such as to render a unilateral change of contracts unprofitable. In other words, it would not be possible for any one firm to improve its mix of worker types by lowering the effort requirement, due to the provoked response from other firms.10

4. Competitive equilibria with unemployment

4.1 Preliminary considerations

The solution discussed in section 3 above implies full employment among both worker types. Technically, at the symmetric market solution represented by the system (16)-(19) and the side constraints (13) and (14) (for i=2), we must have $Q_i \geq B$ for i=1,2 and $n = N/m$: all workers are better off being employed at such a solution, than being unemployed (and generally, except in borderline cases, strictly better off, i.e., the inequalities hold strictly).

We will now study cases where these inequalities no longer hold. In particular, we assume that $Q_2 < B$ at a solution with full employment as described in section 3 above. In this case not all workers will be absorbed in firms at equilibrium, and some workers will be unemployed. An equilibrium with unemployment among some fraction of the total labor force now in general implies that some workers of both types will be unemployed. To see this, assume first that all unemployed workers are of the high-productivity type (2). But this cannot be an equilibrium, since firms would then gain by hiring out of the unemployed; in effect unemployment would serve as an efficient screening device for firms, and the initially suggested solution cannot be stable. Assume next that all unemployed workers are of the low-productivity type (1). This is a very unreasonable candidate for equilibrium,

10 Kreps (1990) discusses the concept of reactive equilibrium in relation to game theory in more detail. His conclusion is that a sound game-theoretic basis for this concept can be secured only in a repeated-game setting, where firms can credibly threaten to punish unilateral defection from the established “equilibrium”. We will here disregard such theoretical problems, and stress that our main interpretation of the established equilibrium is in terms of worker

since low-productivity workers' utilities must be higher in employment than in unemployment ($Q_1 > Q_2 = B$). Type 1 workers then have incentives which are at least as great as those of type 2 workers, to seek paid employment. In this light the following assumption may appear reasonable:

Assumption 6: Consider an equilibrium where some workers are unemployed. Then the ratio of type 1 to type 2 workers, in firms which employ both worker types, is the same as the ratio of the two types of workers in the pool of the unemployed.

This assumption embeds two separate features of the process of worker selection in an economy with unemployment and adverse selection. First, the pool of the unemployed cannot be ”better” than the pool of the employed (i.e., the fraction of type 2 workers cannot be higher in the unemployment pool than in the employment pool) for those firms who experience adverse selection (here, employ both types of workers). This feature ensures that firms which employ both worker types, have no incentives to fire their own workers and instead hire out of the pool of the unemployed. Secondly, the unemployment pool can neither be ”worse” than the employment pool for such firms. This reflects the property of the labor market in this particular case, that firms employing both worker types have no way of selectively attracting preferred workers among those available in the market.

An obvious consequence of assumption 6 is that some workers of both types will be unemployed, at an equilibrium with unemployment. The equilibrium solution must entail $Q_2 = B$ (good workers are indifferent about working and not), and $Q_1 > B$ (unemployed bad workers enjoy strictly lower utilities than employed ones). This opens up for the possibility that low-productivity workers can be attracted selectively by firms who aim to employ only these. It is now namely possible to reduce the wage to these below w_0, paid to all above, and still make these workers prefer such employment, provided that the other relevant option of these workers is unemployment. A firm attracting low-productivity workers selectively would maximize the following lagrangian:

$$H = f(n_1 \alpha e_1) - n_1 w_1 - \eta[v(e_1) + B - w_1],$$

reactions, as indicated in the text above.

where n_1 now is the number of workers in such a firm (all of which are of type 1), w_1 is the wage paid to these, and η a lagrange multiplier associated with the worker expected utility constraint in this case. Maximizing (21) with respect to e_1, w_1 and n yields the set of first-order conditions:

(22) $$\frac{\partial H}{\partial e_1} = n\alpha f'(.) - \eta v'(e_1) = 0$$

(23) $$\frac{\partial H}{\partial w_1} = -n + \eta = 0$$

(24) $$\frac{\partial H}{\partial n_1} = \alpha e_1 f'(.) - w_1 = 0.$$

We may now derive, from (22)-(23), the condition

(25) $$\alpha f'(.) = v'(e_1),$$

which is the first-best condition (and thus identical to (2)), for the low-productivity type. The solution to (22)-(24) does not depend on the number of unemployed workers, the fraction of type 1 workers in the unemployed, nor the solution for firms that employ a mix. We may thus view the profits of firms that attract only type 1 workers, π_1, as an exogenous parameter in the following. π_1 however generally does depend on the parameters B and on α.

To study possible equilibria in such cases it is convenient to first establish two preliminary results.

Lemma 2: *Consider an equilibrium with unemployment. Then the profit π_2 yielded by the system (16)-(19) implies a higher profit than π_1, yielded by the system (22)-(24), provided that the fraction of type 1 workers in the worker pool of firms employing both types, γ_1, is sufficiently small.*

This conclusion follows from the property that when γ_1 is (close to) zero, firms employing a mix of worker essentially experience no adverse selection. Since the value of unemployment is the same for all workers, and type 2 workers are more productive than type 1 workers, the output of firms attracting a mix of workers is always greater than the output of firms attracting only type 1 workers, for any given wage. Thus the (maximal) profits of firms attracting a mix, π_2, must be greater than the (maximal) profits of firms attracting only type 1 workers, π_1.

Lemma 3: *Consider a case where γ_1 is close to one. Then $\pi_2 < \pi_1$.*

This result is also obvious. Also in this case firms attracting a mix of workers "essentially" experience no adverse selection, but now in the sense that there are only "bad" (type 1) workers, and thus in a manner opposite to that under lemma 2. With a homogeneous labor force with only type 1 workers, the system (22)-(24) maximizes profits, and the system (16)-(19) must consequently yield a lower profit.

We are now ready to consider two different cases, as follows.

4.2 Case 1: Unemployment equilibria with only one type of firms

We first consider the possibility of an equilibrium where some workers (of both types) are unemployed, and where there only exist firms which mix, i.e. employ both types of workers. We then immediately have the following result.

Proposition 4: *Assume that we have an equilibrium with unemployment. Then under Assumptions 1-6, all active firms employ a mix of the two types of workers, provided that $\pi_2 > \pi_1$, i.e., for a "sufficiently low" level of γ.*

This result follows directly from Lemma 2, and the property that $\gamma_1 \leq \gamma$, at any possible relevant equilibrium. The latter condition implies that the fraction of low-productivity workers in firms that mix never exceeds γ. From Assumption 6, at an equilibrium with unemployment and all firms attract a mix of workers, the fraction of type 1 workers must equal γ in all firms. The condition $\pi_2 > \pi_1$ then assures that it is never profitable for any firm to offer a contract which attracts only type 1 workers. Thus no such contracts will be offered, and the only possible equilibrim is one where all firms attract a mix.

Intuitively, such a solution occurs when the adverse selection problem facing such firms is ”not too severe”. The contracts offered by firms are designed to fulfill the incentive compatibility and participation constrant of ”good” workers, and the ”contamination” effect from having some ”bad” workers in the economy as well is not particularly serious.

4.3 Case 2: Unemployment equilibria with two types of firms

From Lemma 3, the equilibrium described by Proposition 4 cannot be valid whenever γ is sufficiently high (close to one). The reason is that under such an equilibrium, firms would make higher profits attracting only type 1 workers, instead of attracting a mix where a fraction γ is of type 1. A possible equilibrium must then involve at least some firms attracting type 1 workers only. To study such cases we find it convenient to impose the following condition on workers' beliefs:

Assumption 7: Consider workers of type 1 in firms employing only such workers. At an equilibrium with unemployment, these workers attach probability zero to the event of obtaining employment in firms that employ a mix of the two types of workers, upon quitting their current job.

The beliefs implied by Assumption 7 may be reasonable at an equilibrium where hiring of firms that employ a mix of the two types already has been made. Its implication is that the utility of type 1 workers in firms employing only such workers, is kept at B, i.e. the value of unemployment.

Define now the following variables:

n_1 = the number of workers in each firm which employs only type 1 workers

m_1 = the number of firms employing only type 1 workers.

Thus n_1m_1 equals the number of type 1 workers in firms employing only this type. By Assumption 5, the fraction of type 1 workers in firms that employ both types (given that there exist any such firms) is then given by

(26) $$\gamma_1 = \frac{\gamma N - n_1 m_1}{N - n_1 m_1}.$$

We may then formulate the following result:

Proposition 5: *Under assumptions 1-7, and given that γ is sufficiently high to make $\pi_2 < \pi_1$, there exists a Bayesian equilibrium where a positive fraction of firms, m_1/m, employ type 1 workers only. This fraction equals one given that $\pi_2 < \pi_1$, where π_2 is calculated replacing γ by γ_1 from (26) in the expression for π_2, and setting $m_1 = m$. The fraction m_1/m is less than one given that $\pi_2 > \pi_1$ when calculated at $m_1 = m$. In the latter case a fraction $(m-m_1)/m$ of all firms attract a mix of type 1 and 2 workers, with fraction γ_1 being of type 1, and enjoy profits $\pi_2 = \pi_1$.*

Proposition 5 covers two different classes of cases. First, it covers cases with a ”very severe” adverse selection problem, where there are few high-productivity workers in the economy. In such cases all firms operating at equilibrium employ type 1 workers only. Note that γ_1 from (26) is a decreasing function of m_1, the number of firms which attract type 1 workers only. The lower γ_1, the greater is π_2. Thus π_2 attains it maximum value for $m_1 = m$, i.e. when all active firms attract type 1 workers only. If the maximum value of π_2 falls below π_1**,** no firm will seek to attract type 2 workers (and thus a mix of the two types in accordance with Assumption 5). No type 2 workers will then be employed.11

The other case occurs when this maximum value of π_2 exceeds π_1. Then equilibrium must imply $\pi_2 = \pi_1$, and γ_1 is determined endogenously, thus determining m_1 and thereby the number of firms $m_2 = m - m_1$ which employ a mix of the two types of workers.

Proposition 5 here only asserts the existence of an equilibrium of either of these two types; it does not address how such an equilibrium is established starting from an out-of-equilibrium situation. In particular, it does not address the issue whether type 1 workers are actually willing to accept employment in firms attracting only these, at terms identical to

11 One might argue that this case would be irrelevant under free firm entry, where firms enter whenever profits are positive. Given that firms employing type 1 workers only can make positive profits, a sufficient number of such firms would then enter so as to reduce γ_1 sufficiently, such that production in firms employing both worker types becomes profitable. We will not go in detail on this here, only note that firm entry would modify our results above.

those enjoyed by unemployed workers, given that other firms attract a mix of workers (and provide utilities which are higher than the unemployment value to type 1 workers). It asserts that given that the required number of type 1 workers have already accepted employment in firms employing their own type only, such workers have no incentive to quit their jobs given beliefs represented by Assumption 6.

Note that in cases with coexistence of two types of firms, the presence of firms employing only type 1 workers serves to ameliorate the adverse selection problem for those firms that employ both types. The reason is that this absorbtion lowers the share γ_1 of ”bad” workers in the latter group of firms. As a consequence the equilibrium efforts of both worker types are increased (in the direction of the optimal solution), from (20).

The ”extreme” equilibrium under which no high-productivity worker is employed here corresponds to the separating equilibrium in the Rothschild-Stiglitz model whenever it exists. Note that such an equilibrium exists in ”fewer cases” here, due to our Assumption 5 (which implies that alternative equilibria arise in more cases).

Note finally that in all possile equilibria with unemployment and coexistence of two types of firms, the rate of unemployment will be greater for ”good” than for ”bad” workers. Overall inefficiencies take three different forms: first, effort allocations are inefficient in firms employing both types; secondly, the intrafirm allocation of labor is inefficient; and thirdly, there is inefficient (too high) unemployment of ”good” workers.

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