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Is there a green paradox?[*]

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Abstract

A sufficiently rapidly rising carbon tax may increase near-term emissions compared with the case of no carbon tax. Even so, such a carbon tax path may reduce total costs related to climate change, since the tax may reduce total carbon extraction. A government cannot commit to a specific carbon tax rate in the distant future. For reasonable assumptions about expectation formation, a higher present carbon tax will reduce near-term carbon emissions. Moreover, whatever the expectations about future tax rates are, near-term emissions will decline for a sufficiently high carbon tax. However, if the near-term tax rate for some reason is set below its optimal level, increased concern for the climate may change taxes in a manner that increases near-term emissions.

Keywords: climate change, exhaustible resources, green paradox, carbon tax

JEL classification: Q31, Q38, Q41, Q48, Q54, Q58

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1 Introduction

During the last couple of years, there has been a considerable literature discussing the so-called "green paradox". This term stems from Sinn (2008a,b), who argues that some designs of climate policy, intended to mitigate carbon emissions, might actually increase carbon emissions, at least in the short run. The reason for this possibility is that fossil fuels are nonrenewable scarce resources. For such resources, Sinclair (1992) pointed out that "the key decision of those lucky enough to own oil-wells is not so much how much to produce as when to extract it." Sinn's point is that if e.g. a carbon tax rises sufficiently rapidly, profit maximizing resource owners will bring forward the extraction of their resources. Hence, in the absence of carbon capture and storage (CCS), carbon emissions increase.[1]

A rapidly increasing carbon tax is not the only possible cause of a green paradox. A declining price of a substitute, either because of increasing subsidies or technological improvement, can give the same effect: see e.g. Strand (2007) , Gerlagh (2010) and van der Ploeg and Withagen (2010). In a setting of heterogeneous countries and rising fuel prices, Hoel (2008) showed that carbon emissions may increase also as a consequence of an immediate and once and for all downward shift in the cost of producing a substitute.

As mentioned above, Sinclair (1992) pointed out that the time profile of the carbon tax was important for the development of emissions. A thorough analysis of the effects of taxation on resource extraction was given by Long and Sinn (1985) , but without explicitly discussing climate effects. The optimal design of the carbon tax path in the presence of carbon resource scarcity has since been analyzed by among others Ulph and Ulph (1994), Withagen (1994) , Hoel and Kverndokk (1996) , Tahvonen (1997) , Chakravorty et al. (2006). One of the insights from the literature is that the principles for setting an optimal carbon tax (or price of carbon quotas) are the same as when the limited availability of carbon resources is ignored: At any time, the optimal price of carbon emissions should be equal to the present value of

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[1]Throuout this paper, CCS is ignored. Discussions of climate policy when there is a possibility of CCS and when the carbon resource scarcity is taken into considereation have been given by Amigues et al. (2010), Le Kama et al. (2010) and Hoel and Jensen (2010).

all future climate costs caused by present emissions. A second insight from the literature is that when actual policies deviate from what is optimal, one might get different results than one would find if carbon resource scarcity were ignored.

The present paper focuses on the effects of carbon taxes that are not designed optimally. This includes the case of a carbon tax that is currently set at its optimal level, but without the regulator committing to the future tax development.

Section 2 shows how the carbon extraction path depends on the time path of the carbon tax. In particular, for a rapidly increasing carbon tax, near-term emissions may be *higher* the higher is the *level* of the carbon tax. However, if the level is high enough, near-term extraction is lower with a tax than without, no matter how rapidly the tax increases.

The results of section 2 are generalized to the case of endogenously determined total extraction in section 3. For this case, near-term emissions may decline even if the tax rate rises by a rate higher than the interest rate. Moreover, total extraction is lower the higher is the level of the carbon tax path. An implication of this is that climate change costs may be lower with a tax than without, even if near-term extraction is higher with the tax than without.

Policy makers can in reality not commit to tax rates in the distant future. In the absence of commitment, resource owners must base extraction decisions on their expectations about future tax rates, which may in turn depend on the current carbon tax rate. Section 4 demonstrates that for reasonable assumptions about expectation formation, a higher present carbon tax will reduce near-term carbon emissions.

In section 5 it is shown that introducing a sufficiently high present carbon tax will make near-term emissions decline, no matter what the cost structure of extraction is and no matter what expectations about the future carbon tax rate are. The possibility of a green paradox is thus not an argument against using a carbon tax as the main climate policy instrument. If anything, the possibility of a green paradox suggests that the level of the carbon tax should be set relatively high immediately, and not currently low and

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gradually increasing.

Section 6 finally considers the case in which carbon taxes are set endogenously, depending on the preferences related to climate change. I show that increased concern for the climate issue might increase near-term emissions. However, this can only occur if there is some obstacle that prevents the near-term tax rates being as high as their optimal levels.

Section 7 gives a brief summary of the main results.

2 The green paradox when total carbon extraction is given

Consider the simplest possible model of resource extraction: The available amount of the carbon resource is given by $\bar{A}$, and unit extraction costs are constant equal to c. The consumer price of the resource is $q(t)$, and in the absence of taxes this is also the producer price. Producers are price takers and have an exogenous interest rate r. Producers choose the extraction path $x(t)$ to maximize

$$\Pi = \int_0^\infty e^{-rt} \left[q(t) - c\right] x(t) dt \tag{1}$$

s.t.

$$
\begin{aligned}
\dot{A}(t) &= x(t) \\
A(0) &= 0 \\
x(t) &\geq 0 \quad \text{for all } t \\
A(t) &\leq \bar{A} \quad \text{for all } t
\end{aligned}
$$

In an equilibrium the chosen extraction path must at all dates satisfy $x(t) = D(q(t))$, where D is the demand function, assumed stationary for simplicity. Moreover, provided $D(c) > 0$, total extraction must be equal to

the available amount of the resource:

$$\int_0^\infty x(t)dt = \bar{A} \tag{2}$$

It is well known that the equilibrium of this simple Hotelling model is characterized by

$$\dot{q}(t) = r(q(t) - c) \tag{3}$$

with $q(0)$ determined so the resource constraint (2) is satisfied.

Consider a carbon tax $w(t)$, i.e., a tax equal to $w(t)$ per unit of x. It is useful first to consider a "large" carbon tax, defined as a time path $w(t)$ that satisfies

$$\int_0^\infty D(c + w(t))dt \leq \bar{A} \tag{4}$$

For a carbon tax satisfying (4), the resource constraint is not a binding constraint; the competitive supply of the carbon resource is like the supply of any non-resource good, and the resource rent is therefore zero. For this case there is clearly no green paradox, as the resource extraction at any time is simply equal to demand $D(c + w(t))$, and thus independent of the future carbon tax rate.

It might seem unrealistic to even consider a carbon tax path that is so high that it drives all carbon resource rents to zero. However, in a richer model with heterogeneous resources differing in extraction costs, a carbon tax of the magnitude needed to reach moderately ambitious climate goals may very well drive the resource rent to zero for the resources with the highest costs. This issue is treated in the next section.

Consider next a carbon tax path that does not satisfy (4). Let Ω denote the present value of total carbon taxes:

$$\Omega = \int_0^\infty e^{-rt}w(t)x(t)dt \tag{5}$$

The price to the producer is now $p(t) = q(t) - w(t)$, and instead of maximizing Π producers now maximize $\Pi - \Omega$. Assume that the carbon tax

rises at a constant rate g. From (5) it follows that

$$\Pi - \Omega = \Pi - w(0)\bar{A} - w(0) \int_0^\infty \left[e^{(g-r)t} - 1 \right] x(t) dt \qquad (6)$$

Consider first the case of $g = r$, i.e., the present value of the carbon tax rate is constant. In this case the last of the three terms in (6) is zero. The second term is just like a lump-sum tax (since $\bar{A}$ is given), so that the extraction profile that maximizes Π also maximizes $\Pi - \Omega$. This result generalizes to all cost functions, as long as the total amount extracted is unaffected by the carbon tax.

Consider next the case of $g > r$, implying that the term in square brackets is increasing over time. To maximize $\Pi - \Omega$, resource owners will therefore extract more earlier and less later compared to the case of no taxation. This is the green paradox: We get more extraction and hence also more emissions in the present and the near future than without a carbon tax. Moreover, this effect is stronger the higher is $w(0)$, so that for a given value of $g(> r)$, present and near-term emissions increase as the current carbon tax increases.

Finally, consider the case of $g < r$. Theoretical and numerical models that derive optimal climate policy typically find that it is optimal for the carbon tax to rise at a rate lower than the rate of interest, provided high carbon concentrations in the atmosphere are considered bad also when the carbon concentration is below some exogenously given upper limit.[2] For this case the result is exactly the opposite of the case $g > r$; extraction and hence also emissions are lower in the present and the near future than without a carbon tax. Moreover, this effect is stronger the higher is $w(0)$, so that for a given value of $g(< r)$, present and near-term emissions decline as the current carbon tax increases.

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[2]This result may be found in several contributions to the literature, as examples see Hoel et al. (2009) or Hoel and Kverndokk (1996).

3 Total carbon extraction is endogenous

The model used so far has the unrealistic feature that the available carbon resources are homogeneous and have the same extraction costs. A more interesting case is when the unit cost of extraction is increasing in accumulated extraction, denoted $c(A)$ where A as before is accumulated extraction. This is a specification frequently used in the resource literature, see e.g. Heal (1976) and Hanson (1980). If there is an absolute limit on total carbon extraction also in this case (*i.e.* $A(t) \leq \bar{A}$ for all t), and this limit is binding both with and without the carbon tax, there will be no significant changes compared with the case of constant extraction costs. A more interesting case is when the total amount extracted is determined endogenously. This is the case analyzed below.

To simplify the discussion, it is assumed that demand is zero if the price is sufficiently high. Formally, it is assumed that there is a choke price $\bar{q}$ such that $D(q) = 0$ for $q \geq \bar{q}$, and $D(q) > 0$ and $D'(q) < 0$ for $q < \bar{q}$. This is a purely technical assumption. If it instead had been assumed that $D(q) > 0$ for all q but approached zero as $q \to \infty$, it would nevertheless be true that for some high price $\bar{q}$ (e.g. a million dollars per barrel of oil) demand would be so small that it would be of no practical interest (e.g. 1 barrel of oil per year).

The profit of the resource owners is as before given by (1), except that c must now be replaced by $c(A)$. The first three of the four constraints given earlier remain valid, but there is no longer a binding constraint of the type $A(t) \leq \bar{A}$.[3]

The analysis of the present case is given in the Appendix. Without any carbon tax, the equilibrium is as before characterized by $x(t) = D(q(t))$ and by equations (2) and (3), except that c in (3) is replaced by $c(A)$. Furthermore, total extraction $\bar{A}$ is in the present case not exogenous, but determined by

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[3]The case of such a binding constraint can, however, be approximated by assuming that $c(A) \to \infty$ as $A \to \bar{A}$.

$$c(\bar{A}) = \bar{q} \tag{7}$$

All resources that have an extraction cost below the choke price $\bar{q}$ are thus extracted, and with a positive resource rent.[4]

Introducing a carbon tax $w(t)$, the producer price is changed to $p(t) = q(t) - w(t)$. Equation (3) remains valid, but with q replaced by p, giving

$$\dot{q}(t) = r\left(q(t) - c(A(t))\right) + \left[\dot{w}(t) - rw(t)\right] \tag{8}$$

As before, all resources that have an extraction cost below the price buyers are willing to pay to the resource owners, which is $\bar{q} - w(t^*)$, will be extracted. Instead of (7) and (2) we therefore have

$$c(A^*) = \bar{q} - w(t^*) \tag{9}$$

$$\int_0^\infty x(t)dt = A^* \tag{10}$$

where $w(t^*)$ will depend on the time t^* at which $A(t)$ reaches A^*.

From these equations it is clear that unless $c'(A) = \infty$, the introduction of a carbon tax will reduce total extraction. Some resources that would have been extracted if there were no carbon tax will thus be left unextracted with a positive carbon tax. Total emissions therefore decline as a response to a carbon tax, no matter what time profile the carbon tax has.

What about present and near-term extraction and emissions? Consider first the case in which the carbon tax rises at the rate r. From the previous section we know that the whole extraction profile was unaffected by the carbon tax when total resource extraction was exogenous (provided the carbon tax was not so high that (4) held). When total resource extraction goes down as a response to the carbon tax, emissions must obviously go down in some time periods. Does it go down in the present and near term? In other words, does the initial consumer price $q(0)$ go up as a response to the carbon tax? The answer is yes, and follows from (8) and (9): If $q(0)$ had not increased

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[4]For q to reach $\bar{q}$ we must have $\dot{q} > 0$ for $A < \bar{A}$, i.e. $q > c(A)$ from (3).

as a response to the carbon tax, it would not increase at later dates either as long as $\dot{w}(t) - rw(t) \leq 0$. But if this were the case, the consumer price would not reach the choke level $\bar{q}$ when resource extraction stops (remember that $A^* < \bar{A}$). This would violate the equilibrium conditions.

The argument above applies also to the case in which the carbon tax rises at a rate below r. For $\dot{w}(t) \leq rw(t)$, the introduction of a carbon tax will therefore reduce present and near-term emissions as well as total emissions.

If $\dot{w}(t) - rw(t)$ is positive and sufficiently large, it follows from (8) that q may reach $\bar{q}$ as A reaches A^* even if $q(0)$ is lower with a carbon tax than without. For a sufficiently rapidly rising carbon tax we may thus have a green paradox in terms of present and near-term emissions. However, even in this case the carbon tax may be desirable, since it reduces total emissions.

4   Governments cannot commit to future carbon tax rates

So far, the analysis has been based on an implicit assumption that market participants have full knowledge about the future carbon tax. However, in reality policy makers cannot commit to tax rates in the distant future. It might be possible to make a political commitment for the development of the carbon tax rate for period of up to 10-15 years, but resource owners would like to know the carbon tax for a longer period in order to make optimal decisions regarding their resource extraction. In the absence of commitment, resource owners must base their decisions on their expectations about future tax rates, which may in turn depend on the current carbon tax rate.

To illustrate the above issues, this section considers a two-period model of resource extraction. Period 1 should be interpreted as the near future, for which resource owners have reasonable confidence about the size of the carbon tax. Period 2 is the remaining future. As argued above, 10-15 years might be a crude estimate of the length of period 1.

The assumptions about the extraction cost are the same as in section 3. Formally, let each unit of the resource be indexed by a continuous variable z,

and let $c(z)$ be the cost of extracting unit z, with $c' \geq 0$. In the two-period model x is extraction in period 1 and $A-x$ is extraction in period 2. The cost of extracting x is thus given by $G(x) = \int_0^x c(z)dz$, and the cost of extracting $A-x$ is $\int_x^A c(z)dz = \int_0^A c(z)dz - \int_0^x c(z)dz = G(A) - G(x)$. Notice that these relationships imply that $G'(x) = c(x)$ and $G'(A) = c(A)$. To simplify the expressions in the subsequent analysis, it is assumed that extraction costs are zero for z up to the value of x in all relevant equilibria so that $G(x) = 0^5$. It is also assumed that $G'(A) = c(A) > 0$ and $G''(A) = c'(A) > 0$.

Producers of the carbon resource maximize

$$px + \beta\left[P \cdot (A - x) - G(A)\right]$$

where p and P are the producer prices in period 1 and 2, respectively. This gives the standard Hotelling equation

$$p = \beta P$$

and the equation determining total resource extraction (using $G'(A) = c(A)$)

$$c(A) = P$$

The relationship between prices and extraction rates is given by the following equations, where w is the carbon tax in period 1 and W is the expected carbon tax in period 2:

$$
\begin{aligned}
q &\equiv p + w \\
Q &\equiv P + W \\
x &= f(q) & (11) \\
A - x &= F(Q) & (12)
\end{aligned}
$$

where $f(q)$ and $F(Q)$ are demand functions for the two periods. The six equations above give the following two equations in the two endogenous vari-

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^{5}This simplifying assumption is not important for the results.

ables q and Q:

$$q - \beta Q = w - \beta W \tag{13}$$

$$Q - c\left(F(Q) + f(q)\right) = W \tag{14}$$

It is straightforward to verify that these equations imply that an increase in W (holding w constant) will give a reduction in q, i.e., an increase in x. A more policy relevant question is how a change in w will affect q (and hence x) when the expectation about W might depend on w. Let this expectation be given by some function $W = h(w)$. Inserting this into (13) and (14) and differentiating with respect to w gives

$$\frac{\partial q}{\partial w} = \frac{1}{M}\left[1 + (1 - \beta h')(-F')c'\right]$$

where

$$M = 1 + (-F')c' + \beta(-f')c' > 0$$

What are the conditions for a green paradox, in the sense that an increase in the period 1 carbon tax gives an increase in period 1 emissions? This will occur if and only if the derivative above is negative, i.e. if and only if

$$\beta h' > 1 + \frac{1}{-F'c'}$$

Consider first the case of $c' = "\infty"$, i.e., total resource extraction A is exogenous. In this case a green paradox occurs if and only if $\beta h' > 1$. If this inequality holds, an increased tax in period 1 will give an expectation of an increased tax in period 2 that in present value is at least as large as the tax increase in period 1. This corresponds to the finding in section 2 that an increase in the current carbon tax will increase current extraction and emissions if the tax rate is assumed to grow at a rate larger than the interest rate.

For finite values of c', $\beta h'$ must be higher than some threshold that is

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larger than 1 in order to get a green paradox. This confirms the analysis of section 3, where it was shown that an increase in the current carbon tax would reduce current emissions even if if the tax rate was assumed to grow at a rate slightly larger than the interest rate.

Can we say anything about the expectation function $h(w)$? One possibility would be that expectations are rational in the sense that market participants believe that the government in period 2 will set the carbon tax optimally based on the government's preferences. Assume that climate costs depend on the temperature increase Θ in period 2 (from some base level). Let the climate costs (as perceived by the government) be given by a damage function $\tilde{K}(\Theta)$, which is assumed to be increasing and strictly convex. The climate depends on emissions in both periods:

$$\Theta = \tilde{\Theta}(x, A - x) = \Theta(x, A) \tag{15}$$

The function $\tilde{\Theta}$ is assumed to be increasing in both its arguments. The variable x in $\tilde{\Theta}$ is due partly to the lagged response of temperature to the stock of carbon in the atmosphere, and partly due to the fact that emissions in period 1 affect the stock of carbon in the atmosphere both in period 1 and 2. It is not obvious that the net affect x on Θ for a given A is positive, although this seems reasonable if one cares about how rapidly the climate changes.[6] Although Θ_x has an ambiguous sign, Θ_A is positive since $\tilde{\Theta}$ is increasing in both arguments.

Inserting (15) into $\tilde{K}(\Theta)$ gives us

$$K(x, A) \equiv \tilde{K}(\Theta(x, A))$$

which is increasing in A, while the sigh of K_x will be the same as the sign of Θ_x. To make our derivations slightly simpler without changing anything of substance, I assume that the function $K(x, A)$ takes the simple form

$$K(x, A) \equiv E(A + \gamma x) \tag{16}$$

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[6]Such a consideration cannot be captured in a 2-period model, but see the discussion in Hoel (2008) in a continuous time model.

where γ is a parameter that may be positive ($K_x > 0$) or negative ($K_x < 0$). One case implying $\gamma < 0$ is the (somewhat implausible) case of the environmental concern being limited to the maximal stock of carbon in the atmosphere, without any concern for how rapidly this maximal stock is reached. In the current two-period model this corresponds to being concerned only about the amount of carbon in the atmosphere in period 2. Some fraction δ of the carbon emitted in period 1 will be transferred to the ocean and other carbon sinks at the end of period 1. Since some of the period 1 emissions do not remain in the atmosphere in period 2, emissions in period 1 are in this case therefore considered less harmful than emissions in period 2. Formally, since only $(1 - \delta)x$ of the emissions in period 1 remain in the atmosphere in period 2, and emissions in period 2 are $A - x$, the climate costs in this case are $E((1 - \delta)x + (A - x)) = E(A - \delta x)$, implying $\gamma = -\delta$ in our notation.

The optimal carbon tax in period 2 is the Pigou tax

$$W = E'(A + \gamma x) = E'(A - x + (1 + \gamma)x)$$

Inserting from (11) and (12) gives

$$W = E'(F(Q) + (1 + \gamma)f(q)) \tag{17}$$

Inserting this equation into (13) and (14) and differentiating with respect to w gives

$$\begin{pmatrix} 1 + \beta(1+\gamma)E''f' & -\beta + \beta E''F' \\ -c'f' - (1+\gamma)E''f' & 1 - c'F' - E''F' \end{pmatrix} \begin{pmatrix} \frac{\partial q}{\partial w} \\ \frac{\partial Q}{\partial w} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

and it follows that

$$\frac{\partial q}{\partial w} = \frac{1}{H}(1 - c'F' - E''F') \tag{18}$$

$$\frac{\partial Q}{\partial w} = \frac{1}{H}(c'f' + (1+\gamma)E''f') \tag{19}$$

where

$$H = 1 - c'\left(F' + \beta f'\right) - \left(1 + \gamma\beta c'f'\right)F'E''$$

A sufficient condition for H to be positive is that $1 + \gamma\beta c'f' > 0$. This seems a reasonable assumption, as γ probably is considerably below 1 (1 unit more in future emissions is considerably worse for the climate than one unit emissions moved from the future to the present). Moreover, if the time span from present to future is long, the discount factor β is low.

Assuming $H > 0$ it follows from (18) and (19) that an increase in the carbon tax in period 1 increases the consumer price in this period (and reduces the consumer price in period 2). Use and extraction of the carbon resource therefore decline in period 1, implying that there is no green paradox with this assumption about how expectations of future taxes are created.

Inserting (18) and (19) into (17) give

$$\frac{\partial W}{\partial w} = \frac{E''}{H}\left[(1 + \gamma)f' - \gamma f'F'c'\right]$$

which in *negative* for $H > 0$. In other words, as the present carbon tax increases, the expected future carbon tax declines. Obviously, with such expectations no green paradox can occur.

5 No green paradox with a high carbon tax

For the case of constant unit costs of extraction, I showed in section 2 that if the time path of the carbon tax was sufficiently high, carbon emissions would for sure go down. More generally, a sufficiently high initial carbon tax will make carbon emissions decline, no matter what the cost function is and no matter what expectations about future carbon taxes are. This holds under the mild assumption that resource owners will never sell their resource at a price lower than their extraction costs. If the government introduces a carbon tax that at the initial date is higher than the original resource rent (i.e. the resource rent prior to the introduction of the tax) the consumer price must increase in order for resource owners to cover their extraction costs. The

demand for the resource, and therefore also carbon emissions, must therefore decline.

How high must a carbon tax be for carbon emissions to decline? The answer to this will differ between coal and oil, which are the two most important sources of carbon from fossil fuels. Current coal prices are about 97 dollars per tonne[7]. CO_2 emissions per tonne of coal are approximately 2 tonnes[8], so that 97 dollars per tonne of coal corresponds to about $97/2 \approx$ 49 dollars per tonne of CO_2. This coal price is split between extraction costs and resource rent. The resource rent is probably much lower than 49 dollars per tonne. In any case, a carbon tax above 49 dollars per tonne of CO_2 will for sure increase the consumer price of coal, and therefore also reduce CO_2 emissions from the use of coal.

Turning next to oil, current oil prices are about 77 dollars per barrel. CO_2 emissions per barrel of oil are approximately 0.43 tonnes[9], so that 77 dollars per barrel of oil corresponds to about $77/0.43 = 179$ dollars per tonne of CO_2. This oil price is split between extraction costs and resource rent. The resource rent is probably much lower than 179 dollars per tonne. In any case, a carbon tax above 179 dollars per tonne of CO_2 will for sure increase the consumer price of oil, and therefore also reduce CO_2 emissions from the use of oil.

A carbon tax above about 179 dollars per tonne of CO_2 will for sure reduce carbon emissions. Since extraction costs for oil are not zero, the threshold is in reality lower. With an extraction cost of oil of e.g. 30 dollars per barrel, this threshold is reduced from 179 to 109 dollars per tonne of CO_2. Even this value is much higher than carbon tax rates or emission quota prices in most countries. For instance, the quota price in EU is only about 19 dollars per tonne of CO_2. However, there are also cases of explicit or implicit carbon taxes well above 109 dollars per tonne of CO_2 in some

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[7] Coal and oil prices are averages for the first half of 2010, obtained from http://www.worldbank.org/prospects/pinksheets, "commodity price data".

[8] The exact amount of CO_2 per tonne of coal depends on the type of coal. Dividing total world CO_2 emissions from coal consumption by total world coal consumption gives a factor of 1.97 (numbers from http://www.eia.doe.gov/emeu/iea/coal.html for 2006).

[9] See e.g. http://www.epa.gov/greenpower/pubs/calcmeth.htm.

countries (e.g. Sweden), at least for some sectors of the economy. Most integrated assessment models suggest an optimal current price of emissions clearly below 100 dollars per tonne of CO_2 (see e.g. Hoel et al., 2009, for an overview).

For carbon taxes below about 100 dollars per tonne of CO_2 we cannot rule out the possibility of emissions from the use of oil increasing (compared to emissions without any tax). However, emissions from the use of coal will for sure go down provided the carbon tax is above about 49 dollars per tonne of CO_2. Since extraction costs for coal are not zero, the threshold is in reality lower. With an extraction cost of coal of e.g. 50 dollars per tonne, this threshold is reduced from 49 to 24 dollars per tonne of CO_2. Optimal near-term carbon prices derived from integrated assessment models are in many cases above 24 dollars per tonne of CO_2, at least for the more ambitious climate goals. Introducing a world wide carbon tax at a level above 24 dollars per tonne of CO_2 is therefore likely to reduce emissions from the use of coal. However, since we cannot rule out the possibility of oil extraction increasing as a response to a global carbon tax in the range of about 20-100 dollars, we cannot rule out the possibility of near-term emissions increasing as a consequence of introducing a carbon tax in this range.

6 A green paradox with endogenous carbon taxes

So far the carbon tax rate, at least in period 1, has been considered exogenous. In reality, tax rates will be determined endogenously, with the government's preferences being an important factor. What are the effects in this case of an increased concern for the climate? I analyze this below, and show that a green paradox may occur if the tax in the first period is lower than its ideal level.

Let the government's preferences be represented by the function $E(A + \gamma x)$, as discussed in the previous section. Moreover, let let $b(x)$ and $B(A-x)$ be the consumer benefit of using the resource in the two periods, with $q = b'$

and $Q = B'$. The fist best optimum is found by maximizing

$$b(x) + \beta\left[B(A - x) - G(A) - E(A + \gamma x)\right]$$

and the first order conditions are (using $q = b'$, $Q = B'$ and $c(A) = G'(A)$)

$$q - \beta Q = \beta\gamma E'(A + \gamma x)$$
$$Q - c(A) = E'(A + \gamma x)$$

Comparing with (13) and (14), it is clear that the first best optimum is achieved if

$$w = \beta(1 + \gamma) E'$$
$$W = E'$$

These two equations show how the optimal carbon taxes in the two periods depend on the preferences of the government, represented by the function $E(A + \gamma x)$. A slightly generalized version of these equations is

$$w = \mu\beta(1 + \gamma) E' \tag{20}$$
$$W = E' \tag{21}$$

where the positive parameter $\mu \leq 0$ represents the possibility that the tax rate in period 1 is set at a level below its optimal level. There could be several reasons why $\mu < 1$. One obvious possibility is that the present model represents the global economy, and that E' thus represents global marginal climate costs. If these costs are not fully internalized in period 1 due to the lack of an international climate agreement, taxes throughout the world would typically be set below their optimal values.

It follows from (20) and (21) that

$$\frac{W}{w} = \frac{1}{\mu\beta(1+\gamma)}$$

implying that the growth rate of the tax rate is lower than the interest rate (β^{-1}) if and only if $\mu(1+\gamma) > 1$. If this inequality holds, it follows from the previous analysis that there will be no green paradox: Carbon emissions in period 1 are lower when taxes are given by (20) and (21) than they would have been if there were no taxes. The opposite is true if $\mu(1+\gamma) < 1$, which will occur if $\gamma < 0$ or μ sufficiently small. For the case of $\gamma < 0$ it is of course not really a paradox that early emissions increase as a response to climate policy, since $\gamma < 0$ means that early emissions are *good* for the climate given the total amount of emissions. In the rest of this section I therefore assume $\gamma > 0$.

To see in more detail what the consequences are of a positive shift in the marginal climate costs function E', I insert (20) and (21) back into the equilibrium conditions (13) and (14). Using the demand functions (11) and (12) this gives

$$q - \beta Q = \beta\left[\mu(1+\gamma) - 1\right]\left[E'(F(Q) + (1+\gamma)f(q)) + s\right] \qquad (22)$$

$$Q - c\left(F(Q) + f(q)\right) = E'(F(Q) + (1+\gamma)f(q)) + s \qquad (23)$$

where s is a shift parameter.

Differentiating (22) and (23) with respect to s gives

$$\begin{pmatrix} 1 - \beta\left[\mu(1+\gamma) - 1\right](1+\gamma)E''f' & -\beta - \beta\left[\mu(1+\gamma) - 1\right]E''F' \\ -c'f' - (1+\gamma)E''f' & 1 - c'F' - E''F' \end{pmatrix} \begin{pmatrix} \frac{\partial q}{\partial s} \\ \frac{\partial Q}{\partial s} \end{pmatrix}$$
$$= \begin{pmatrix} \left[\mu(1+\gamma) - 1\right]\beta \\ 1 \end{pmatrix}$$

Solving gives

$$\frac{\partial q}{\partial s} = \frac{\beta}{J}\left[\mu(1+\gamma) + (1 - \mu(1+\gamma))c'F'\right] \qquad (24)$$

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and

$$\frac{\partial Q}{\partial s} = \frac{1}{J} \left[1 - \beta \left(1 - \mu \left(1 + \gamma\right)\right) c' f'\right] \tag{25}$$

where

$$
\begin{aligned}
J =\ & 1 - c'F' - \beta c'f' - F'E'' - \beta\gamma c'f'F'E'' \\
& + \mu \left[(1 + \gamma)\, \beta\gamma c'f'F'E'' - \beta f'E'' - \beta\gamma^2 f'E'' - 2\beta\gamma f'E''\right]
\end{aligned}
$$

is an increasing function of μ. Even for $\mu = 0$ it is reasonable to assume that $J > 0$. A *sufficient* condition for this is that $-F'c' - \beta c'f' - \beta\gamma c'f'F'E'' > 0$, i.e. that $\gamma E''(-F') < 1 + \frac{F'}{\beta f'}$, which is reasonable to assume: In the climate cost function, it seems reasonable to assume that γ is relatively small, certainly below 1. Moreover, $E''(-F') < 1$ if $E'' < (-F')^{-1}$, which says that a decline in the consumer price of carbon by e.g. one dollar, giving an increased use of carbon, increases marginal environmental damages by less than one dollar.

If $\mu = 1$ we have

$$\frac{\partial q}{\partial s} = \frac{\beta}{J} \left[(1 + \gamma) - \gamma c'F'\right]$$

and

$$\frac{\partial Q}{\partial s} = \frac{1}{J} \left[1 + \beta\gamma c'f'\right]$$

An positive shift in the function E' thus for sure makes q larger and therefore near-term emissions decline. However, it is not obvious that future emissions decline: If $\gamma c'$ is sufficiently large, Q will decline and future carbon emissions will increase. Nevertheless, total emissions $A = f(q) + F(Q)$ will go down:

$$\frac{\partial A}{\partial s} = f'\frac{\partial q}{\partial s} + F'\frac{\partial Q}{\partial s} = \frac{1}{J} \left[\beta \left(1 + \gamma\right) f' + F'\right] < 0$$

The last result also valid for $\mu < 1$. From $A = f(q) + F(Q)$ and the expressions (24) and (25) it follows that

$$\frac{\partial A}{\partial s} = f' \frac{\partial q}{\partial s} + F' \frac{\partial Q}{\partial s} = \frac{1}{J} \left[\beta \mu \left(1 + \gamma \right) f' + F' \right] < 0 \qquad (26)$$

Although total emissions decline with increasing climate concern even if $\mu < 1$, it is not obvious that near-term emissions decline. The term in square brackets in (24) is positive for $\mu = 1$, but is declining in μ and becomes negative for sufficiently low positive values of μ . Formally,

$$\frac{\partial q}{\partial s} < 0 \text{ for } \mu < \frac{c' F'}{\left(1 + \gamma \right) \left(1 + c' F' \right)} \qquad (27)$$

Notice that the threshold value of μ for the green paradox case of $\frac{\partial q}{\partial s} < 0$ to occur is higher the larger is c', with the threshold being $(1 + \gamma)^{-1}$ for the limiting case of $c' \to \infty$.

Finally, consider the two limiting case of $c' = 0$ and $c' = "\infty"$. The case of $c' = 0$ means that there is no scarcity of the resource, neither of a physical or economic type. If $c' = 0$ it follows from (24) and (25) that

$$\frac{\partial q}{\partial s} = \frac{\beta}{J} \left[\mu \left(1 + \gamma \right) \right] > 0$$
$$\frac{\partial Q}{\partial s} = \frac{1}{J} > 0$$

Hence, in this case emissions unambiguously decline in both periods as a response to increased concern for climate change.

For the case of $c' = "\infty"$ it follows from (24) and (25) that

$$\frac{\partial q}{\partial s} = \frac{\beta F'}{\tilde{J}} \left(1 - \mu \left(1 + \gamma \right) \right)$$

and

$$\frac{\partial Q}{\partial s} = \frac{-\beta f'}{\tilde{J}} \left(1 - \mu \left(1 + \gamma \right) \right)$$

where

$$\tilde{J} = -F' - \beta f' - \beta \gamma f' F' E'' \left(1 - \mu \left(1 + \gamma\right)\right)$$

I assume $\tilde{J} > 0$ for the same reason as J was assumed positive.

By assumption, total emissions are not affected by preferences in this case. Formally, this follows from (26) and the fact that $J \to \infty$ as $c' \to \infty$. Moreover, from the equations above we see that $\frac{\partial q}{\partial s}$ and $\frac{\partial Q}{\partial s}$ have opposite signs. If $\mu > (1 + \gamma)^{-1}$, $\frac{\partial q}{\partial s} < 0$ and $\frac{\partial Q}{\partial s} > 0$, while the opposite is true if $\mu < (1 + \gamma)^{-1}$. If there are no obstacles preventing the near-term tax rate being equal to its optimal value, increased concern for the environment thus gives a postponement of extraction and emissions in this case.

7 Concluding remarks

There are six important lessons from this paper:

1. Analyses of climate policy without taking into consideration the fact that fossil fuels are scarce non-renewable resources can give misleading conclusions. Although the principles for the design of an optimal carbon tax are not affected, the consequences of deviating from the optimum may be different than one might believe if the scarcity of carbon resources is ignored.

2. A rapidly rising carbon tax may give a green paradox in the sense that near-term emissions become higher than they would be without any carbon tax. The threshold of how rapidly the tax must increase is higher when the resource is not limited in an absolute physical sense, but more realistically by extraction costs increasing with accumulated extraction.

3. If the resource is not limited in an absolute physical sense, but by extraction costs increasing with accumulated extraction, total climate change costs may go down even if the carbon tax path gives increased near-term emissions.

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4. In reality, governments do not set carbon tax paths extending into the distant future. Instead, they set a carbon tax for a relatively short period, and market participants form expectations about the carbon tax in the more distant future. For reasonable modeling of these expectations, a higher current carbon tax will reduce near-term emissions.

5. If a sufficiently high carbon tax is introduced, emissions will for sure decline. The possibility of a green paradox is therefore not an argument against the use of a carbon tax, but rather an argument against setting the carbon tax too low.

6. If the near-term tax rate for some reason is set below its optimal level, increased concern for the climate may change taxes in a manner that increases near-term emissions.

Appendix: Endogenous total extraction

The simplest way to analyze the market equilibrium is to consider this equilibrium as the outcome of maximizing the sum of consumer benefits of using the resources and the costs, including taxes, of extracting the resource. Let $B(x)$ be the consumer benefit, with $q = B'(x)$ and $\bar{q} = B'(0)$. I assume that $c(0) + w(0) < B'(0)$ and $c(A) > B'(0)$ for sufficiently high values of A (where $w(0)$ is the initial carbon tax). Moreover, I restrict the analysis to the case of a non-decreasing carbon tax path $w(t)$, so that extraction will be declining.

The objective function of the private sector is

$$V = \int_0^\infty e^{-rt} \left[B(x(t)) - c(A(t))x(t) - w(t)x(t) \right] dt$$

This objective function is maximized subject to

$$\dot{A}(t) = x(t) \tag{28}$$
$$x(t) \geq 0$$
$$A(0) = 0$$

The current value Hamiltonian is (written so the shadow price of A, denoted π, is positive, and ignoring time references where this cannot cause misunderstanding)

$$H = B(x) - c(A)x - wx - \pi x$$

The optimum conditions are

$$B'(x) - c(A) - w - \pi \leq 0 \ \ [= 0 \text{ for } x > 0] \tag{29}$$

$$\dot{\pi} = r\pi - xc'(A) \tag{30}$$

$$Lim_{t \to \infty} \left[e^{-rt} \pi(t) \right] = 0 \tag{31}$$

Using (28) and $q = B'$ it follows from (29) and (30) that the consumer price development is given by

$$\dot{q} = r\left(q - c(A)\right) + \left[\dot{w} - rw\right] \tag{32}$$

which corresponds to equation (8) in the text.

It is useful to distinguish between the case of w constant $(= \bar{w})$ and w increasing. For $w = \bar{w}$ (which may be zero or positive) carbon extraction is positive for all t. To see this assume the opposite, i.e. that $x(t) = 0$ for $t \geq T$. From (30) this implies that $\dot{\pi} = r\pi$ for $t \geq T$. From (31) it follows that $\pi(T) = 0$, so that (29) implies

$$B'(0) - c(A(T)) - \bar{w} \leq 0$$

Going backwards in time from T, we see from the differential equations (28) and (30) that $\pi(t) = 0$ and $B'(0) - c(A(t)) - \bar{w} \leq 0$ will hold also for all $t < T$. But this violates the assumption $c(0) + w(0) < B'(0)$. This completes the proof that $x(t) > 0$ for all t when $w(t) = \bar{w}$.

Although $x(t) > 0$ for all t when $w(t) = \bar{w}$, $x(t)$ will asymptotically approach zero. To see this, assume instead that $x(t) > \delta > 0$ for all t. Then $A(t)$ become so large that $c(A) > B'(0)$, so that (29) would be violated for any non-negative $w + \pi$.

As $x(t)$ approaches 0 asymptotically, $\pi(t)$ approaches 0, and from (29) it follows that $A(t)$ approaches $\bar{A}$ given by

$$c(\bar{A}) + \bar{w} = B'(0) \tag{33}$$

The case of $w(t)$ increasing over time is not much different from w constant. However, if w is unbounded, extraction cannot be positive extraction for all t, since eventually we would have $B'(0) - c(A) - w(t) < 0$ for any value of A. In the present case there is thus a date t^* at which extraction stops. At this date we have $\pi(t^*) = 0$, as (31) otherwise would be violated. Since $x(t)$ is positive immediately prior to t^*, it therefore follows from (29) that

$$c(A^*) + w(t^*) = B'(0) \tag{34}$$

Since the time path of extraction depends on the carbon tax also prior to t^*, the values A^* and $w(t^*)$ are determined endogenously by the condition (34) in combination with the differential equations (28) and (32) as well as $q = B'(x)$.

References

Amigues, J.-P., G. Lafforgue and M. Moreaux (2010). Optimal capture and sequestration from the carbon emission flow and from the atmospheric carbon stock with heterogeneous energy consuming sectors. Paper presented at the SURED conference 2010.

Chakravorty, U., B. Magne and M. Moreaux (2006). A Hotelling model with a ceiling on the stock of pollution. *Journal of Economic Dynamics & Control*, *30*(12), 2875–2904.

Gerlagh, R. (2010). Too Much Oil. *CESifo Economic Studies*, *forthcoming*.

Hanson, D. A. (1980). Increasing Extraction Costs and Resource Prices: Some Further Results. *The Bell Journal of Economics*, *11*(1), 335–342.

Heal, G. (1976). The Relationship between Price and Extraction Cost for a Resource with a Backstop Technology. *The Bell Journal of Economics*, *7*(2), 371–378.

Hoel, M. (2008). Bush Meets Hotelling: Effects of Improved Renewable Energy Technology on Greenhouse Gas Emissions. CESifo Working Paper 2492.

Hoel, M., M. Greaker, C. Grorud and I. Rasmussen (2009). Climate policy - costs and design. TemaNord 2009:550, Nordic Council of Ministers.

Hoel, M. and S. Jensen (2010). Cutting Costs of Catching Carbon: Intertemporal effects under different climate policies. Unbuplished work in progress, Department of Economics, University of Oslo and Ragnar Frisch Centre for Economic Research.

Hoel, M. and S. Kverndokk (1996). Depletion of fossil fuels and the impacts of global warming. *Resource and Energy Economics*, *18*(2), 115–136.

Le Kama, A., M. Fodha and G. Lafforgue (2010). Optimal carbon capture and storage policies. Paper presented at the SURED conference 2010.

Long, N. and H.-W. Sinn (1985). Surprise Price Shifts, Tax Changes and the Supply Behavior of Resource Extracting Firms. *Australian Economic Papers*, *24*(45), 278–289.

Sinclair, P. (1992). High does nothing and rising and worse: carbon taxes should be kept declining to cut harmful emissions. *Manchester School of Economic and Social Studies*, *60*, 41–52.

Sinn, H. (2008a). *Das Grüne Paradoxon. Plädoyer für eine Illusionsfreie Klimapolitik*. Econ.

Sinn, H. (2008b). Public Policies against Global Warming: a supply side approach. *International Tax and Public Finance*, *15*, 360–394.

Strand, J. (2007). Technology Treaties and Fossil Fuels Extraction. *The Energy Journal*, *28 (4)*, 129–142.

Tahvonen, O. (1997). Fossil Fuels, Stock Externalities, and Backstop Technology. *Canadian Journal of Economics*, *30*, 855–874.

Ulph, A. and D. Ulph (1994). The optimal time path of a carbon tax. *Oxford Economic Papers*, *46*, 857–868.

van der Ploeg, F. and C. A. Withagen (2010). Is There Really a Green Paradox? CESifo Working Paper 2963.

Withagen, C. (1994). Pollution and exhaustibility of fossil fuels. *Resource and Energy Economics*, *16*, 235–242.