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On The Origins of Data Envelopment Analysis

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ON THE ORIGINS OF DATA ENVELOPMENT ANALYSIS[1]

by

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Abstract. The concept "Data Envelopment Analysis" (DEA) was introduced in the journal literature by the highly influential 1978 paper of Charnes, Cooper, and Rhodes. In the subsequent literature the development of research leading up to this paper tended to be forgotten. However, studying this diffusion of ideas may still give valuable insights into research issues still unexplored and insight in the research process itself. A natural starting point is Farrell's seminal 1957 paper on concepts of efficiency and their computation. The richness of ideas presented in Farrell is demonstrated by the fact that the developments in the following two decades were based on aspects and ideas there. The origins of the main developments are identified, and the connections to Charnes, Cooper, and Rhodes are explored.

Keywords: Farrell efficiency measures, frontier function, Linear Programming, DEA.

JEL classification: B21, D24.

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[1] The paper is a further development of "The diffusion of research on productive efficiency: The economist's guide to DEA evolution" by Finn R. Førsund and Nikias Sarafoglou, Discussion Paper #D-02/1999, Department of Economics and Social Sciences, NLH, and first draft presented at the Sixth European Workshop on Efficiency and Productivity Analysis, Copenhagen October 29 – 31, 1999. Tor Jakob Klette has given valuable comments on the last version.

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1. Introduction

Economics and operational research have common interests as to several research fields, one of the prominent ones being analyses of the production possibilities for micro units. The specific research strand of efficiency measurement for production units in the field of Operational Research took off with ”Measuring the efficiency of decision making units” by Abraham Charnes, William.W. Cooper and Edwardo Rhodes (1978) (CCR) as the seminal paper. The increasingly popular empirical use of linear programming techniques for calculating efficiency scores is due to the *DEA model* introduced to the general research public in CCR. Per 1999 the CCR paper has over 700 citations in the Social Sciences Citation Index[2].

New frontiers of research usually have explicit shoulders to stand on. In CCR, and in working papers leading to it (see Charnes, Cooper and Rhodes (1976, 1977)) a key inspiration is the paper “The measurement of productive efficiency” by Michael James Farrell (1957)[3].

It seems to be a widespread misunderstanding, and not only in the operational research and management science (OR/MS) community, that Farrell (1957) was forgotten until CCR was published. A typical quotation from a person outside the OR/MS community, is:

> Farrell’s contribution was itself ignored for more than two decades. It was rediscovered by Charnes, Cooper, and Rhodes (1978), who referred to the mathematical-programming method of measuring technical efficiency as data envelopment analysis (DEA), an appellation that seems to have stuck. Their paper

[2] OR scientists could argue that this paper is not the most important work of William W. Cooper. His work on Stochastic programming in the late 1950s is recognised as path breaking. However, no other paper of W.W. Cooper has achieved such a high citation as CCR within his over 2500 lifetime citations.

[3] It is interesting to note that Abe Charnes and Bill Cooper knew Mike Farrell well before 1957. Farrell was a visiting scholar for about two years on the faculty of the Graduate School of Industrial Administration at Carnegie Institute of Technology (now Carnegie Mellon University) in the early 1950s, where Abe Charnes and Bill Cooper were teaching. Farrell went back one more time before 1957 and unfortunately contracted polio, forcing him to use crutches. He worked with Abe Charnes, but not on efficiency-related topics. Bill W.Cooper’s Ph.D. student Edwardo Rhodes brought Farrell’s paper to the attention of Bill W. Cooper, and Charnes was brought in at a later stage of Rhode’s Ph.D. (Private communication from Bill W. Cooper.)

> has led to a flood of papers applying DEA, most of them in management-science/operations-research journals."
>
> (Russell (1998), p. 28.)

On the contrary, compared with an average paper, Farrell received quite a widespread attention *prior* to the publication of CCR, and citations of Farrell continued to outnumber those of CCR in the late 70s through the 80s, and into early 90s.

The main purpose of the paper is to show the development in the economics literature in the period between Farrell (1957) and the publication of CCR in 1978, and to elaborate on the connection between the former and CCR. The richness of ideas presented in Farrell's seminal 1957 paper (including the *"Discussion on Mr. Farrell's Paper"* (Farrell (1957), pp.282-290), is demonstrated by the fact that the developments in the following three decades were based on aspects and ideas there. Problems and points discussed there became key research questions. The explicit use of Farrell's ideas in the developments within economics will be studied in detail.

Within economics Farrell's efficiency analyses were followed up by means of general or parametric frontier functions, being restricted to single output in the development of econometric techniques. CCR and the subsequent literature within OR/MS took up the development of linear programming applied to non-parametric specifications of the production possibilities. Although statistical inference should not be the only interpretation of econometrics, it seemed that way in the two decades between the seminal papers.

Independent discoveries are rather the rule than exception in research. A group of agricultural economists at Berkeley, with James Boles as the main person, pioneered the linear programming approach in 1967, and in Boles (1971) the complete CCR model is found. We will discuss the contribution of the agricultural economists and reasons for being overlooked by the profession.

The plan of the paper is as follows: Farrell's contributions are reviewed in Section 2, and some bibliometric information based on citation data of SSCI as to the diffusion pattern presented in Section 3. The development of Farrell's ideas and the use of insights of Farrell are elaborated upon in Section 4. Concluding remarks are offered in Section 5.

2. The Farrell approach to efficiency analysis

In the immediate post-war years there was a general interest in growth and productivity, and Solow's most influential paper on these issues within a macro setting appeared in 1957. At the same time Farrell laid the foundation for new approaches to efficiency and productivity studies at the micro level, involving new insights on two issues: how to define efficiency and productivity, and how to calculate the benchmark technology and the efficiency measures.

The fundamental assumption was the possibility of inefficient operations, immediately pointing to a *frontier production function* concept as the benchmark, as opposed to a notion of *average performance* underlying most of the econometric literature on the production function up to the time of the seminal contribution.

2.1. Farrell's contributions

The contribution of Farrell was path breaking as to three aspects:

(i) efficiency measures were based on radial uniform contractions or expansions from inefficient observations to the frontier,

(ii) the production frontier was specified as the most pessimistic piecewise linear envelopment of the data,

(iii) the frontier was calculated through solving systems of linear equations, obeying the two conditions on the unit isoquant:

> (i) that its slope is not positive;
> (ii) that no observed point lies between it and the origin.
> (Farrell (1957), p.256).

Efficiency and productivity are core concepts of economics. The new aspect of Farrell was to offer a decomposition into technical efficiency[4], price (or allocative) efficiency and overall efficiency at the micro level of a firm (or production unit). The radial contraction/expansion connecting inefficient observed points with (unobserved) reference points on the production frontier as the basis for the measures is the hallmark, and due to fundamental duality between production– and cost functions identical measures can also be defined using the latter.[5] The Farrell efficiency measures are shown in the original illustration in Figure 1. The definitions are:

> *Technical efficiency*: inputs needed at best practice to produce observed outputs relative to observed input quantities, keeping observed input ratios; OQ/OP.
>
> *Price efficiency*: costs of producing observed output at observed factor prices assuming technical efficiency, relative to minimised costs at the frontier; OR/OQ.
>
> *Overall efficiency*: costs of producing observed output if both technical efficiency and price efficiency are assumed relative to observed costs;
> OR/OP = (OQ/OP) (OR/OQ).

In the choice of a production frontier benchmark Farrell adopts a most practical approach, starting with engineering considerations and ending up with recommending observed best practice.[6] Inspired by the activity analysis of Koopmans (1951) his contribution was to introduce a *piecewise linear* envelopment of the data as the most pessimistic specification of the frontier, in the sense of the function being as close to the observations "as possible" (see Figure 2 for his original illustration), and to show how the frontier could be established by solving linear equations.

---

[4] Farrell himself points to Debreu's (1951) concept of "coefficient of resource utilisation" as inspiration for his measure of technical efficiency. This has led some to adopt the name "Debreu – Farrell measures of efficiency" (see e.g. Färe and Lovell (1978), Färe, Grosskopf and Lovell (1985), Russell (1998)). However, studying Debreu reveals that he was concerned with an economy-wide characterisation of resource utilisation. Furthermore, he worked only from the resource cost side, defining his coefficient as the ratio between minimised resource costs of obtaining a given consumption bundle and actual costs, for given prices and a proportional contraction of resources.

[5] Debreu's coefficient of resource utilisation is just built up from the cost function side.

[6] "Although there are many possibilities two at once suggest themselves – a theoretical function specified by engineers and an empirical function based on the best results observed in practice." (Farrell (1957), p. 255).

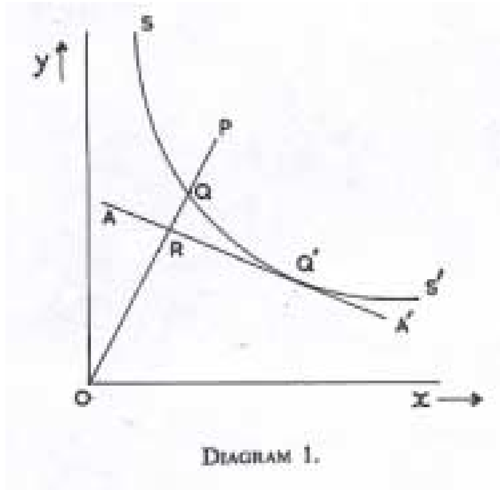

Figure 1. The definition of Farrell's technical efficiency measure. (Note that y and x are input coefficients.)

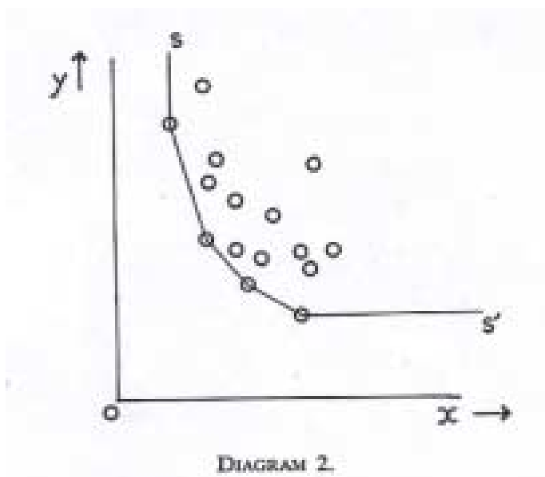

Figure 2. The piecewise linear production frontier (unit isoquant). (Note that y and x are input coefficients.)

2.1. Shoulders to stand on

Research is not carried out in isolation. As already mentioned Farrell points to Debreu and Koopmanns as inspirations. Otherwise it may be of interest to note that Farrell himself lists only eight references. This may be partly due to the paper's intended public:

> It is hoped that the paper will be of interest to a wide range of economic statisticians, businessmen and civil servants, many of who have little knowledge

> of economic theory or mathematics. For their benefit, the main exposition is in the sort of terms used by elementary economic textbooks, and such elementary mathematics as is necessary for discussion of the general case or of the computing problems is confined to sections 2.3 and 5. Similarly, although the treatment of the efficient production function is largely inspired by activity analysis, no reference is made to this in the exposition. The professional economist can easily draw the necessary parallels for himself, as indeed, he can note the similarity of the measure of "technical efficiency" and Debreu's coefficient of resource utilization…
>
> (Farrell (1957), pp. 253-254.)

However, he overlooked one link with the axiomatic production theory that would have been very appropriate as motivating his choice of the radial contraction or expansion from an observation to the frontier, namely the distance function concept introduced to economists by Shephard (1953). Another contribution overlooked, making use of proportional changes, was Malmquist (1953). The latter paper inspired Caves, Christensen and Diewert (1982) to launch the *Malmquist productivity index* based on ratios of Farrell efficiency indices.[7]

In fact, Farrell did not motivate much his choice of efficiency measure definition. Shephard used the distance function as a crucial device when establishing the fundamental duality between production and costs; i.e. proving the unique correspondence between the production technology and the cost function. Farrell actually used this feature when he showed how his efficiency measures also had a cost interpretation.

3. The diffusion of Farrell's ideas

Main factors influencing citations, apart from the quality of the research contribution, are reputation of journal, reputation of author, number of close followers; colleagues, "cadres of proteges" (Zuckerman (1987, p.332)), Ph.D. students, and extent of network ("invisible college"). The journal Farrell published in was well known in England, but not so in U. S., where the journal was not included in the social science databases. He himself had a good reputation, although not a prolific journal contributor, but he had no group of Ph.D. students around him. The Cambridge

[7] However, no reference was made to Farrell, only to distance functions.

location should not be a disadvantage from a network point of view, but as a person he would probably not be a pushy marketing man for his own paper or ideas. In fact, the first self-citation, a possible sign of advertising, came quite naturally in a follow-up paper five years later on scale issues (Farrell and Fieldhouse (1962)).

Economist both from Europe and US did early citations of Farrell. The most central are set out in Table 1. The first citation came in 1959, and was not very enthusiastic, as revealed by the evaluation:

> Mr. Farrell's work has the virtue of making a start – though perhaps a false start – where previously little had been done. (Hall and Winsten (1959, p. 85)).

The next reference was in 1961[8] and then 1962 by Farrell himself.

Another early citation came in 1964 by an OR scientist (Amey), and was quite interestingly a consequence of 1963 being officially designated as a National Productivity Year in England. The conference theme of the National Conference of the Operational Research Society was "Productivity Criteria: Their use and abuse". Amey (1964) was read at the meeting.

The group of researcher was heterogeneous, ranging from OR - people, econometricians, agricultural economists and mainstream economists to management scientists. The journals involved reflect the heterogeneity, but have on average high "impact factors".

[8] Ruist (1961) is a translation from Swedish of Ruist (1960).

Table 1. Early citations of Farrell (selection)

Year	Author(s)	Journal
1959	Hall & Winsten*)	Economic Journal
1961	Ruist*)	Productivity Measurement Review
1962	Farrell & Fieldhouse*)	Journal of Royal Statistical Society
1964	Amey	Operational Research Quarterly
1967	Boles, Bressler *)	Western Farm Economics Association Proceed.
1968	Aigner & Chu	American Economic Review
1970	Nadiri	Journal of Economic Literature
1970	Seitz	American Journal of Agricultural Economics
1971	Førsund	Swedish Journal of Economics
1971	Timmer	Journal of Political Economy
1971	Seitz	Journal of Political Economy
1972	Afriat *)	International Economic Review
1972	Carlsson	Swedish Journal of Economics
1972	Hanoch & Rothschild	Journal of Political Economy
1973	Leibenstein	Journal of Political Economy
1974	Førsund & Hjalmarsson	Swedish Journal of Economics
1975	Färe	Zeitschrift für Nationalekonomie
1976	Weston	European Journal of Marketing
1977	Aigner, Lovell & Schmidt	Journal of Econometrics
1977	Meeusen & Broeck	International Economic Review
1978	Charnes, Cooper & Rhodes	European Journal of Operations Research
1978	Färe & Lovell	Journal of Economic Theory
1979	Førsund & Hjalmarsson	Economic Journal

*) Not in SSCI

Farrell struggled to compute the efficiency measures for his illustration, consisting of only 48 units and five variables. Employing the first electronic computer in Europe, the ESDAC computer in Downing Street (the ESDAC has its 50ies jubilee in May 1999), up to 60 hours for one run is reported (Farrell (1957), p.265). (ESDAC had probably less computing capacity than a modest pocket calculator of today!).

4. Developing Farrell's ideas

Within economics two strands of development can be distinguished; the development of estimation methods for a parametric frontier production function and the theoretical underpinnings of the Farrell efficiency measures. Thompson and Thrall (1993) have the following comment as to classifications of contributions after Farrell:

Farrell's seminal paper was followed by a relatively large numbers of refinements and extensions, which may be broadly classified into three schools of thought (where the initial contributor's name provides a convenient reference here) (Thompson and Thrall (1993) pp.13-14).

The three schools identified by Thompson and Thrall (1993) were the *Afriat School*, the *Charnes School* (more appropriately termed the *Charnes – Cooper School* hereafter) and the *Shephard School*. The first one covers the econometricians' parametric estimation approach, while the last one may more accurately be termed the axiomatic production theory school.

4.1. Developing estimation procedures for parametric frontiers

On the estimation from a statistical viewpoint Farrell observed:

> There exists some efficient function, from which all the observed points deviate randomly but in the same direction." (p. 263),

and then he referred to the analogy of estimating the parameters of extreme distributions. Some of the milestones directly building upon Farrell (see e.g. Førsund, Lovell and Scmidt (1980) for a survey) are the deterministic approach of Aigner and Chu (1968), Afriat (1972) on statistical foundation of frontier estimation, Richmond (1974) introducing "corrected ordinary least squares" (COLS) and the composed error introduced by Aigner, Lovell and Schmidt (1977) and independently by Meeusen and Broeck (1977). Later econometric developments are the panel data model (see Cornwell, Schmidt and Sickles (1990) and Battese and Coelli (1992)), and works on giving non-parametric models statistical interpretations (see e.g. Banker (1993), Simar (1996) and Simar and Wilson (1998, 2000).

4.1.1.The deterministic parametric approach

The stated purpose in "On estimating the industry production function." by Aigner and Chu (1968) (AC) was to provide an estimation technique for a deterministic parametric frontier production function. The connection to Farrell was made quite clear in the introduction:

> In the spirit of M. J. Farrell [5] [6] [Farrell (1957) and Farrell and Fieldhouse (1962)] who constructs an envelope isoquant for the industry, the "industry" production function is conceptually a frontier of potential attainment for given input combinations. Aigner and Chu (1968), p. 826.

This was the first time econometric estimation of an alternative concept to the *average* function was offered. Although sympathetic to Farrell's non-parametric method, this approach was discarded for the standard parametric approach of economists:

> .., M. J. Farrell [5] [6] defined an "efficient production function" which resembles our industry function, and devised an ingenious way of estimating this efficient production function through constructing isoquants. The difficulty with Farrell's method, however, is that it is not general enough. Many types of production cannot be characterized within his models. For example, it is not possible to estimate a production function with his method which conforms to the Law of Variable Proportions. Aigner and Chu (1968), p. 830.

AC introduced the standard Cobb – Douglas function as the benchmark for efficiency estimation, and within a deterministic setting linear and quadratic programming was used to calculate the frontier. The focus was not on efficiency as such, but on the frontier production concept. No new material or insights on efficiency was offered. The connection to the linear programming techniques used in Farrell and Fieldhouse and by the agricultural economists' group at Berkeley was not recognised.

4.1.2. Utilising the average

Mr. C. B. Winsten (1957, p. 283), after complementing Mr. Farrell on an ingenious method of estimating the frontier function, wondered:

> It would also be interesting to know whether in practice this efficient production function turned out to be parallel to the average production function, and whether it might not be possible to fit a line to the averages, and then to shift it parallel to itself to estimate the efficient production function.

Such a procedure was some 20 years later introduced by Richmond (1974) but he had no reference to Farrell or the Discussion at all (only to Afriat). Richmond's approach is termed "Corrected Ordinary Least Squares" (COLS) in Førsund, Lovell and Schmidt (1980), and we also have the related "shifted ordinary least squares" of Gabrielsen (1975) and Greene, 1980).

Another way of utilising averages when estimating frontier functions is provided by Mr. Sturrock (1957, p. 285) in the discussion of Farrell's paper:

> It would be better to have either "average results" or "premium results" (say the average of the upper 10 or 20 per cent.).

The latter procedure is related to what is called estimating a "thick frontier", see Berger and Humphrey (1991).

4.1.3. The stochastic frontier function

The interventions by Mr. Sturrock about problems with joining up the *best* results, and concern of Mr. J. A. C. Brown (1957) about errors of observation and possible bias corrections, inspired Farrell to exclaim: "Indeed, I still hope that some theoretical statistician will take up the challenge offered by this knotty little problem." (p. 290). The development of estimation procedures for stochastic frontiers, elaborated in Afriat (1972), can be seen as response to the interventions quoted above and Farrell's own prayer. However, his use of Farrell was very brief. In the first sentence Afriat states:

> The method for production analysis which is to be studied has relationship to the approach Farrell [18] has made to production efficiency measurement with constant returns to scale,.. Afriat (1972), p. 568.

Afriat does not return to the specific observations made by Farrell and in the *Discussion* about statistical estimation of frontier functions. However, ideas proposed by Afriat about efficiency distributions picked up by Aigner, Lovell and Schmidt (1977) are clearly inspired by Farrell. Afriat's framework for testing the consistency of a data set with hypothesis of productive efficiency or cost minimising and profit maximising is also developed in Hanoch and Rothschild (1972). Their measure of failure to meet economic efficiency criteria may be related to the Farrell measures.

Extensive statistical inference was made possible by the introduction of the composed error term in parametric models in "Formulation and estimation of stochastic frontier production function models." by Aigner, Lovell and Schmidt (1977) (ALS), and independently in "Efficiency estimation from Cobb-Douglas production functions with composed errors." by Meeusen and Broeck (1977), thus opening up for more rigorous econometric analyses of frontier functions.[9]

In the discussion we will focus on ALS because it is by far the most cited of the papers. This may be the *"Matthew effect"* (see e.g. Merton, 1968) at work: ALS were by far more established, and they also published in a journal with a higher impact factor at that time. In the period 1981-1985 there were 19 citations of Meeusen and Broeck, and 28 of ALS, (only one reference to Battese and Corra), while in the period 1986-1990 the former got *zero* citations, as was also the case in 1991 and 1996 (and also the case for Battese and Corra) (see Førsund and Sarafoglou, 1999).

The connection to Farrell was made clear in the first paragraph of ALS as to the usefulness of the frontier concept[10]:

> .., econometricians have been estimating *average* production functions. It has only been since the pioneering work of Farrell (1957) that serious consideration has been given to the possibility of estimating so-called frontier production functions,...
> (ALS (1977), p. 21.)

Other sources of inspiration are Aigner and Chu, Afriat and Richmond. The composed error consisted of two parts, a stochastic component symmetrically distributed, catching "white noise." and a stochastic component with a one-sided distribution, representing inefficiency. Within the single-output model the connection between the one-sided term and the Farrell measure of (output-oriented) technical efficiency was direct. Note, however, Mr. Sturrock's (1957, p. 285) interventions after Farrell's presentation:

> Not only are there errors but there are chance variations that have nothing to do with efficiency. Every business has good years and bad years depending on the state of the market, chance variation in prices and other factors over which the manager has no possible control. To call only these freakishly good results "100 per cent. efficient" would result in hanging the carrot too high and the donkey would be discouraged.

We see that the approach is a direct follow-up of the general problems stated already then.

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[9] There is also a third paper published in 1977 on the composed error frontier model (Battese and Corra), but they refer to ALS. They knew about the ALS paper because George Battese was one of the referees for the paper (personal communication from George Battese).
[10] Meeusen and Broeck (1977) also only offers a passing reference to Farrell. Their inspirations are Afriat and Richmond.

4.2. Testing the model specification

Farrell also had ideas as to how to test the specification of a model. Firstly, he pointed out

> …, the introduction of a new factor of production into the analysis cannot lower, and in general, raises the technical efficiency...”. Farrell (1957), pp. 269-270)

Then he suggested how to use this property:

> It is to such frequency distributions [of efficiency scores] that one must look for a measure of the success of the analysis (p. 270).
>
> …….
>
> In going from $x_1x_2x_3$ to $x_1x_2x_3x_4$ [inputs] by introducing capital (another relevant factor) only a little of the apparent variation in efficiency is explained. Thus although in principle $x_1x_2x_3x_4$ is the better analysis , it seems that in fact relatively little difference is made to the efficiency estimates omitting x_4 . (This may perhaps be thought of as analogous to a regression analysis where the introduction of a variable does not significantly improve the fit; although here there is no objective measure of significance.) (p. 272).

A statistical testing procedure was developed by Banker (1993) (see also Banker (1996) and Kittelsen (1998) for further improvement of the testing procedures), but without pointing to Farrell’s ideas presented above.

4.3. The development of linear programming methods

In the *Discussion* Dr. A. J. Hoffman (1957) pointed out (p. 284) that the newly established LP solution algorithm, the dual simplex method of C. E. Lemke (the first Ph.D. student of Abraham Charnes), could be applied. This turned out to be a very practical suggestion, and adopted by Farrell himself in Farrell and Fieldhouse (1962).

4.3.1. The Berkeley agricultural economists

The first to follow up Hoffman’s suggestion after Farrell and Fieldhouse (1962) were some agricultural economists at Berkeley. This group of agricultural economists used explicit LP modelling developed by Boles (1967) in a symposium volume of Western Farm Economic Association published in 1967. Since the agricultural economists’ revival effort remain largely unnoticed, we may briefly note that R. G. Bressler (Seitz’ first thesis advisor, but he unfortunately died of cancer soon after[11]) introduced the special session devoted to applying the Farrell approach, James N. Boles presented the FORTRAN LP routine, Wesley D. Seitz and Bistok L. Sitorus applied it to data, and William G. Brown did a summing-up of the session. However, there are no formal references in Brown, Seitz and Sitorus, only references to the Farrell method and/or approach.

We should note that the LP model used by Boles (1967) was the same as proposed by Hoffman (1957) and used in Farrell and Fieldhouse (1962), focussing on constant returns to scale and using the unit isoquant when formulating the model. Strangely enough these connections and references are not mentioned in Boles (but obviously known to him). Also, only one output was used, but the correct ideas for generalisation were indicated in Boles (1967). How to implement a generalisation in the case of constant returns to scale within a LP model was actually pointed out already in Farrell and Fieldhouse (1962).

In Boles (1967) the dual to the linear programme for calculating the Farrell efficiency score was also presented, and given a cost interpretation:

> An economic interpretation of the dual is to select a set of nonnegative factor prices to minimize the cost of production of one unit of the jth activity subject to the condition that the cost of production of each of the n activities is greater than or equal to 1.0, and the efficient facet coincides with a unit isocost hyperplane with all points, P_k, on the hyperplane or on the side away from the origin. (Boles, 1967, p. 138.)

James N. Boles (the department computer guru according to W. Seitz) continued to work on the LP model according to ideas sketched in the 1967 paper. Indeed, in Boles (1971, p. 1) it is announced that:

> … a system of computer programs has been developed and tested to handle three types of problems: single product with no economies or diseconomies of scale, multiple product with no economies or diseconomies of scale, and single product with economies of scale.

In fact, in Boles (1971) there are complete computer codes in FORTRAN written for an IBM 1130 covering the three models. Moreover, the model with multiple outputs and inputs is *identical* to the model termed the CCR model now, i.e. the "*ordinary* linear programming problem" (CCR, p. 431) corresponding to the ratio

[11] Personal communication from Wesley D. Seitz

form or fractional programming problem[12]. Outputs from the programme include both slacks and shadow prices on the constraints. The programmes were made more user friendly in Carlson (1976), where an interesting extension to quality "factors" can also be found.

The efforts of the Berkeley agricultural economists to improve the Farrell estimation method already in 1966, by employing a LP model that was to become the centrepiece of the DEA method introduced in CCR, failed completely to become acknowledged.

The journal outlet was one factor for the lack of attention, and the low productivity of the group in terms of journal papers another. Only one member of the group, Seitz, had a publication record afterwards that could make some impact, but after two more publications (Seitz (1970) and 1971) he left the field[13].

4.3.2. The DEA approach

The 1978 paper "Measuring the efficiency of decision making units" by Abraham Charnes, William.W. Cooper and Edwardo Rhodes (CCR) covered the same ground as regards the efficiency measure concept as Farrell. Both the proposed efficiency measures and the framework of a piecewise linear production technology were identical. But the linear programming model formulated was a generic one, and was quite superior to Farrell's unit isoquant approach in the case of a single output, and the change of origin approach proposed in the multiple output case in Farrell and Fieldhouse (1962) (for constant returns to scale). In CCR it is shown how the Farrell

[12] Since this is, to our knowledge, the first time this connection is pointed out, the Boles model will be stated with his own symbols:

$Max \ \theta$

s.t.

$\sum_{j=1}^{N} X_j Q_j \geq (1+\theta) Q_t, \sum_{j=1}^{N} X_j F_j \leq F_t, X_j \geq 0$, where X_j is the coefficient vector (intensity variables), Q_j the output vector, F_j the input vector and N the number of units.

[13] This was a pity for efficiency and productivity studies, because in Seitz (1971) one can find the first use of the Malmquist index, defined theoretically in 1982 and applied empirically in 1989! Seitz actually used a comparison of Farrell efficiency measure for two cross section data sets by first calculating the scores for the first year, and then calculating the score for the second year with reference to a frontier based on the pooled data sets.

unit isoquant model is a special case of the "*ordinary* linear programming problem" (CCR, p. 431) corresponding to the ratio form or fractional programming problem[14].

The CCR model was readily computable, either using standard LP codes on mainframes or developing more efficient tailor-made software. However, as just documented above, the CCR linear programming model is identical to one of the models in Boles (1971).

As mentioned in the introduction, Charnes and Cooper knew Farrell well personally, and they and Rhodes were very familiar with the 1957 Farrell paper as documented in Charnes, Cooper and Rhodes (1976). Indeed, in a first version of the 1978 paper (Charnes, Cooper and Rhodes (1977)) they found it natural to address economists and show how their approach fitted in with standard production theory in general, and the Farrell paper in particular. However, the almost 60 pages long paper was turned down first by the American Economic Review[15], and then by Quarterly Journal of Economics. They then revised and drastically shortened the paper for an OR public (choosing the newly started European Journal of Operations Research), dropping the connection to production theory (and thereby dropping a reference to Førsund (1974), which would have been the only reference to his paper as far as we know[16]), *and the rest is history,* as the saying goes.

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[14] Actually the transformation of Farrell's LP isoquant approach to a standard form in inputs and outputs was done in Charnes, Cooper, and Rhodes (1976). The paper documents an intimate knowledge and understanding of Farrell's approach, and discusses several improvements, such as for Farrell's ad hoc use of "points of infinity", and generalises the LP model to multiple outputs. A hand written comment by Abe Charnes in a copy of this paper contains the first formulation of the "ratio model" that was to be developed in CCR (personal communication from Bill Cooper).

[15] Personal communication from Bill Cooper. As to the turndown by AER, Abe Charnes would not, true to his character, give up without a fight. In a letter (made available to us by Bill Cooper) to the editor, Professor George H. Borts, of June 15, 1977 Page 4, he states: "There presently seems to be a special group of American economists who seem to us to be overly zealous in their use of "duality relations" of Shephard's variety This also seems to color their attribute toward other, possibly different, approaches. If it is in order to do so, we would therefore like to suggest that someone outside this circle be used for the new referee we are requesting. …..
Two possible persons whom we know only through their work are Professor Finn Førsund and Professor Leif Johansen, both at The Insitute of Economics, University of Oslo…."

[16] The paper is given the following flattering reference on p.3 of Charnes, Cooper and Rhodes (1977): "An excellent discussion of Frisch's concept as well as other approaches to empirical uses of production studies may be found in [20]." (Førsund, F. R. Studies in the Neo-Classical Theory of Production). This was Førsund's Thesis for the Licentiate Degree.

In the OR/MS community there seems to be one widespread misunderstanding (it should be mentioned that this misunderstanding is not made in CCR) as to one aspect of Farrell's contribution and consequently the contribution of CCR. In the influential Charnes et al. (1994, p. 4) it is stated[17]:

> CCR used the optimization method of mathematical programming to generalize the Farrell (1957) single-output/input technical efficiency measure to the multiple output/multiple-input case….

The sub-heading "2.3. Generalization to the Case of Many Inputs and Outputs" (Farrell (1957, p. 256) should make it quite clear that multiple outputs and inputs were covered by Farrell in theory, extending his facet algorithm, and in Farrell and Fieldhouse (1962) the LP programme is also generalised in a special way in the case of constant returns to scale by moving the origin to the output levels observed for the unit in question for each calculation of an efficiency score (see Førsund and Kittelsen, 1999).

Charnes and Cooper (1985), p. 90 made the following observation:

> Almost all of the work in "Farrell Efficiency" has been restricted to single output situations. Farrell, as well as others working in the tradition he initiated, describe what is to be done in extending their methods for use in the case of multiple outputs (see Farrell[55], p. 257). They do not, however, supply what is needed in the way of precise mathematical details with accompanying definitions and interpretations.

This seem to be quite fair and accurate observations, with the notable exception of the oversight of Boles (1971)[18].

At first within the OR and MS communities, but later also within economics, CCR started up a new active research field (the bibliography in Seiford (1996) contains over 700 references), popularly called *DEA* (Data Envelopment Analysis).[19] For the applied economist the great advantage was the feasibility of carrying out studies with

[17] The OR/MS community is in good company. In Färe, Grosskopf and Lovell (1985, p.7) it is stated: "However, Farrell confined his attention to a single output technology…" .

[18] In Charnes, Cooper and Rhodes (1977) Seitz (1971) is quoted. Since Boles (1967) is quoted in the latter, the trio is very close indeed to discovering the works of Boles.

[19] The name was actually not coined in CCR, but as stated in the former during the work on the dissertation of E. Rhodes accepted the same year. But both in Charnes, Cooper and Rhodes (1977), CCR and Charnes, Cooper and Rhodes (1981) it seems that the meaning of DEA was more special than later use of the concept, cf. the statement in CCR, p. 432 : "This [DEA] is a method for adjusting data to prescribed theoretical requirements such as optimal production surfaces, etc., *prior* to undertaking various statistical tests for purposes of public policy analysis."

multiple outputs. The econometric framework for parametric frontier production functions were, up to the recent time of directly estimating distance functions (see e.g. Grosskopf et al. (1997) for a recent application), limited to only single output on primal form, or dual cost functions had to be used in the case of multiple outputs. But for the most interesting applications in the public sector prices were not available for outputs.

4.3.3. Contributions of Charnes, Cooper and Rhodes (1978) beyond Farrell

One unique contribution of CCR is the explicit connection made between a productivity index in the form of a weighted sum of outputs on a weighted sum of inputs, and the Farrell technical efficiency measure (in the case of constant returns to scale). This was the starting point in CCR: Finding weights by maximisation of such a productivity ratio subject to best practice and normalisation constraints, the so-called *ratio form* of CCR, corresponds to the natural science- engineering definition of efficiency. A bridge was offered between the engineering concepts of micro productivity ratios and economists' concept of efficiency.

Farrell's own interest in the connection with productivity indices was simply not there, cf.:

> attempts have been made to construct "indices of efficiency." in which a weighted average of inputs is compared with output. These attempts have naturally run into all the usual index number problems.
> It is the purpose of this paper to provide a satisfactory measure of productive efficiency - one which takes account of all inputs and yet avoids index number problems-…
>
> (Farrell (1975), p. 253).

He left it there. The insight the ratio form and its linear programming equivalent in the form of the Farrell technical efficiency measure yield, due to CCR, is that the dual solution to calculating the measure by LP provides us with shadow prices that are the *endogenously* determined weights for a productivity index formulation (see also Førsund (1998) for an exposition of this point[20]).

[20] When explaining the Farrell approach Ruist (1960) points out that Farrell's measure of input efficiency is identical to a general index problem. Moreover, he refers to Malmquist (1953) as having used the same method (footnote, p. 21)

Neither Hoffman (1957) nor Farrell and Fieldhouse(1962) stated the dual problem or utilised the dual form (but as pointed out above Boles (1967) and (1971) did, although in a much more limited way than CCR). Building on earlier work on fractional programming (see Charnes and Cooper (1962)) CCR demonstrated the conversion of the problem on ratio form , i.e. maximising the productivity of a unit subject to the constraint that no productivity could be higher than on the frontier, to an ordinary linear programming problem of maximising the Farrell efficiency score, subject to the frontier constraint.

Farrell consciously made his exposition simple, to the benefit of “statisticians, businessmen and civil servants, many of whom have little knowledge of economic theory or mathematics.” A contribution of CCR was then that the efficiency calculation problem was presented in stringent mathematical form more readily understood and absorbed by the research community.

Making explicit the interpretation of primal and dual solutions showed how to calculate useful features like marginal productivities, and in later development when the constant returns to scale format of CCR was extended to variable returns to scale, also scale elasticities (Banker, Charnes, and Cooper (1984)). Farrell was focussed on portraying isoquants and never made reference to or made use of dual formulations. So it was left to the “economist” Cooper and the “mathematician” Charnes to make explicit use of key economic concepts like shadow prices[21]. The insights of the dual proved to provide rich inspirations to further developments.

In view of another Farrell oversight it is also interesting to note that CCR (p.442) explicitly point to the connection between Farrell’s technical efficiency measure and the distance function employed by Shephard, claiming, quite correctly, to have tied together “..two previously separate strands of economic research...” (p. 443).[22]

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[21] We are using characterisations due to Baumol. In discussing linear programming Baumol (1977) makes the following observations: “Several economists have made important contributions. Notable among these are T. C. Koopmans, R. Dorfman, and W. W. Cooper…..Important contributions have been made by mathematicians such as the Russian L. V. Kantorovich, who first formulated the problem, H. W. Kuhn, A. W. Tucker, A. Charnes, and others.”

[22] An Austin working paper from 1977 involving Charnes and Cooper is referred to. This is probably the first paper connecting the Farrell measure and the distance function. Another early reference is Färe and Lovell (1978). In fact, in Färe, Grosskopf and Lovell (1985) the latter is credited demonstrating the connection; CCR is overlooked.

4.4. Developing the theoretical underpinnings

In contrast to the papers above representing advances as to methods of estimating the frontier function the axiomatic approach associated with Shephard has contributed to the understanding of the nature of the efficiency concept, in a formal sense. But since most of the advances have been done after the publication of CCR, we will be rather brief. But it is interesting to note that work on axiomatic production theory related to the piecewise linear frontier production function was mostly inspired by CCR and not Farrell. Färe and Lovell (1978) were the first to formulate axioms efficiency measures should satisfy, see Russell (1998) for an overview. Extensive accounts of the axiomatic approach can be found in Färe, Grosskopf and Lovell (1985, 1994).

As already indicated a crucial feature of the Farrell efficiency measures is that they can be given a dual interpretation: The input oriented technical efficiency measure has a cost saving interpretation, and the output oriented efficiency measure has a revenue increasing interpretation. The various duality relations are portrayed and analysed in Färe and Primont (1995), introducing the "duality diamond".

Farrell and Fieldhouse (1962) tried to extend the framework to variable returns to scale. However, their proposal of grouping data did not catch on, and was limited to a single output. The follow-ups in Seitz (1970), (1971) where scale efficiency considerations are discussed, suffer also from this limitation and a lack of connection to production theory[23]. Førsund and Hjalmarsson (1974), (1979) provide consistent scale efficiency definitions for the multiple output - variable returns to scale case.

As to the communication between the "*Shephard*- and the *Charnes-Cooper Schools*" it may be of interest to note that the piecewise linear frontier exhibiting variable returns to scale is attributed Banker, Charnes and Cooper (1984) by the latter. However, the so-called BCC model is clearly stated on p. 581 in Afriat (1972)

[23] Seitz (1970) tries to generalise the unit isoquant concept to variable returns to scale, but in the case of a single output only, and is implicitly assuming homotheticity. See Førsund (1971) for a more general analysis.

(although in the case of one output), and the general version both stated and implemented in Färe, Grosskopf and Logan (1983) .

5. Concluding remarks

The importance of Farrell (1957) was not really seen by his contemporaries and not even 20 years later. Both in the obituary in The Times 1975 and in Fisher (1976) a paper on how increasing returns to scale or non-convexities in general can be reconciled with competitive equilibrium is viewed as Farrell's most important contribution.

The second decade after the publication of Farrell's paper saw major developments in estimation methods all based on his ideas, as well as further understanding of theoretical issues, published in high-ranking economics journals. The spread of journals and geographical origin of authors were quite wide. But all the methodological developments were concerned with econometric methods based on statistical inference. Economists did not follow up on the non-parametric method, with the exception of the group of agricultural economists at Berkley. For people that may have tried to publish programming applications they may have encountered that same negative attitude to analysis not based on statistical inference as Charnes, Cooper and Rhodes (1977) themselves experienced when they first tried to publish their paper in economics journals[24].

The advances made by agricultural economists on the programming approach for piecewise linear frontier production functions went unnoticed by the research community. It was the Charnes, Cooper and Rhodes (1978) paper that started the development of programming methods. At first contributions came from the OR/MS community, but in the 1990s more and more economists have also adapted the programming approach, especially for empirical applications, and contributions have

[24] According to personal communication from Douglas Todd, who managed to get out a report using the Farrell approach probably for the first time in England (see Todd (1971)), there were several attempts in the 1960s in England to apply the Farrell approach, but papers could not get published.

also been appearing from the axiomatic Shephard school, contributing to our understanding of underlying theoretical issues.

Until recently the parametric econometric approach only allowed single output, or multiple outputs using a cost function if price data were available, which often they were not in public sector applications. But since Farrell efficiency measures (identical to distance functions or their inverse) contains full production function estimation, a possible approach is to estimate parameters of a parametric efficiency (distance) function directly (see e.g. Grosskopf et al. (1997) for a recent application).

The econometrics approach is usually associated with statistical inference, while the programming approach has been labelled deterministic. There is now exciting ongoing research of merging the two approaches, making statistical inference also possible for the non-parametric approach, see Banker (1993), Simar (1996), Kittelsen (1998), Simar and Wilson (1998, 2000).

A final reflection of the nature of Farrell's contribution is in place. Already Hall and Winsten (1959) noted:

> However, his paper still suffers from a central weakness: that he does not analyse the concept of efficiency.." (p. 85).

In the case of the axiomatic Shephard school there have been contributions as to the nature of the Farrell efficiency measures, but not really as to the nature of inefficiency as such.

In the *Discussion* Mr Colin Clark (1957) expressed:

> I think we will agree, having heard this paper, that Mr. Farrell has already reached some interesting and successful results, and has come nearer than any previous investigator to a true measure of agricultural efficiency, which figures in their turn will send the economists further down the road hunting for the social and other factors which lie behind them.
>
> (Discussion on Mr. Farrell's Paper, p.282, also quoted in Burley, 1994)

However, the hunts down the road have not been too successful. It is obvious that the Xefficiency concept of Leibenstein may serve as a link to the nature of efficiency (see Burley (1994) for an attempt), but not too much has come out of this line of research. Førsund and Hjalmarsson (1987) have explored the vintage structure as an explanation for inefficiency. Bogetoft (1994, 2000) has applied the DEA model in a agent – principal game framework, where the unobservable true nature of efficiency can be concealed, and must be induced to be revealed by the proper incentive structures. But explanations of efficiency differences are still dominated by ad hoc speculations about organisational factors, environmental factors, non-measured quality aspects of outputs and inputs, etc. The two-stage approach of first calculating efficiency scores and then seeking to correlate these scores with various explanatory variables, is still the applied approach.[25] Let this be a challenge to future research.

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[25] It may be interesting to note that this approach was pioneered within a non-parametric frontier setting already by Seitz (1967), (1971) and Sitorus (1967), and Nerlove (1965) in a parametric setting. The latter argues for including explanatory variables in the first stage as the preferred model. This has been followed up in e.g. Ray (1988), and Battese and Coelli (1995) within a parametric panel data model. Banker (1999) provides a comprehensive review of the statistical foundations for the two-stage approach within a non-parametric setting.

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